Süsteemide diagnostika

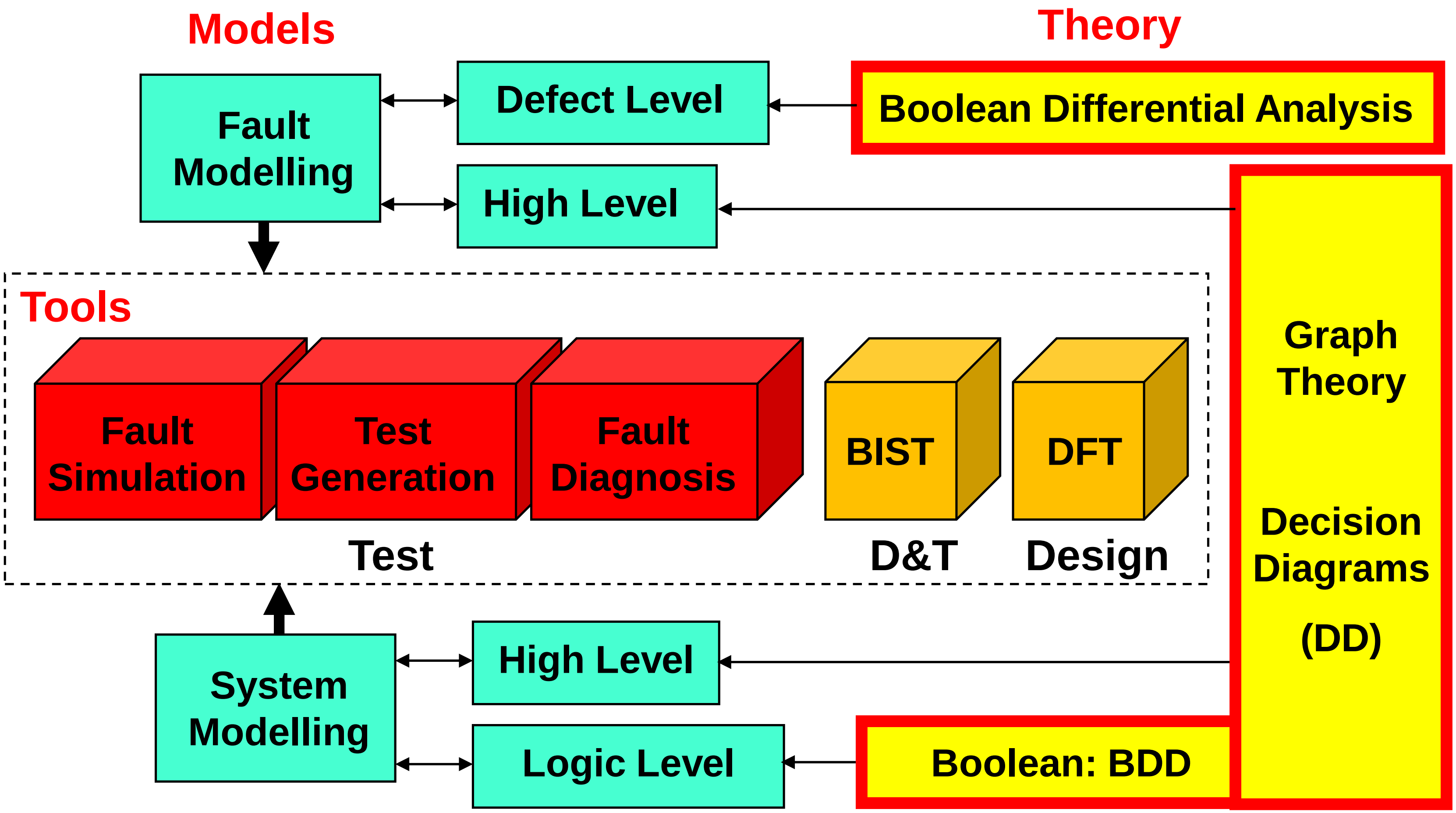
2. Teoreetilised alused

2.1. Boole'i differentsiaalalgebra
- Boole'i funktsioonide differentseerimine
- Boole'i differentsiaalvõrrandid

2.2. Binaarsed otsustusdiagrammid (BDD)

2.3. Kõrgtasandi otsustusdiagrammid

Introduction to Theories: The Course Map



Models

Theory

Fault Modelling

Defect Level

High Level

Boolean Differential Analysis

Tools

Fault Simulation

Test Generation

Fault Diagnosis

BIST

DFT

Test

D&T

Design

Graph Theory

Decision Diagrams

(DD)

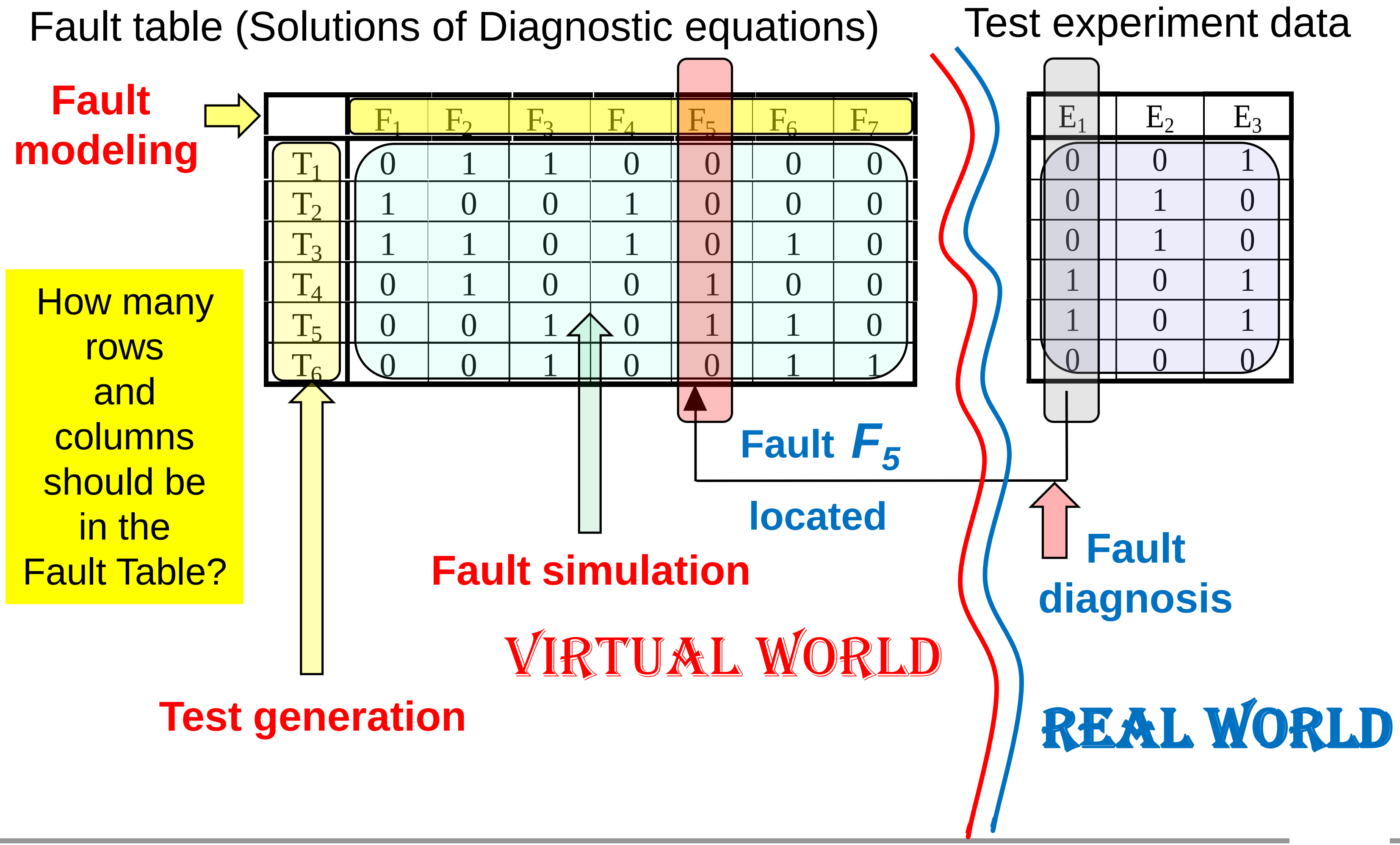
System Modelling

High Level

Logic Level

Boolean: BDD

Test Related Basic Problems



Fault table (Solutions of Diagnostic equations)

	F_1	F_2	F_3	F_4	F_5	F_6	F_7
T_1	0	1	1	0	0	0	0
T_2	1	0	0	1	0	0	0
T_3	1	1	0	1	0	1	0
T_4	0	1	0	0	1	0	0
T_5	0	0	1	0	1	1	0
T_6	0	0	1	0	0	1	1

Test experiment data

E_1	E_2	E_3
0	0	1
0	1	0
0	1	0
1	0	1
1	0	1
0	0	0

Fault modeling

How many rows and columns should be in the Fault Table?

Fault simulation

Test generation

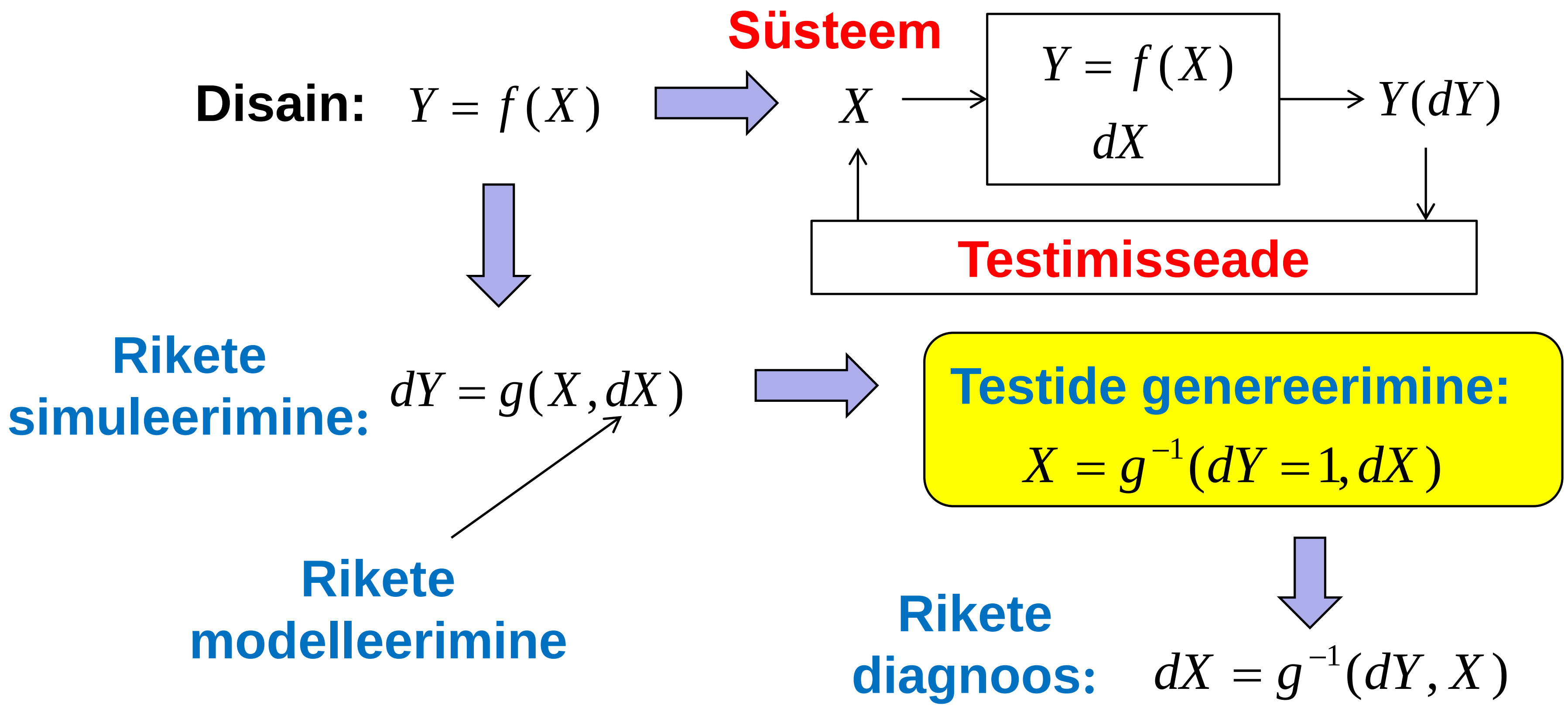
Fault F_5 located

Fault diagnosis

VIRTUAL WORLD

REAL WORLD

Formaalne testimine ja diagnostika



Disain: $Y = f(X)$

Süsteem

$$X \rightarrow \boxed{\begin{array}{c} Y = f(X) \\ dX \end{array}} \rightarrow Y(dY)$$

Testimisseade

Rikete simuleerimine: $dY = g(X, dX)$

Rikete modelleerimine

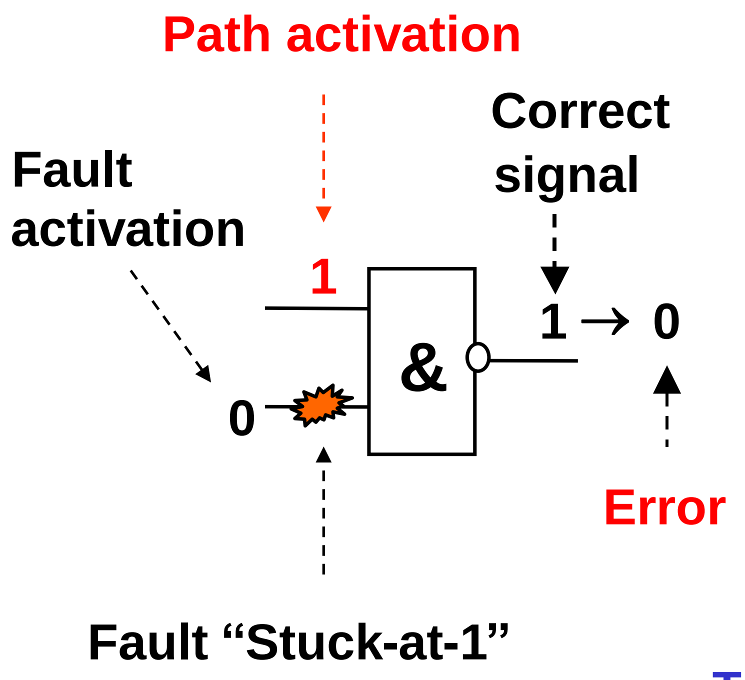
Testide genereerimine:

$$X = g^{-1}(dY = 1, dX)$$

Rikete diagnoos: $dX = g^{-1}(dY, X)$

Basics: Fault Propagation Problem

Single logic gate:



Path activation

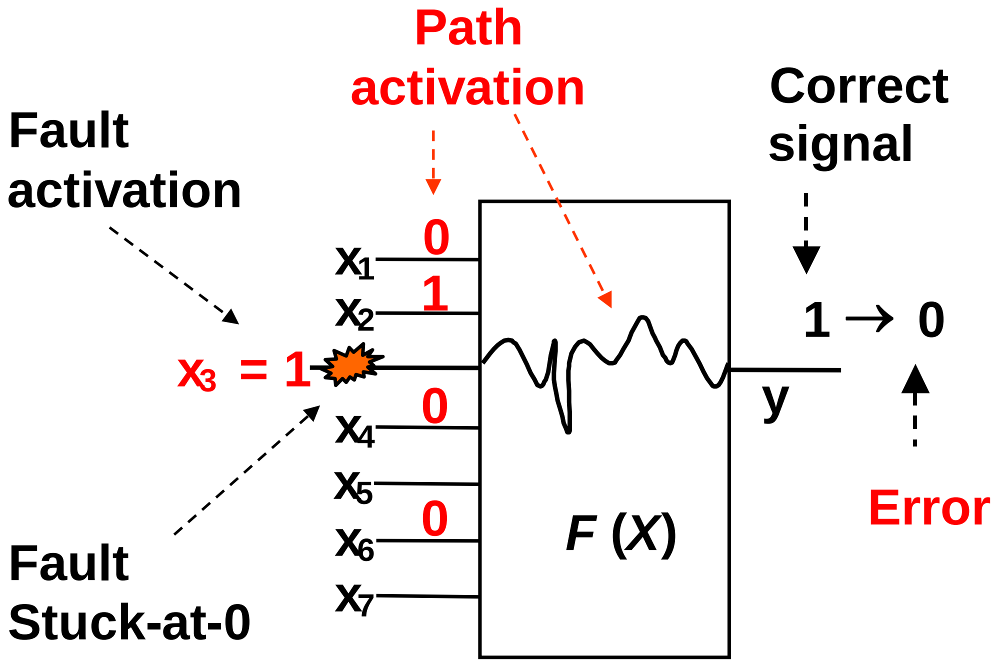
Fault activation

Correct signal

Error

Fault “Stuck-at-1”

Logic circuit:



Path activation

Fault activation

Correct signal

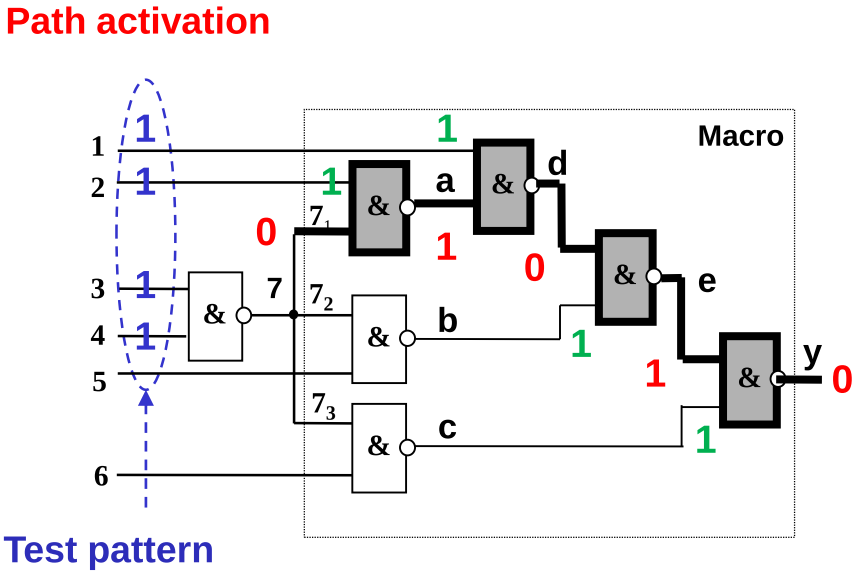
$x_3 = 1$

Error

Fault Stuck-at-0

Testide genereerimine: $X = g^{-1}(dY = 1, dX)$

Gate-Level Structural Test Generation



How about test generation for $x_{7,1} \equiv 0$

$x_{7,1} = 1 \rightarrow \{b = 0, c = 0\}$
$b = 0 \rightarrow x_5 = 1$
$c = 0 \rightarrow x_6 = 1$

Gate level test generation:

Fault sensitization (for $x_{7,1} \equiv 1$):

$x_{7,1} = 0$

Fault propagation:

$x_2 = 1, x_1 = 1, b = 1, c = 1$

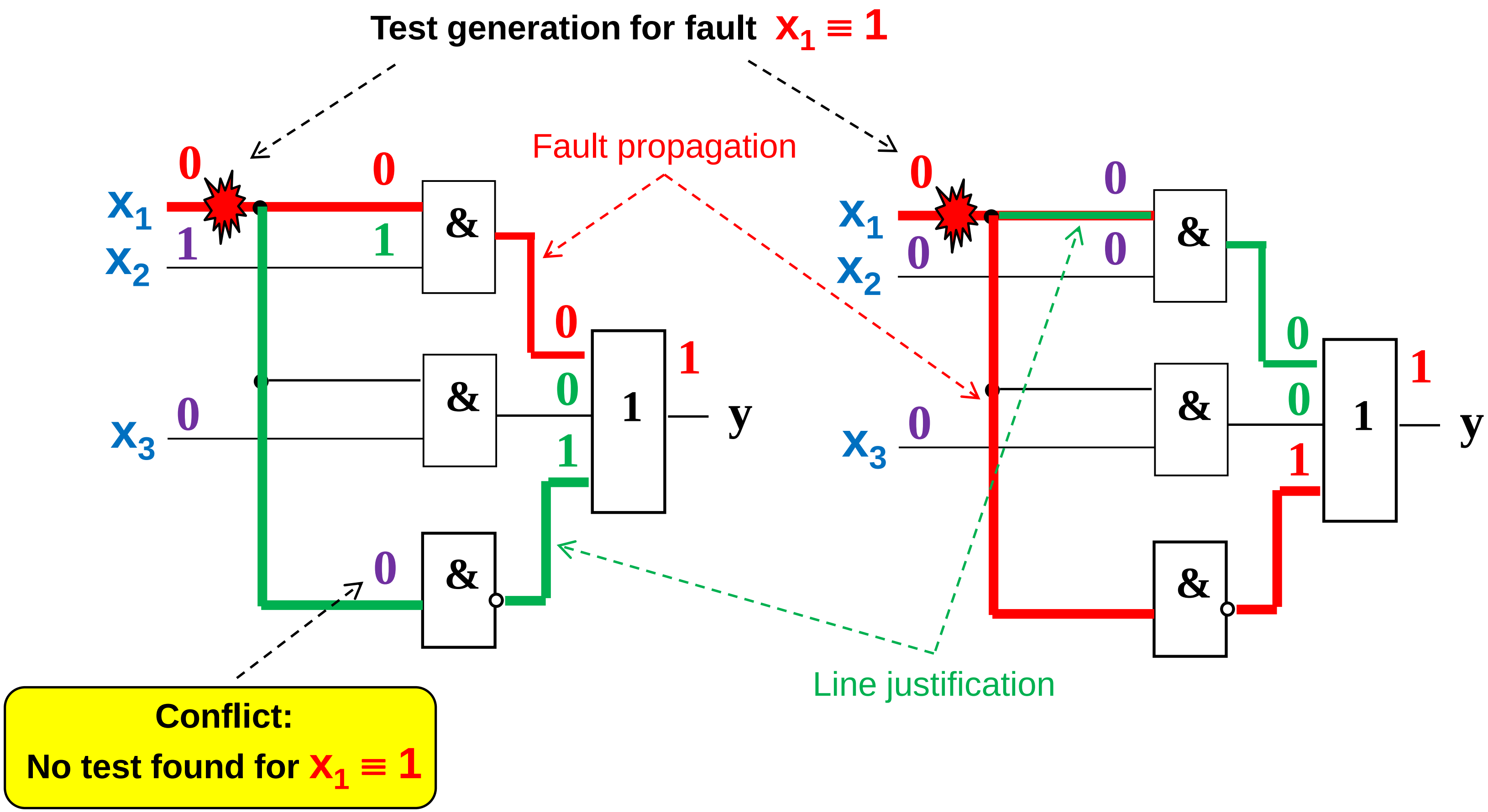
Line justification:

$x_{7,1} = 0 \rightarrow x_7 = 0 \rightarrow \{x_3 = 1, x_4 = 1\}$
$b = 1 \rightarrow$ (already justified)
$c = 1 \rightarrow$ (already justified)

The expected result:

$y = 0$ - if fault is missing
$y = 1$ - if fault is present

Backtracking in Test Generation



Test generation for fault $x_1 \equiv 1$

Fault propagation

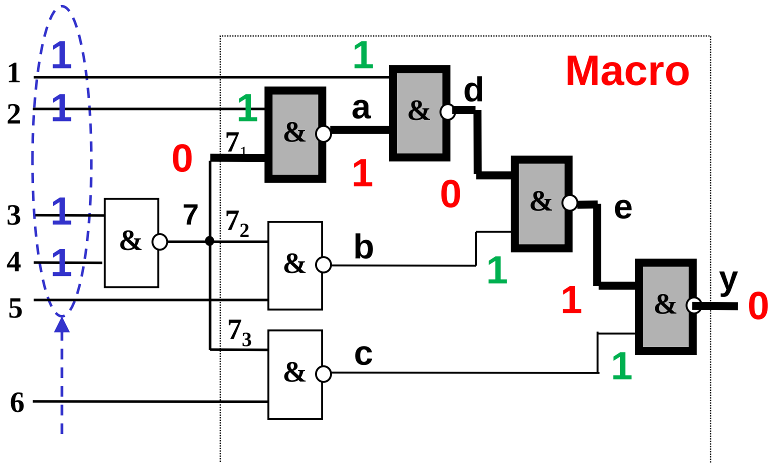
Line justification

Conflict:

No test found for $x_1 \equiv 1$

Gate-Level Structural Test Generation

Path activation



Test pattern

Gate level test generation:

Fault sensitization (for $x_{7,1} \equiv 1$):

$x_{7,1} = 0$

Fault propagation:

$x_2 = 1, x_1 = 1, b = 1, c = 1$

Line justification:

$x_{7,1} = 0 \rightarrow x_7 = 0 \rightarrow \{x_3 = 1, x_4 = 1\}$

$b = 1 \rightarrow$ (already justified)

$c = 1 \rightarrow$ (already justified)

The expected result:

y = 0 - if fault is missing
y = 1 - if fault is present

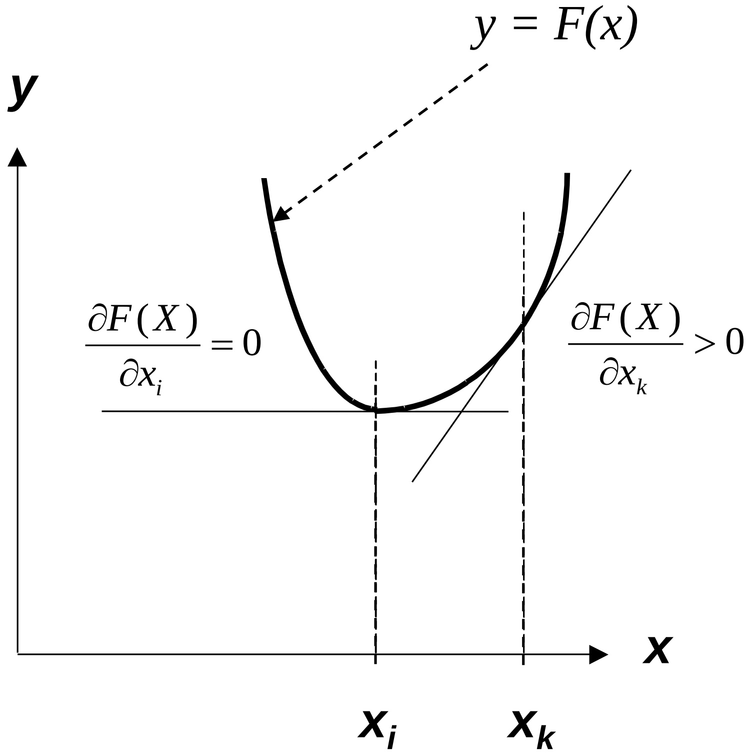
Another possibility: Macro level test generation

$$y = x_6 x_{7,3} \vee (\overline{x_1} \vee x_2 x_{7,1})(\overline{x_5} \vee \overline{x_{7,2}})$$

$$\frac{\partial y}{\partial x_{7,1}} = (\overline{x_6} \vee \overline{x_{7,3}})(\overline{x_5} \vee \overline{x_{7,2}}) x_1 x_2 = x_1 x_2 \overline{x_7} = 1$$

Boolean Derivatives

Traditional algebra: **speed**



Testide genereerimine:

$$Y = f(X) \Rightarrow X = g^{-1}(dY = 1, dX)$$

Boolean algebra: **change**

$$x \in \{0,1\}, \; F(X) \in \{0,1\}$$

$$\frac{\partial F(X)}{\partial x_i} = 1$$

$F(X)$ **will change**
if x_i changes

$$\frac{\partial F(X)}{\partial x_i} = 0$$

$F(X)$ **will not change**
if x_i changes

Boolean derivatives

Boolean function:

$$Y = F(x) = F(x_1, x_2, \dots , x_n)$$

Boolean partial derivative:

$$\frac{\partial F(X)}{\partial x_i} = F(x_1, \dots x_i, \dots x_n) \oplus F(x_1, \dots \bar{x}_i, \dots x_n) \quad = 1$$

Simplified equation:

$$\frac{\partial F(X)}{\partial x_i} = F(x_1, \dots x_i = 0, \dots x_n) \oplus F(x_1, \dots x_i = 1, \dots x_n) \quad = 1$$

Testide genereerimine: $X = g^{-1}(dY = 1, dX)$

Boolean Derivatives

Useful properties of Boolean derivatives:

For *F(x)* not depending on *x_i*

$$\frac{\partial[F(X)G(X)]}{\partial x_i} = F(X)\frac{\partial G(X)}{\partial x_i}$$

$$\frac{\partial[F(X)\vee G(X)]}{\partial x_i} = \overline{F(X)}\frac{\partial G(X)}{\partial x_i}$$

Example: $x_1x_4(x_2x_3 \vee x_2\overline{x_3})$

$$F(X) = x_1x_4 \quad G(X) = x_2x_3 \vee x_2\overline{x_3}$$

$$\frac{\partial[F(X)G(X)]}{\partial x_2} = x_1x_4\frac{\partial G(X)}{\partial x_i}$$

These properties allow to simplify the Boolean differential equation to be solved for generating test pattern for a fault at *x_i*

Calculation of Boolean Derivatives

Special cases for differential equations:

$$\frac{\partial F(X)}{\partial x_i} \equiv 0$$ - **if** *F(x)* **is independent of** x_i

$$\frac{\partial F(X)}{\partial x_i} \equiv 1$$ - **if** *F(x)* **depends always on** x_i

$$\frac{\partial F(X)}{\partial x_i} = F(x_1, \ldots x_i = 0, \ldots x_n) \oplus F(x_1, \ldots x_i = 1, \ldots x_n)$$

Examples:

$$F(X) = x_1(x_2 x_3 \vee x_2 \overline{x_3}): \qquad \frac{\partial F(X)}{\partial x_3} = x_1 x_2 \oplus x_1 x_2 \equiv 0$$

$$F(X) = x_1 \oplus x_2: \qquad \frac{\partial F(X)}{\partial x_1} = \overline{x_2} \oplus x_2 \equiv 1$$

Calculation of Boolean Derivatives

Example:

Given: $y = x_1 x_2 \vee x_3(\overline{x}_2 x_4 \vee \overline{x}_1(x_4 \vee (x_5 \vee \overline{x}_2 x_6))) \vee x_1 \overline{x}_3$

Calculation of the Boolean derivative:

$$\frac{\partial y}{\partial x_5} = \overline{x_1 x_2 \vee x_1 \overline{x}_3}\, \frac{\partial (x_3(\overline{x}_2 x_4 \vee \overline{x}_1(x_4 \vee (x_5 \vee \overline{x}_2 x_6))))}{\partial x_5} =$$

$$= \overline{x_1 x_2 \vee x_1 \overline{x}_3}\, x_3 \frac{\partial (\overline{x}_2 x_4 \vee \overline{x}_1(x_4 \vee (x_5 \vee \overline{x}_2 x_6)))}{\partial x_5} =$$

$$= \overline{x_1 x_2 \vee x_1 \overline{x}_3}\, x_3 \overline{\overline{x}_2 x_4}\, \frac{\partial (\overline{x}_1(x_4 \vee (x_5 \vee \overline{x}_2 x_6)))}{\partial x_5} =$$

$$= \overline{x_1 x_2 \vee x_1 \overline{x}_3}\, x_3 \overline{\overline{x}_2 x_4}\, \overline{x}_1 \frac{\partial (x_4 \vee (x_5 \vee \overline{x}_2 x_6))}{\partial x_5} = \overline{x_1 x_2 \vee x_1 \overline{x}_3}\, x_3 \overline{\overline{x}_2 x_4}\, \overline{x}_1 \overline{x}_4 \overline{\overline{x}_2 x_6} \frac{\partial x_5}{\partial x_5}$$

Calculation of Boolean derivatives

Finding a solution of the differential equation:

$$\frac{\partial y}{\partial x_5} = (\overline{x_1 x_2 \vee x_1 \overline{x_3}})x_3(\overline{\overline{x_2} x_4})\overline{x_1}\,\overline{x_4}(\overline{\overline{x_2} x_6})\frac{\partial x_5}{\partial x_5} =$$

$$= (\overline{x_1} \vee \overline{x_2})(\overline{x_1} \vee x_3)x_3(x_2 \vee \overline{x_4})\overline{x_1}\,\overline{x_4}(x_2 \vee \overline{x_6}) =$$

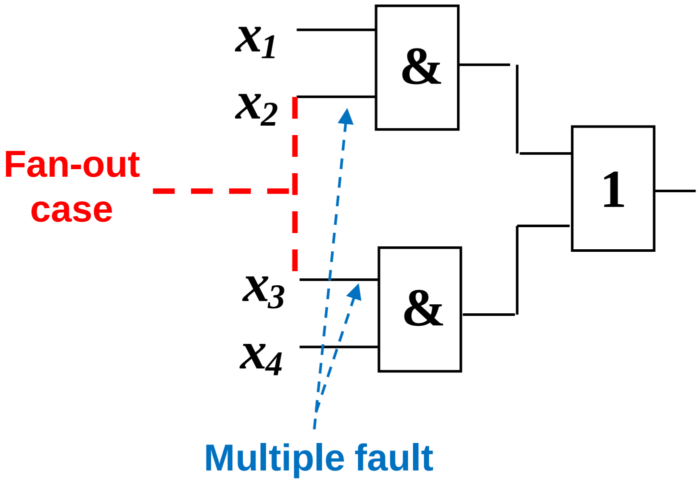
$$= \overline{x_1}\,\overline{x_4}x_3x_2 \vee \ldots = 1$$

Boolean vector derivatives

Multiple fault cases and fan-outs:

$$\frac{\partial F(X)}{\partial(x_i, x_j)} = F(x_1,\ldots x_i, x_j,\ldots x_n) \oplus F(x_1,\ldots \bar{x}_i, \bar{x}_j,\ldots x_n)$$

Example:



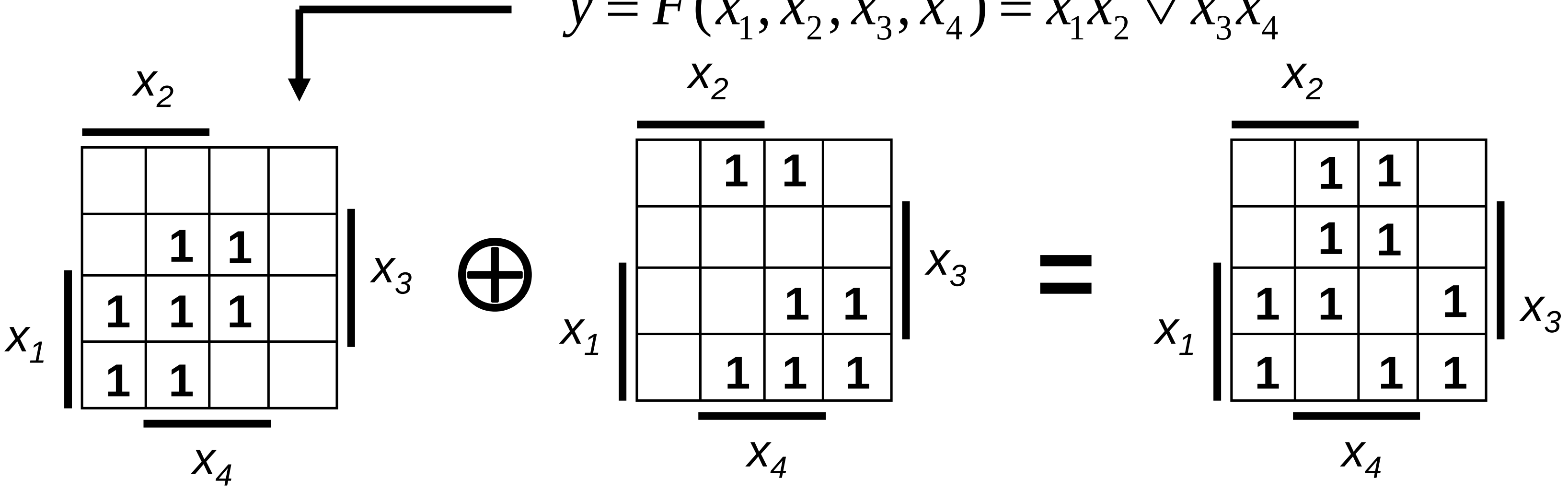
$$y = F(x_1, x_2, x_3, x_4) = x_1 x_2 \vee x_3 x_4$$

$$\frac{\partial F(X)}{\partial(x_2, x_3)} = x_1 \bar{x}_4 \vee \bar{x}_1 x_4 \vee x_1 x_4 (\bar{x}_2 \bar{x}_3 \vee x_2 x_3)$$

Boolean vector derivatives

Calculation of the vector derivatives by Carnaugh maps:

$$y = F(x_1, x_2, x_3, x_4) = x_1x_2 \vee x_3x_4$$



	1	1	
1	1	1	
1	1		

	1	1	
		1	1
	1	1	1

	1	1	
	1	1	
1	1		1
1		1	1

$$x_1x_2 \vee x_3x_4$$

$$x_1\overline{x}_2 \vee \overline{x}_3x_4$$

$$\frac{\partial F(X)}{\partial(x_2, x_3)} = x_1\overline{x}_4 \vee \overline{x}_1x_4 \vee x_1x_4(\overline{x}_2\overline{x}_3 \vee x_2x_3) = 1$$

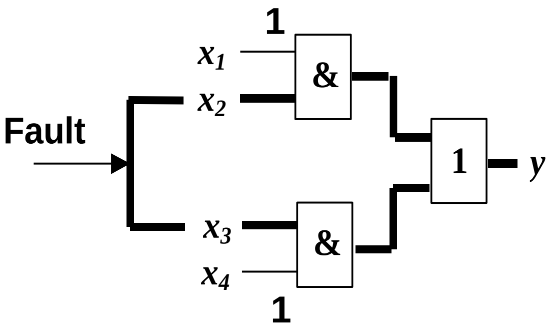
Boolean vector derivatives

Interpretation of three solutions:

$$\frac{\partial F(X)}{\partial(x_2, x_3)} = x_1 \bar{x}_4 \vee \bar{x}_1 x_4 \vee x_1 x_4 (\bar{x}_2 \bar{x}_3 \vee x_2 x_3) = 1$$

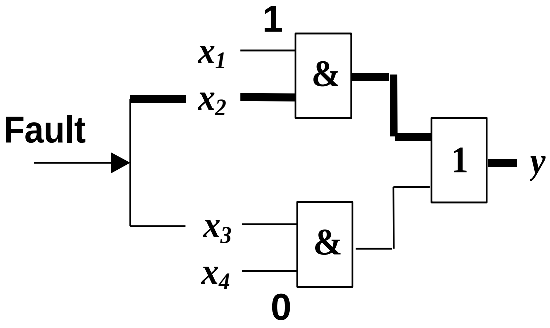
$$x_1 x_4 = 1$$

Two paths activated



$$x_1 \bar{x}_4 = 1$$

Single path activated



Multiple Boolean derivatives

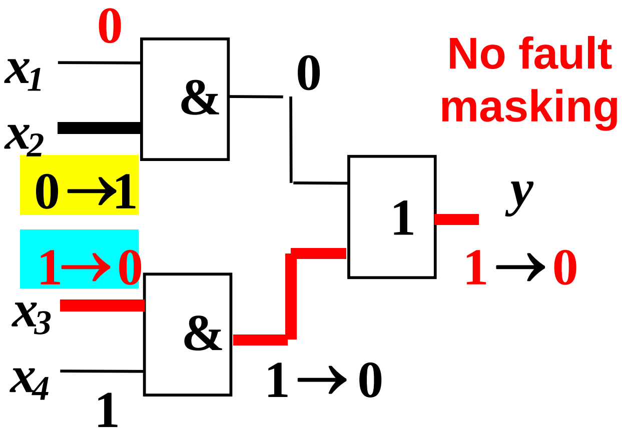
$$y = x_1x_2 \vee x_3x_4$$

Test for x_3

$$\frac{\partial y}{\partial x_3} = \bar{x}_1 x_4 \vee \bar{x}_2 x_4 = 1$$

$$\frac{\partial^2 y}{\partial x_2 \partial x_3} = \frac{\partial}{\partial x_2}\left(\frac{\partial y}{\partial x_3}\right) = x_1x_4 = 0$$

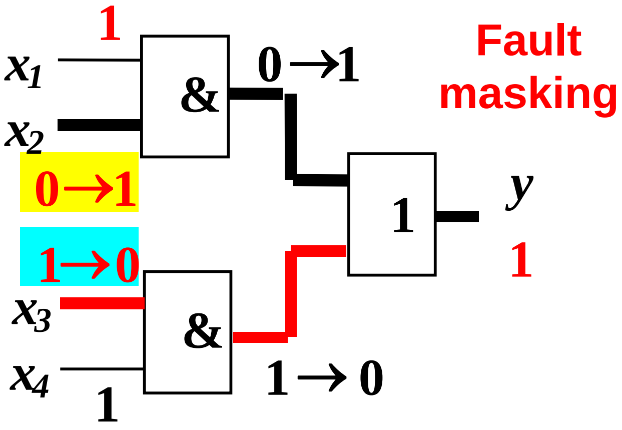
Two faults



Fault in x_2 cannot mask the fault in x_3

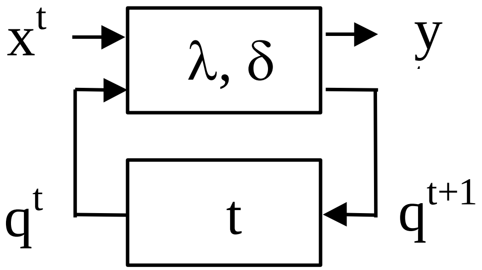
Faults

$$x_1x_4 = 1$$



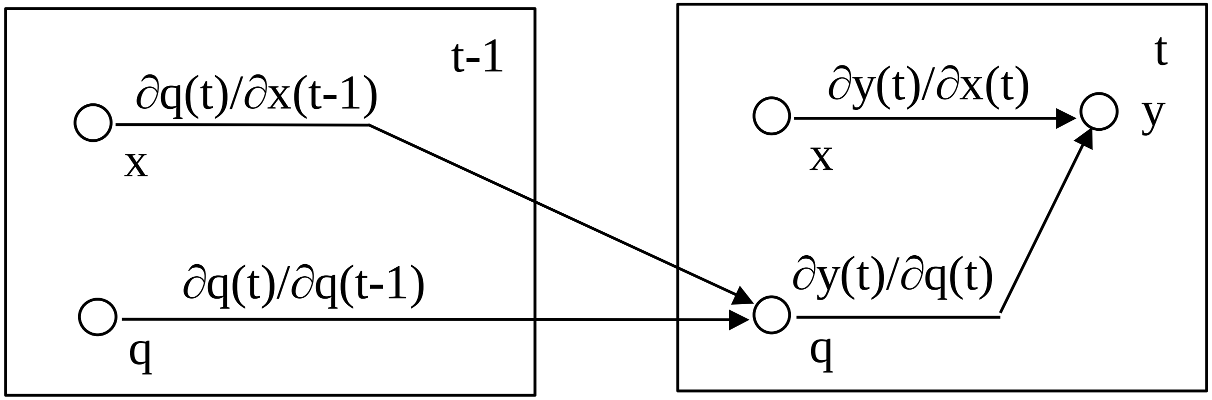
Bool. derivatives for sequential circuits

Boolean derivatives for state transfer and output functions of FSM:



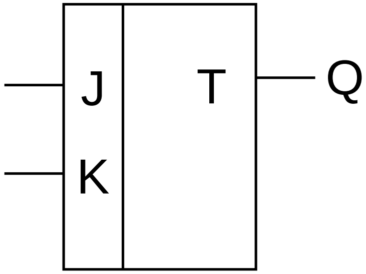
$$y(t) = \lambda[x(t),q(t)]$$

$$q(t+1) = \delta[x(t),q(t)]$$



Bool. derivatives for sequential circuits

Boolean derivatives for JK Flip-Flop:



$$Q(t) = J(t)\overline{Q}(t-1) \vee \overline{K}(t-1)Q(t-1)$$

$$\frac{\partial Q(t)}{\partial J(t-1)} = \overline{Q}(t-1)$$

$$\frac{\partial Q(t)}{\partial K(t-1)} = Q(t-1)$$

The erroneos signal will propagate from inputs to output

$$\frac{\partial Q(t)}{\partial Q(t-1)} = K(t-1)J(t-1) \vee \overline{K}(t-1)\overline{J}(t-1)$$

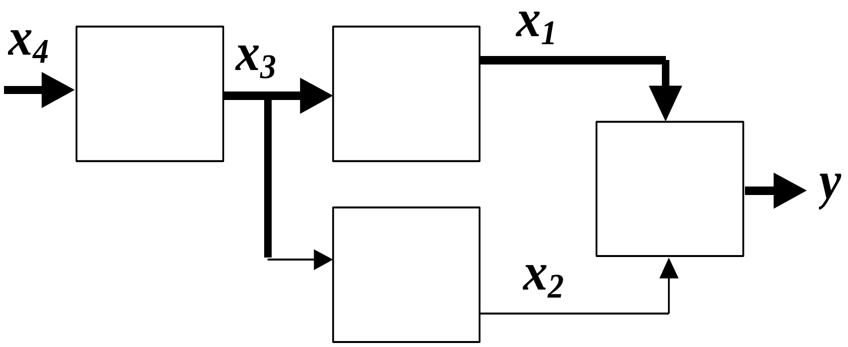
The erroneous signal was stored in the previous clock cycle

Derivatives for complex functions

Boolean derivative for a complex function:

$$\frac{\partial F_k(F_j(X),X)}{\partial x_i} = \frac{\partial F_k}{\partial F_j}\frac{\partial F_j}{\partial x_i}$$

Example:



Hierarchical solution:

$$\frac{\partial y}{\partial x_4} = \frac{\partial y}{\partial x_1}\frac{\partial x_1}{\partial x_3}\frac{\partial x_3}{\partial x_4}$$

Additional condition:

$$\frac{\partial x_2}{\partial x_3} = 0$$

Overview about Applications of BDs

- **Fault simulation**
  - Calculate the value of: $\frac{\partial F(X)}{\partial x_i}$
- **Test generation**
  - Single faults: Find the solution for: $\frac{\partial F(X)}{\partial x_i} = 1$
  - Multiple faults: Find the solution for: $\frac{\partial F(X)}{\partial (x_i, x_j)} = 1$
  - Decompositional approach (complex functions):

$$\frac{\partial F_k(F_j(X), X)}{\partial x_i} = \frac{\partial F_k}{\partial F_j} \frac{\partial F_j}{\partial x_i}$$

- **Fault masking analysis:** $\frac{\partial y}{\partial x_i} = 1; \frac{\partial^2 y}{\partial x_j \partial x_i} = \frac{\partial}{\partial x_j}\left(\frac{\partial y}{\partial x_i}\right) = 0$

Süsteemide diagnostika

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2.1. Boole'i differentsiaalalgebra

- Boole'i funktsioonide differentseerimine
- Boole'i differentsiaalvõrrandid

2.2. Binaarsed otsustusdiagrammid (BDD)

2.3. Kõrgtasandi otsustusdiagrammid

Boolean Differentials and Fault Diagnosis

dx - fault variable, *dx* ∈ *(0,1)*

***dx* = 1**, if the value of *x* has changed because of a fault

Partial Boolean differential (for fault **simulation**):

$$d_{x_i}F = F(x_1,\ldots,x_i,\ldots,x_n) \oplus F(x_1,\ldots,x_i \oplus dx_i,\ldots,x_n) = \frac{\partial F}{\partial x_i} dx_i$$

Full Boolean differential (for fault **diagnosis**):

$$dF = F(x_1,\ldots,x_i,\ldots x_n) \oplus F(x_1 \oplus dx_1,\ldots,x_i \oplus dx_i,\ldots x_n \oplus dx_n)$$

$$dF = F(X) \oplus F(X \oplus dX)$$

Fault diagnosis: $dX = g^{-1}(dY, X)$

Boolean Differentials and Fault Diagnosis

$$y = \bar{x}_1(\bar{x}_2 \vee x_3) \qquad dy = y \oplus (\bar{x}_1 \oplus dx_1)((\bar{x}_2 \oplus dx_2) \vee (x_3 \oplus dx_3))$$

Diagnostic experiment ⇨ **Fault diagnosis:** $dX = g^{-1}(dY, X)$

$x_1 = 0$
$x_2 = 1$
$x_3 = 1$
$dy = 0$

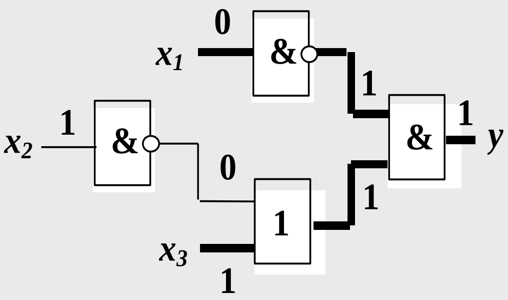
} **Test pattern**

- **Correct reaction**

Substitution of values:

$$dy = 1 \oplus \overline{dx_1}(dx_2 \vee \overline{dx_3}) = 0$$

$$dy = \overline{dx_1}(dx_2 \vee \overline{dx_3}) = 1$$



Adjusting for SAF faults:

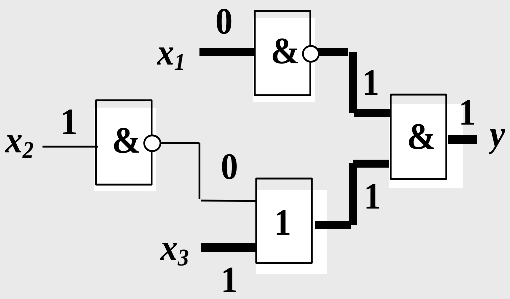
$$\overline{dx_1}^{\,0}(dx_2^1 \vee \overline{dx_3}^{\,1}) = 1$$

Partial diagnosis: $\overline{dx_1}^{\,0} = 1$

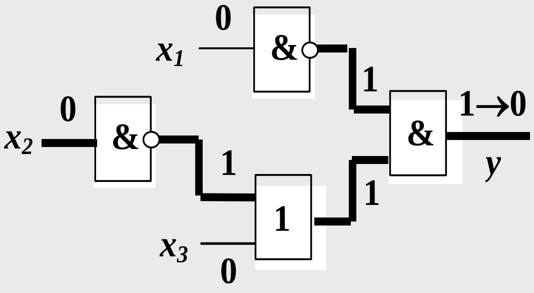
Boolean Differentials and Fault Diagnosis

$$y = \bar{x}_1(\bar{x}_2 \vee x_3) \qquad dy = y \oplus (\bar{x}_1 \oplus dx_1)((\bar{x}_2 \oplus dx_2) \vee (x_3 \oplus dx_3))$$

Two diagnostic experiments:



$x_1 = 0$
$x_2 = 1$
$x_3 = 1$
$dy = 0$



$x_1 = 0$
$x_2 = 0$
$x_3 = 0$
$dy = 1$

1) Correct output signal:

$$\overline{dx_1}^{\,0}(dx_2^1 \vee \overline{dx_3}^{\,1}) = 1 \longrightarrow \boxed{\overline{dx_1}^{\,0} = 1}$$

2) Erroneous output signal:

$$dy = 1 \oplus \overline{dx_1}(\overline{dx_2} \vee dx_3) = 1$$

$$dx_1^0 \vee dx_2^0\,\overline{dx_3}^{\,0} = 1$$

Diagnosis from two experiments:

$$\overline{dx_1}^{\,0}(dx_2^1 \vee \overline{dx_3}^{\,1})(dx_1^0 \vee dx_2^0\,\overline{dx_3}^{\,0}) = 1$$

Boolean Differentials and Fault Diagnosis

Diagnosis from two experiments:

$$\overline{dx_1}^0 (dx_2^1 \vee \overline{dx_3}^1)(dx_1^0 \vee dx_2^0 \overline{dx_3}^0) = 1$$

Rule: $\overline{dx_k}^0 dx_k^0 = 0$

$$\overline{dx_1}^0 (\underset{= 0}{dx_2^1 dx_2^0} \overline{dx_3}^0 \vee dx_2^0 \overline{dx_3}^0 \overline{dx_3}^1) = 1$$

Rule: $dx_k^0 dx_k^1 = 0$

Final diagnosis:

$$\overline{dx_1}^0 dx_2^0 \overline{dx_3}^0 \overline{dx_3}^1 = 1$$

The line x_3 works correctly

There is a fault: $x_2 \equiv 1$

The fault $x_1 \equiv 1$ **is missing**

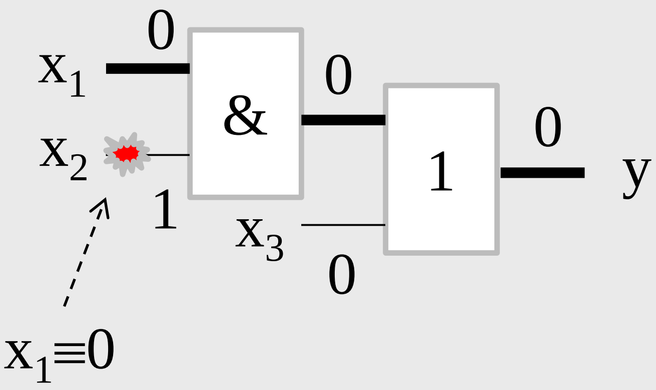
Question:

1) Which question regarding the possible present faults is still open?

Correctness Proof with Test Pairs

$$y = x_1 x_2 \vee x_3$$

$$dy = y \oplus (x_1 \oplus dx_1)(x_2 \oplus dx_2) \vee (x_3 \oplus dx_3)$$



We will test the fault $x_1 \equiv 1$

The test is successful
So, we conclude that x_1 is correct

Let us use now mathematics:

$$dy = dx_1 \overline{dx_2} \vee dx_3 = 0$$

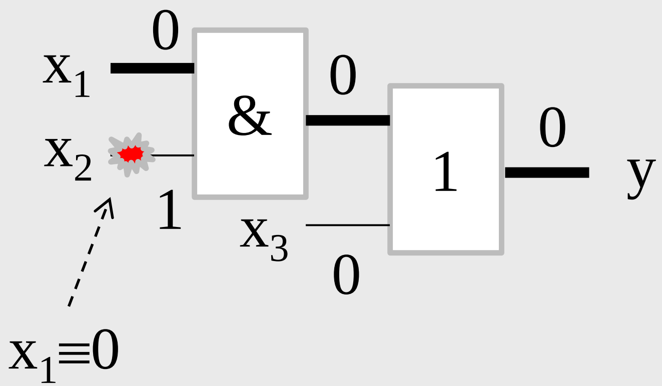
$$(\overline{dx_1} \vee dx_2)\overline{dx_3} = 1$$

Now we are not any more very sure about x_1

Correctness Proof with Test Pairs

$$y = x_1 x_2 \vee x_3$$

$$dy = y \oplus (x_1 \oplus dx_1)(x_2 \oplus dx_2) \vee (x_3 \oplus dx_3)$$



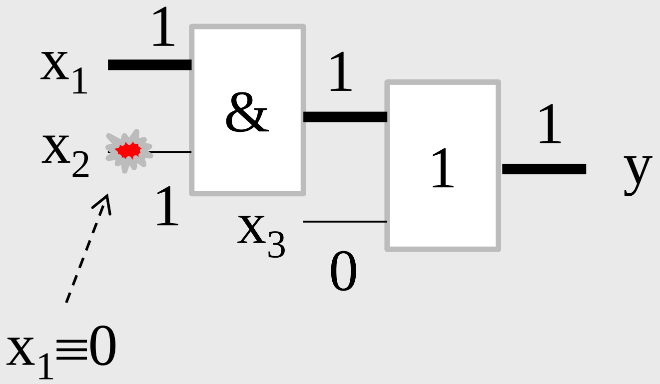
We have: $dy = dx_1 \overline{dx_2} \vee dx_3 = 0$

$$(\overline{dx_1} \vee dx_2)\overline{dx_3} = 1$$

2. test: we change the value of x_1 to have a test pair

$$dy = 1 \oplus (\overline{dx_1}\,\overline{dx_2} \vee dx_3) = 0$$

$$(\overline{dx_1}\,\overline{dx_2} \vee dx_3) = 1$$



Both experiments together:

$$(\overline{dx^1{}_1}\,\overline{dx_2} \vee dx_3)(\overline{dx^0{}_1} \vee dx_2)\overline{dx_3} =$$

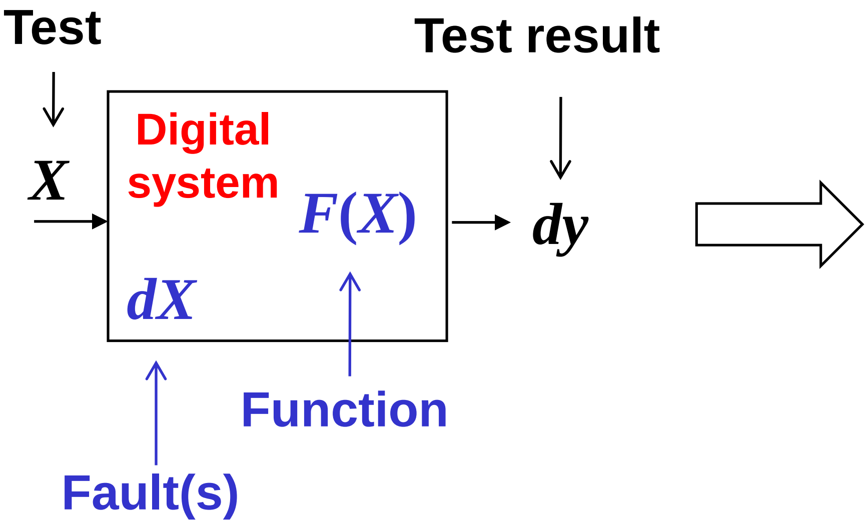
$$= \overline{dx^1{}_1}\,\overline{dx^0{}_1}\,\overline{dx^1{}_2}\,\overline{dx^0{}_3} = 1$$

Diagnostic Equation

Digital circuit: $X \Rightarrow$ $y=F(X)$ $\quad y = \bar{x}_1(\bar{x}_2 \vee x_3)$ $\Rightarrow y$

$X,dX \Rightarrow$ **Diagnostic model:** **Full diferential equation** $dy=F(X,dX)$ $\Rightarrow dy$

$$dy = y \oplus (\bar{x}_1 \oplus dx_1)((\bar{x}_2 \oplus dx_2) \vee (x_3 \oplus dx_3))$$



Diagnostic equation:

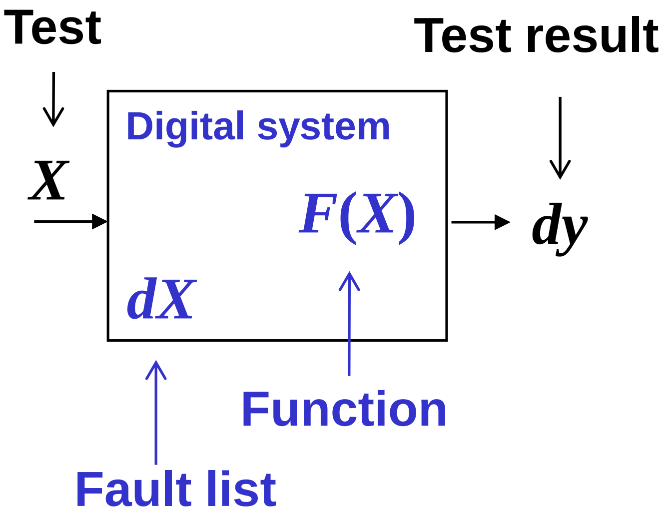
$$F(X,dX) = dy$$

Test

Fault(s)

Test result

How the Test Tasks are Related



Diagnostic equation:

$$F(X,dX) = dy$$

Test

Fault(s)

Test result

Task	Given		Find
Test generation	dx	dy = 1	X
Fault diagnosis	X	dy	{dx}
Fault simulation	X	dy = 1	{dx}

Special case of fault diagnosis

Both equations can be presented as SAT tasks