

Built-In Self-Test

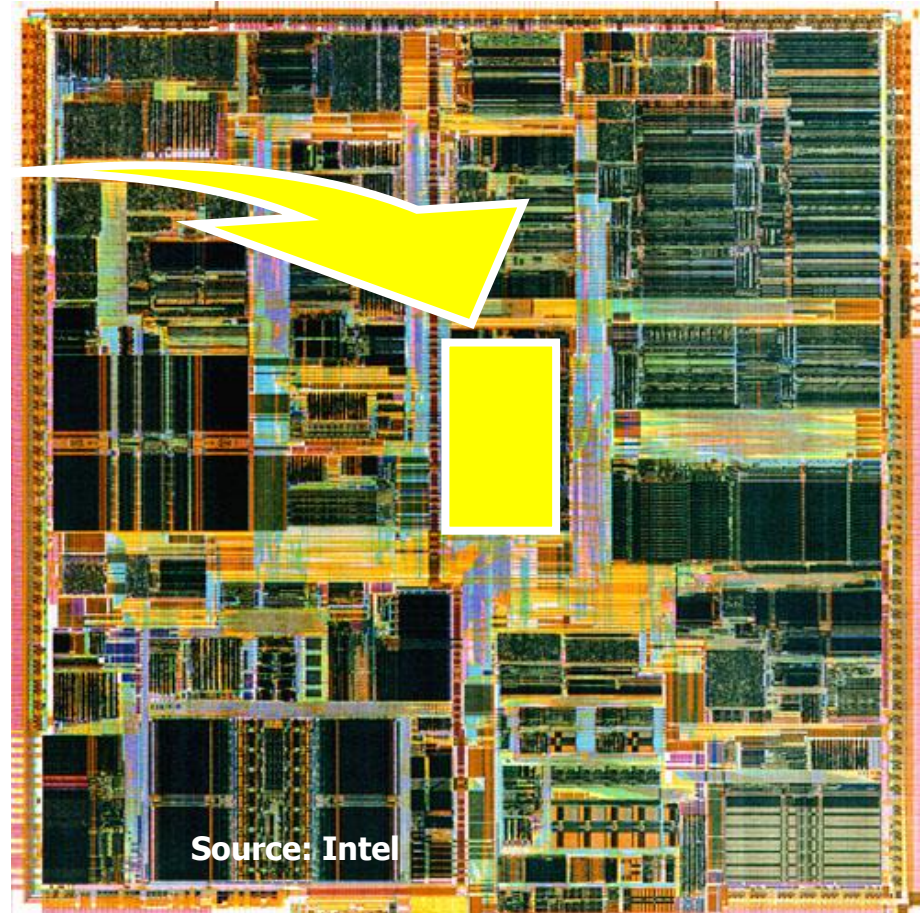
Outline

- **Motivation for BIST**
- **Testing SoC with BIST**
- **Test per Scan and Test per Clock**
- **HW and SW based BIST**
- **Exhaustive and pseudoexhaustive test generation**
- **Pseudorandom test generation with LFSR**
- **Hybrid BIST**
- **Response compaction methods**
- **Signature analyzers**

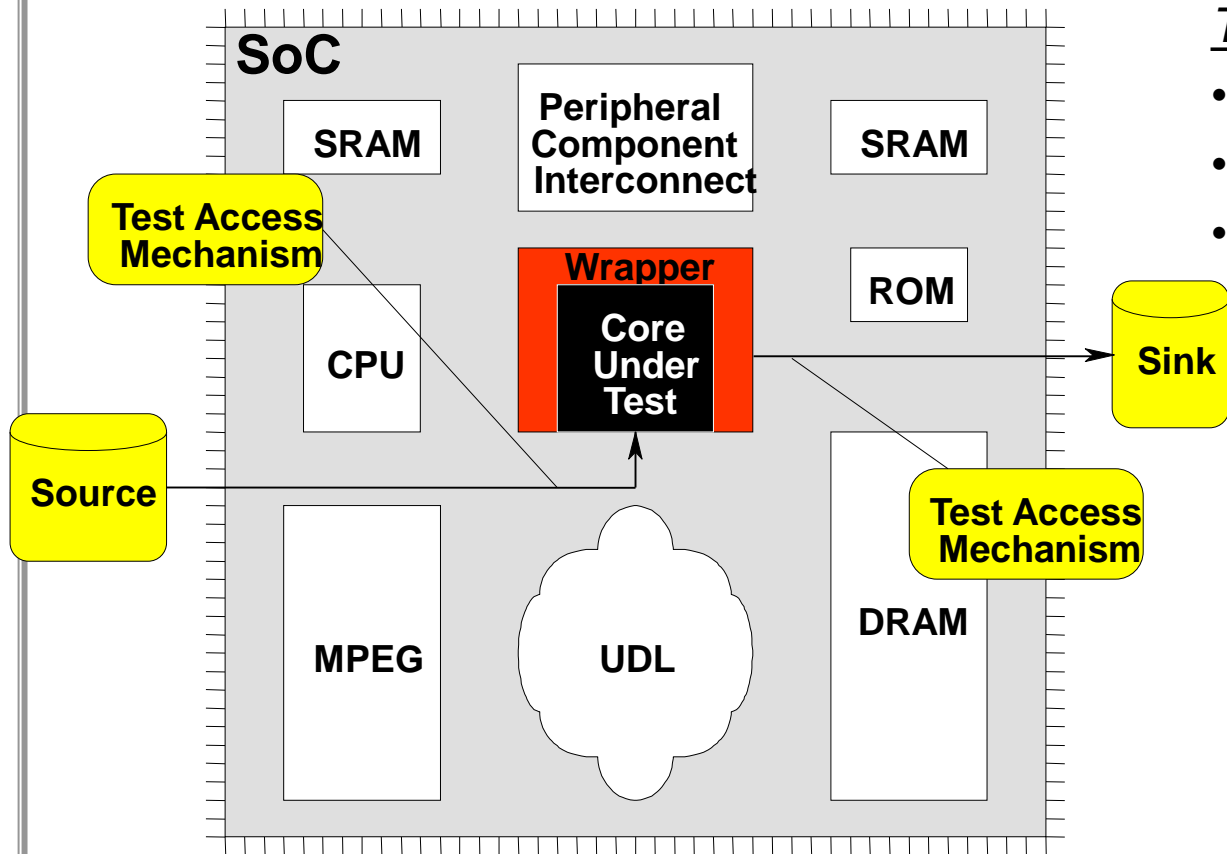
Testing Challenges: SoC Test



Cores have to be tested on chip



Self-Test in Complex Digital Systems



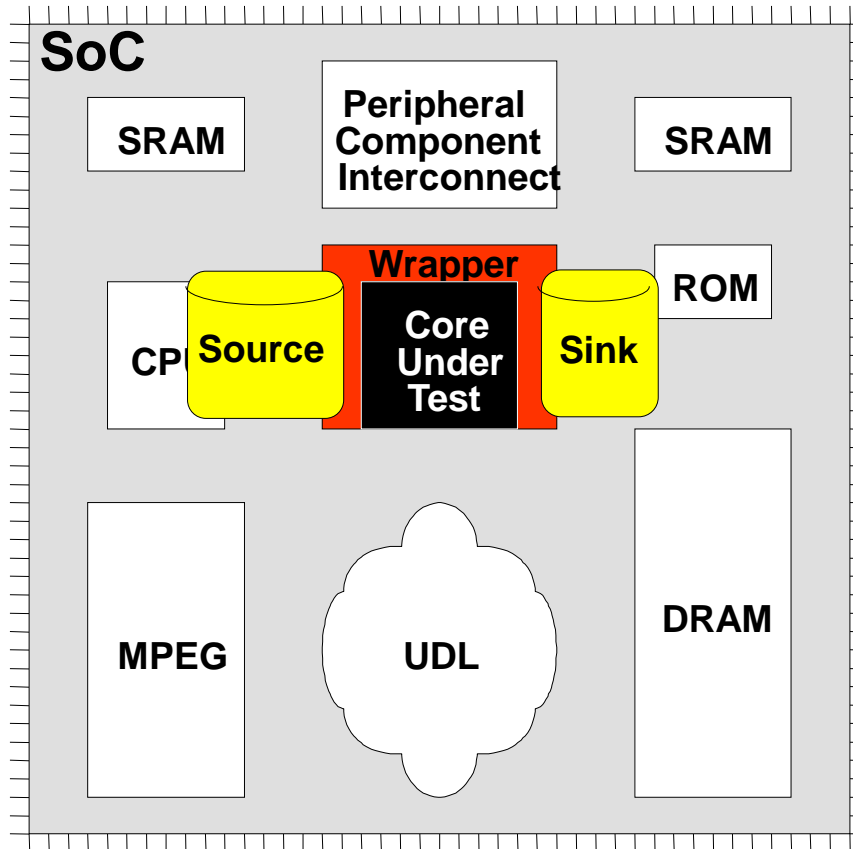
Test architecture components:

- Test pattern source & sink
- Test Access Mechanism
- Core test wrapper

Solutions:

- Off-chip solution
 - need for external ATE
- Combined solution
 - mostly on-chip, ATE needed for control
- On-chip solution
 - BIST

Self-Test in Complex Digital Systems



Test architecture components:

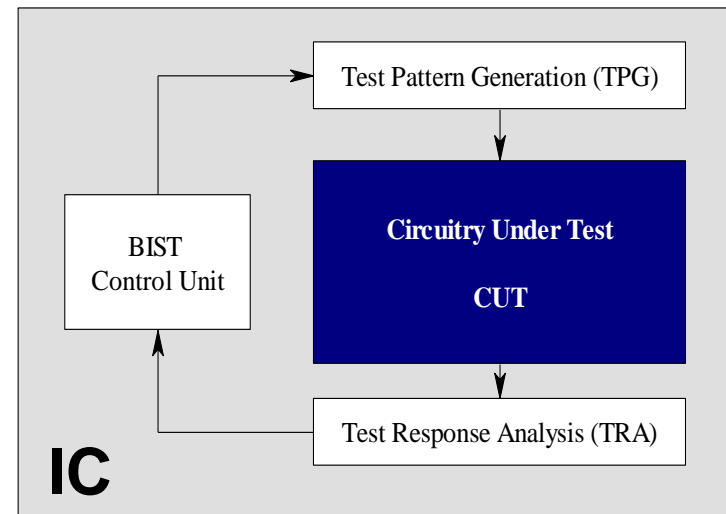
- Test pattern source & sink
- Test Access Mechanism
- Core test wrapper

Solutions:

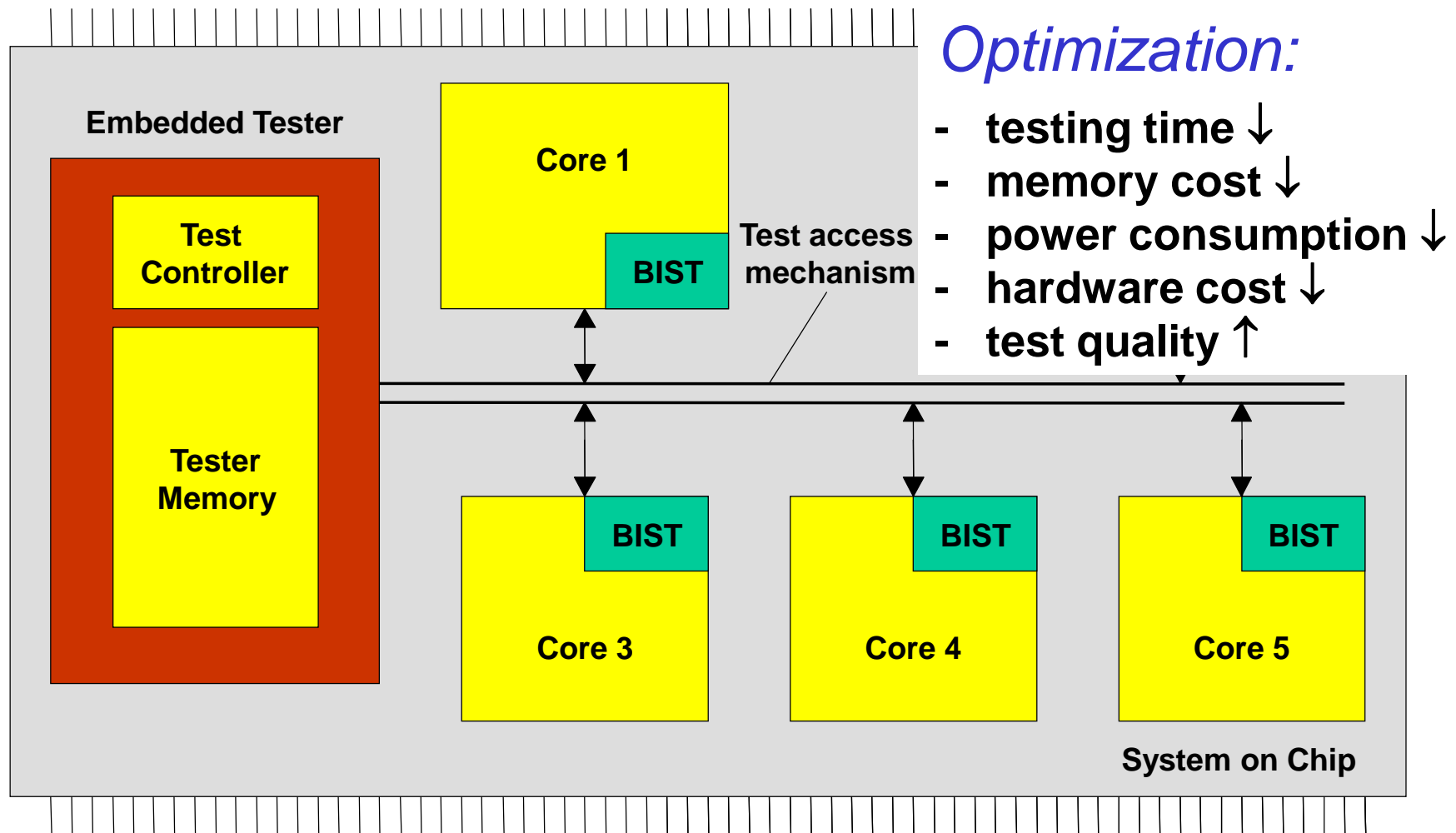
- Off-chip solution
 - need for external ATE
- Combined solution
 - mostly on-chip, ATE needed for control
- On-chip solution
 - BIST

What is BIST

- **On circuit**
 - Test pattern generation
 - Response verification
- **Random pattern generation, very long tests**
- **Response compression**



SoC BIST



Built-In Self-Test

- **Motivations for BIST:**

- **Need for a cost-efficient testing** (general motivation)
- Doubts about the stuck-at fault model
- Increasing difficulties with TPG (Test Pattern Generation)
- Growing volume of test pattern data
- Cost of ATE (Automatic Test Equipment)
- Test application time
- **Gap between** tester and UUT (Unit Under Test) **speeds**

- **Drawbacks of BIST:**

- **Additional pins** and silicon area needed
- Decreased reliability due to increased silicon area
- **Performance impact** due to additional circuitry
- Additional design time and cost

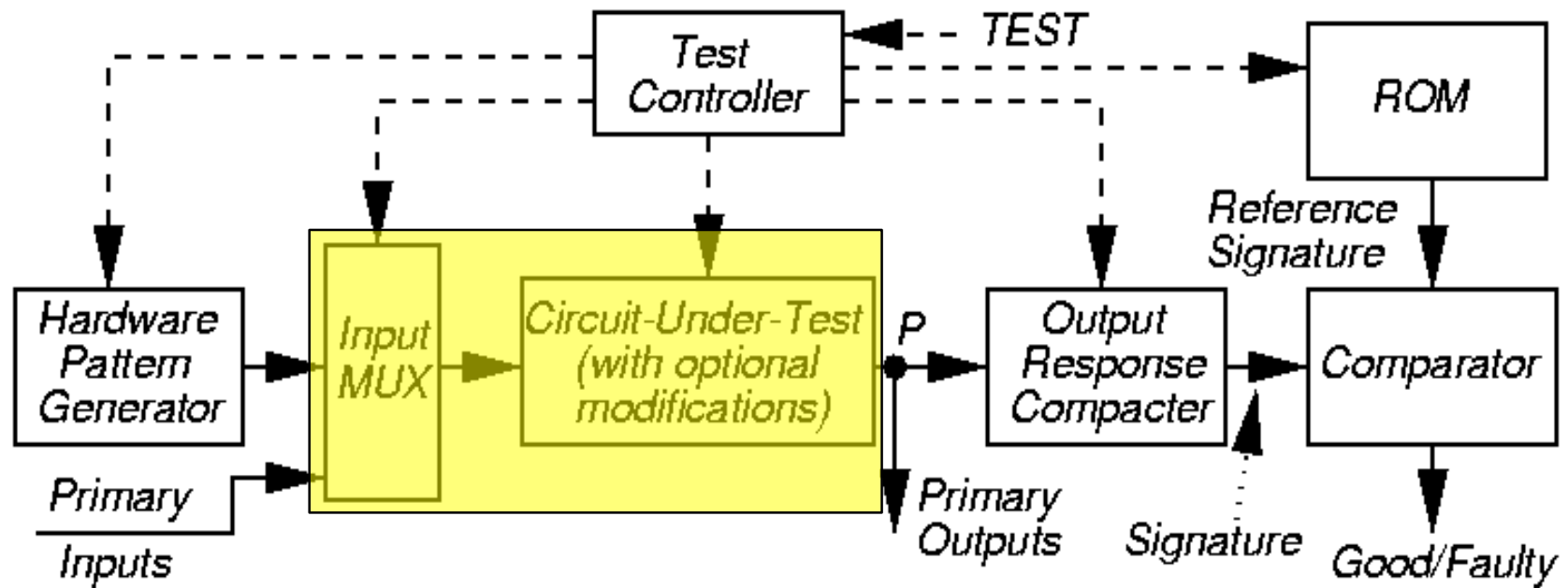
BIST in Maintenance and Repair

- **Useful** for field test and diagnosis (less expensive than a local automatic test equipment)
- To overcome the **disadvantages** of software tests for field test and diagnosis (nonBIST):
 - Low hardware fault coverage
 - Low diagnostic resolution
 - Slow to operate
- **Hardware BIST benefits:**
 - Lower system test effort
 - Improved system maintenance and repair
 - Improved component repair
 - Better diagnosis
 - Possibility to use the functionality of microprocessors

BIST Techniques

- **BIST techniques are classified:**
 - **on-line BIST** - includes concurrent and nonconcurrent techniques
 - **off-line BIST** - includes functional and structural approaches
- **On-line BIST** - testing occurs during normal functional operation
 - **Concurrent on-line BIST** - testing occurs simultaneously with normal operation mode, usually **coding techniques or duplication** and comparison are used
 - **Nonconcurrent on-line BIST** - testing is carried out while a system is in an **idle** state, often by executing diagnostic software or firmware routines
- **Off-line BIST** - system is not in its normal working mode, usually on-chip test generators and output response analyzers or microdiagnostic routines
 - **Functional off-line BIST** is based on a functional description of the Component Under Test (CUT) and uses functional high-level fault models
 - **Structural off-line BIST** is based on the structure of the CUT and uses structural fault models (e.g. SAF)

Detailed BIST Architecture



BIST: Test Generation Methods

Universal test sets

1. Exhaustive test (trivial test)
2. Pseudo-exhaustive test

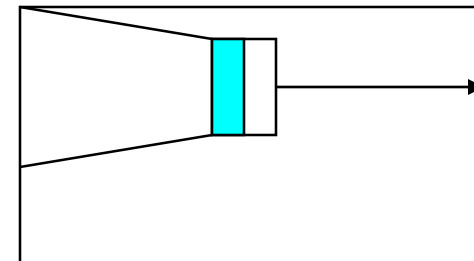
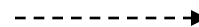
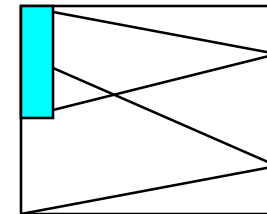
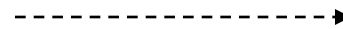
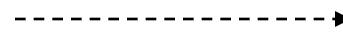
Properties of exhaustive tests

1. **Advantages** (concerning the stuck at fault model):
 - test pattern generation is not needed
 - fault simulation is not needed
 - no need for a fault model
 - redundancy problem is eliminated
 - single and multiple stuck-at fault coverage is 100%
 - easily generated on-line by hardware
2. **Shortcomings:**
 - long test length (2^n patterns are needed, n - is the number of inputs)
 - CMOS stuck-open fault problem

Exhaustive and Pseudo-Exhaustive Testing

Exhaustive combinational **fault model**:

- exhaustive test patterns
- pseudoexhaustive test patterns
 - exhaustive **output line** oriented test patterns
 - exhaustive **module** oriented test patterns



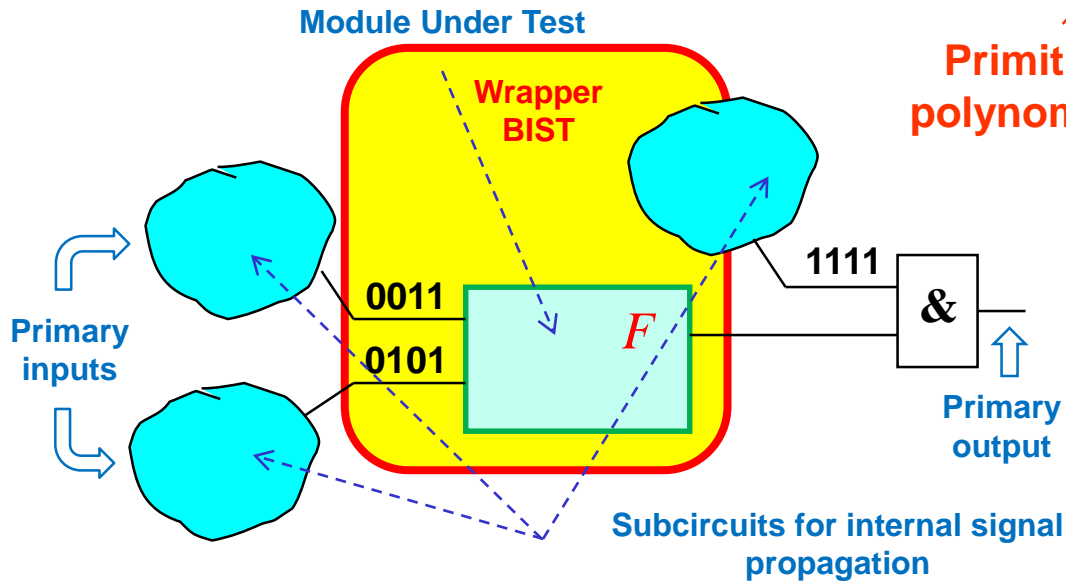
BIST: Pseudoexhaustive Testing

Pseudo-exhaustive test sets:

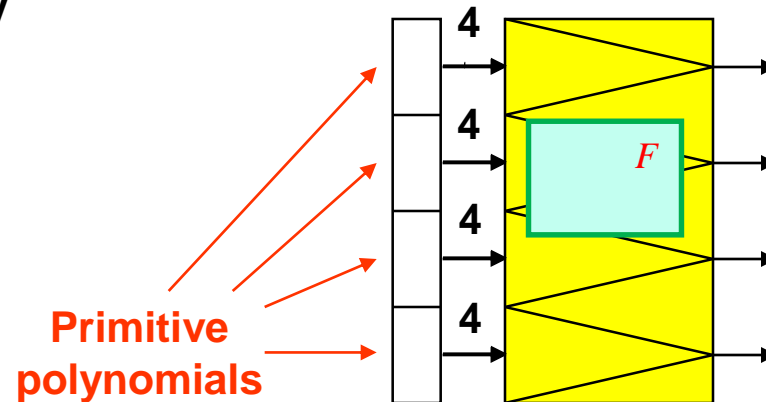
Output function verification

- maximal parallel testability
- partial parallel testability

Module function verification



Output function verification



$$2^{16} = 65536 \gg 4 \times 16 = 64 > 16$$

Exhaustive test	Pseudo-exhaustive sequential	Pseudo-exhaustive parallel
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Testing ripple-carry adder

Output function verification (maximum parallelity)

Exhaustive test generation for n-bit adder:

Good news:

Bit number n - arbitrary

Test length - **always 8 (!)**

Bad news:

The method is correct

only for ripple-carry adder

	c_0	a_0	b_0	c_1	a_1	b_1	c_2	a_2	b_2	c_3	...
1	0	0	0	0	0	0	0	0	0	0	
2	0	0	1	0	0	1	0	0	1	0	
3	0	1	0	0	1	0	0	1	0	0	
4	0	1	1	1	0	0	0	1	1	1	
5	1	0	0	0	1	1	1	0	0	0	
6	1	0	1	1	0	1	1	0	1	1	
7	1	1	0	1	1	0	1	1	0	1	
8	1	1	1	1	1	1	1	1	1	1	

0-bit testing

1-bit testing

2-bit testing

3-bit testing

... etc

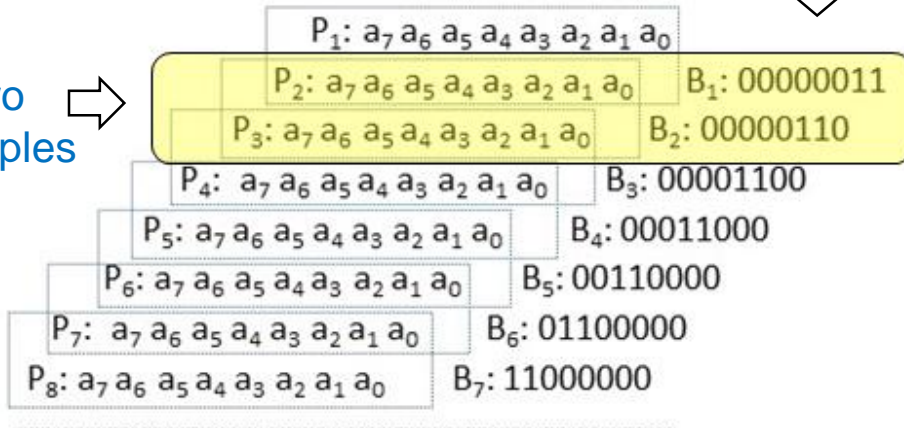
Pseudo-Exhaustive Test for Multiplier

Selectable multiplicands

Multipliers

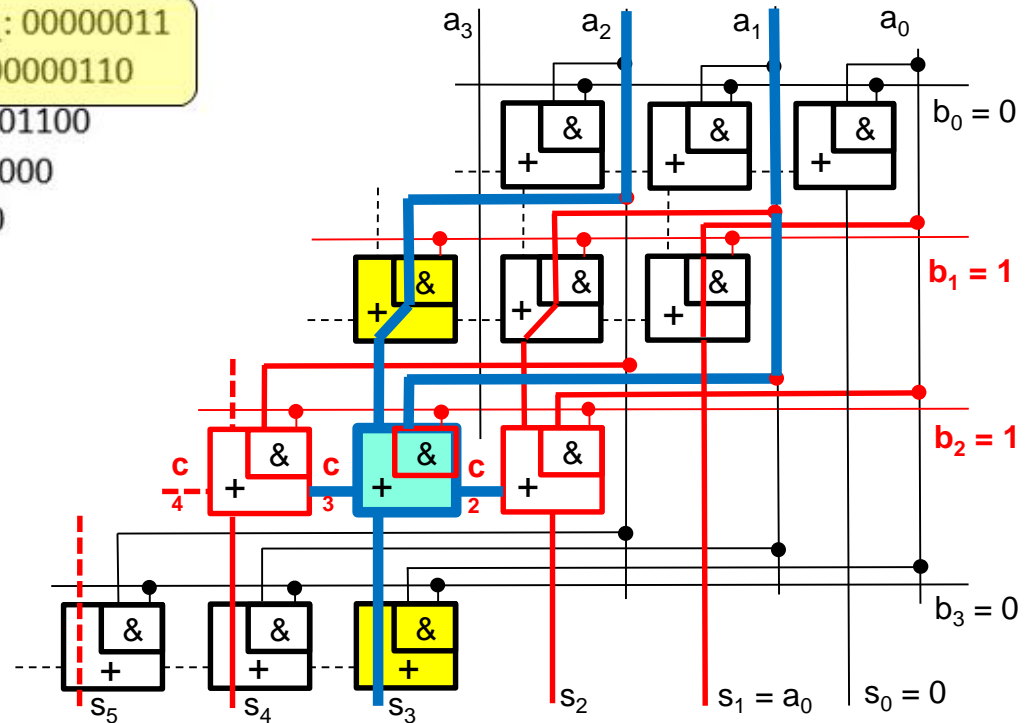
Multiplier array

Two examples



$S_{15} S_{14} S_{13} S_{12} S_{11} S_{10} S_9 S_8 S_7 S_6 S_5 S_4 S_3 S_2 S_1 S_0$

Multiplication with traditional "paper and pencil" method




Pseudo-Exhaustive Test for Multiplier

Replication of columns with pseudo-exhaustive patterns for

This table is replicated and all replications are repeated for all shifted **b = (...11...)**

Adder 

Multiplier 

No	...	4-bit	3-bit	2-bit	1-bit	0-bit
		a ₄ b ₄ c ₄	a ₃ b ₃ c ₃	a ₂ b ₂ c ₂	a ₁ b ₁ c ₁	a ₀ b ₀
1	...	0 0 0	0 0 0	0 0 0	0 0 0	0 0
2	...	0 1 0	0 1 0	0 1 0	0 1 0	0 1
3	...	1 0 0	1 0 0	1 0 0	1 0 0	1 0
4	...	1 1 0	0 0 1	1 1 0	0 0 1	1 1
5	...	0 0 1	1 1 0	0 0 1	1 1 0	0 0
6	...	0 1 1	0 1 1	0 1 1	0 1 1	1 1
7	...	1 0 1	1 0 1	1 0 1	1 0 1	1 1
8	...	1 1 1	1 1 1	1 1 1	1 1 1	1 1

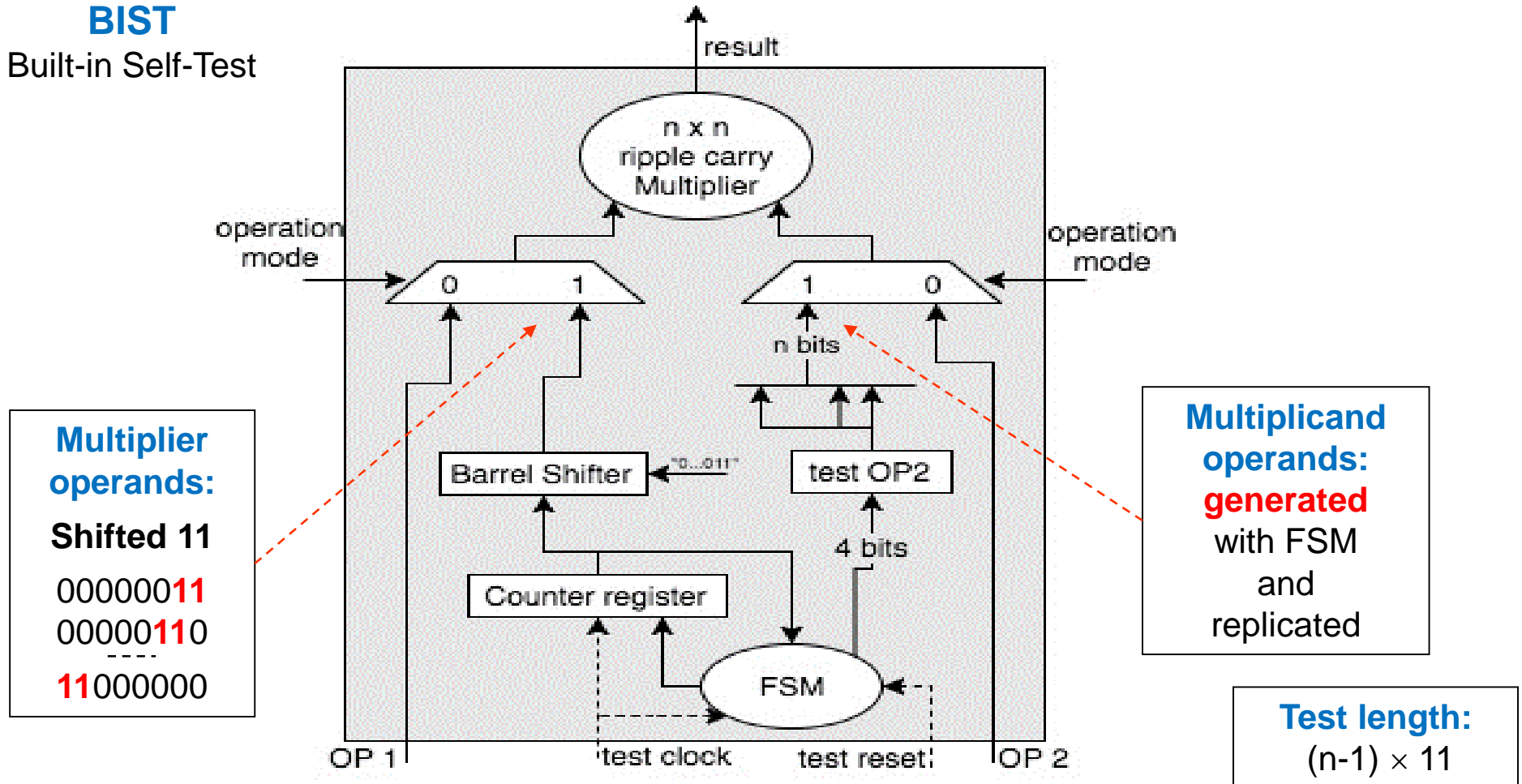
carry multiplier array

N	6-bit	5-bit	4-bit	3-bit	2-bit	1-bit	0-bit
	c ₆ a ₇ a ₆	c ₅ a ₆ a ₅	c ₄ a ₅ a ₄	c ₃ a ₄ a ₃	c ₂ a ₃ a ₂	c ₁ a ₂ a ₁	a ₁ a ₀
1	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0
2	0 1 0	0 0 1	0 1 0	0 0 1	0 1 0	0 0 1	1 0
3	0 0 1	0 1 0	0 0 1	0 1 0	0 0 1	0 1 0	0 1
4	1 0 1	0 1 1	0 1 0	1 0 0	1 0 1	0 1 1	1 0
5	1 1 0	1 0 1	1 1 0	1 0 1	1 1 0	1 0 1	1 1
6	1 0 1	1 1 1	1 1 1	1 1 0	1 0 1	1 1 1	1 1
7	0 1 1	0 1 0	1 0 0	1 0 1	0 1 1	0 1 0	0 0
8	1 0 0	1 0 1	0 1 1	0 1 0	1 0 0	1 0 1	1 1
9	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1
10	0 1 0	1 0 0	1 0 1	0 1 1	0 1 0	0 1 0	1 0
11	1 1 1	1 1 0	1 0 1	1 1 1	1 1 1	1 1 1	1 1

Exhaustively Self-Testing Multiplier

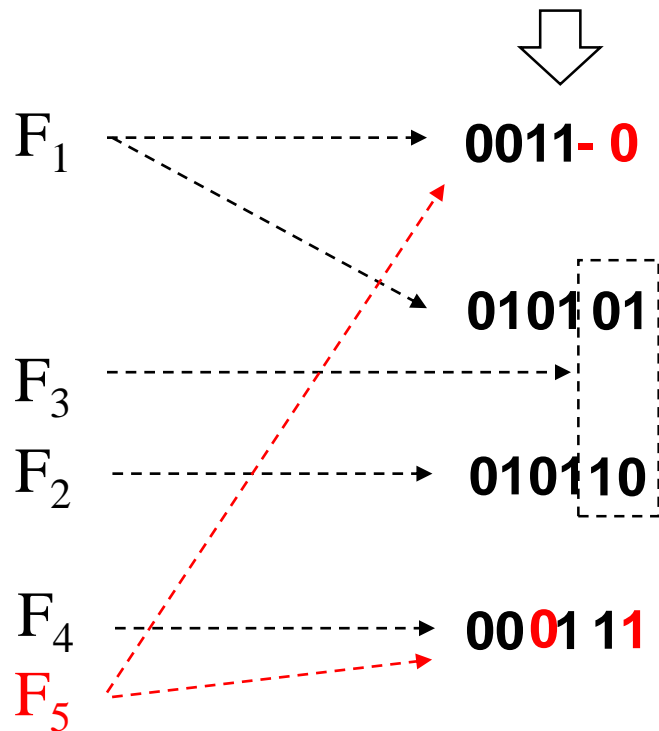
BIST

Built-in Self-Test

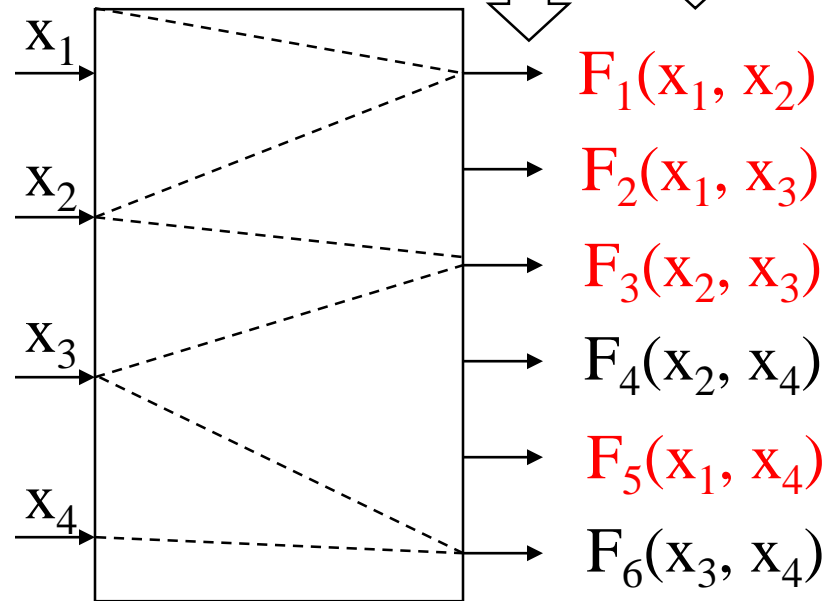


Pseudoexhaustive Test Optimization

Simple iterative algorithm for test pattern generation:



Output function verification



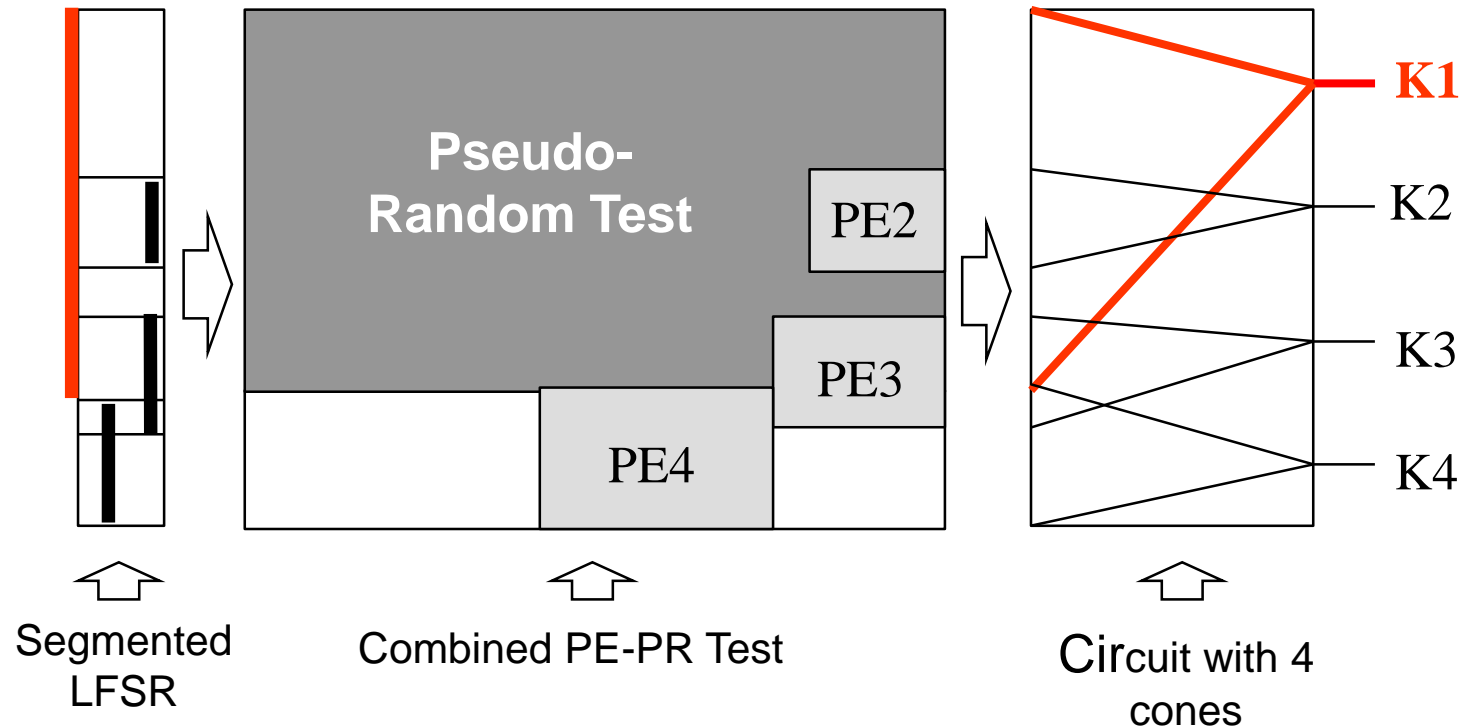
Partial parallelism

Exhaustive testing - 16

Pseudo-exhaustive, full parallel – 4 (not possible)

Pseudo-exhaustive, partially parallel - 6

Combined Pseudo-Exhaustive-Random Testing



A set of Partial Pseudo-Exhaustive tests can be combined with

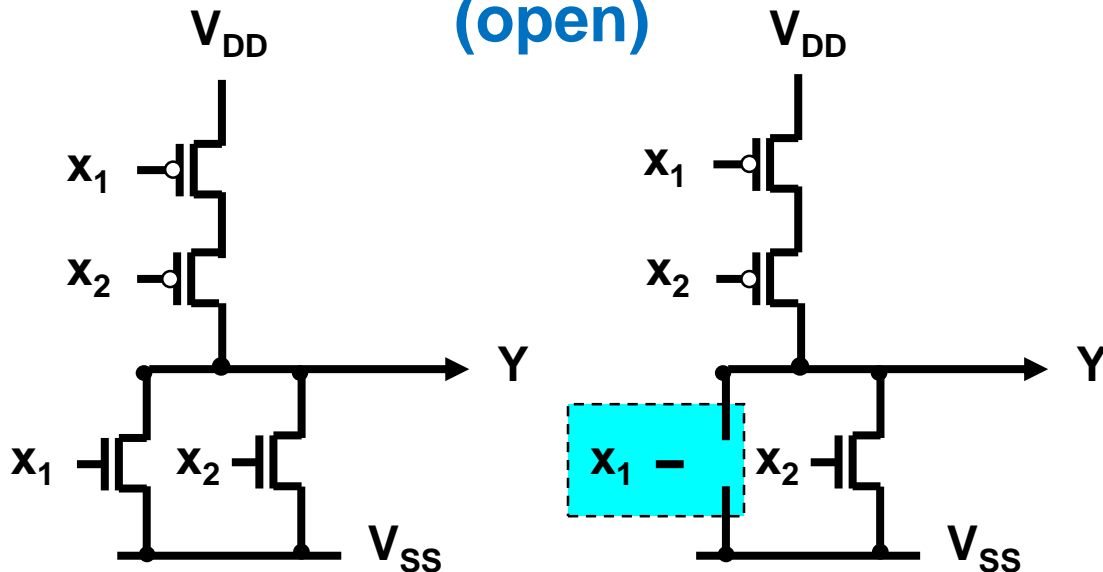
- (1) Pseudorandom BIST or
- (2) Stored Deterministic test set

Problems with Exhaustive Testing

Problem: Sequential fault class - **Transistor Level Stuck-off Faults**

NOR gate test:

**Stuck-off
(open)**



No conducting path from V_{DD} to V_{SS} for "10"

x_1	x_2	y	y^d
0	0	1	1
0	1	0	0
1	0	0	Y'
1	1	0	0

**Test sequence
is needed:
00,10**

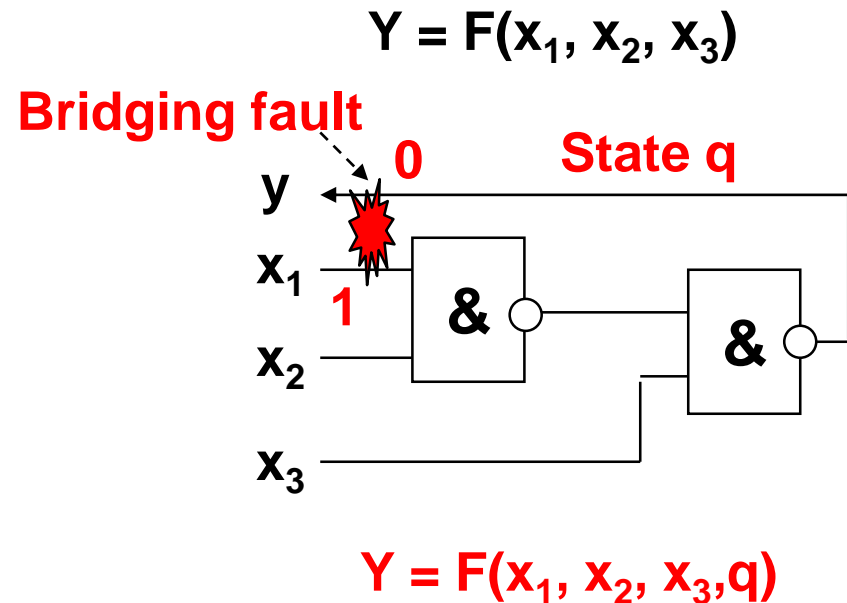
Problems with Exhaustive Testing

Problem: Sequential fault class - **Bridging Fault Sequentiality**

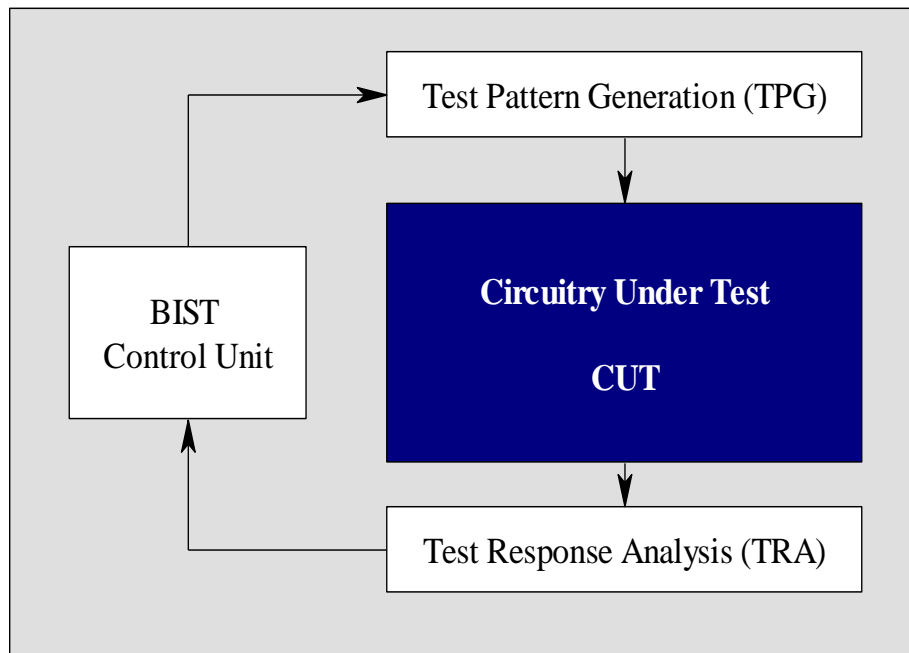
A short will change the circuit into sequential one,
and you will need because of that

$2^4 = 16$ input patterns

Instead of $2^3 = 8$

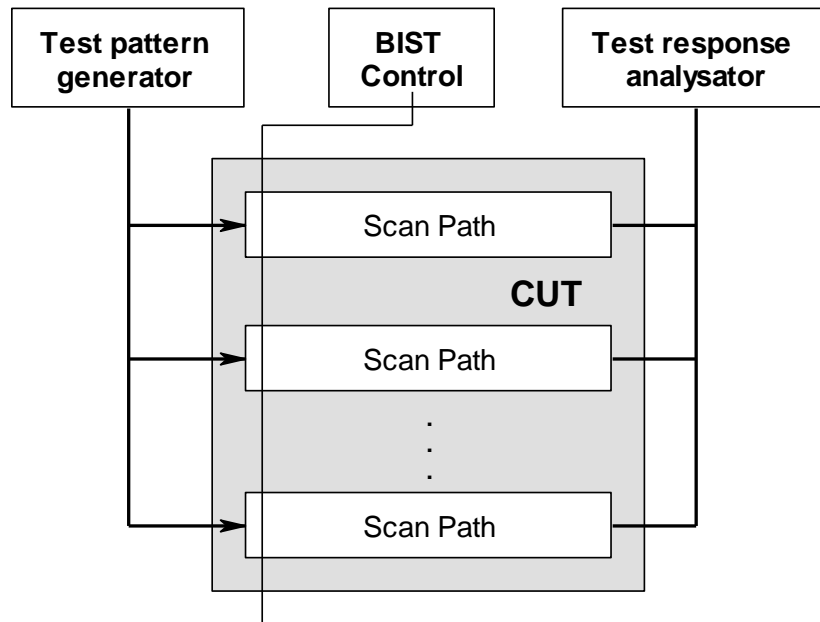


General Architecture of BIST



- **BIST components:**
 - Test pattern generator (TPG)
 - Test response analyzer (TRA)
- **TPG & TRA are usually implemented as linear feedback shift registers (LFSR)**
- **Two widespread schemes:**
 - test-per-scan
 - test-per-clock

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Test per Scan:

Initial test set:

T1: 1100

T2: 1010

T3: 0101

T4: 1001

Test application:

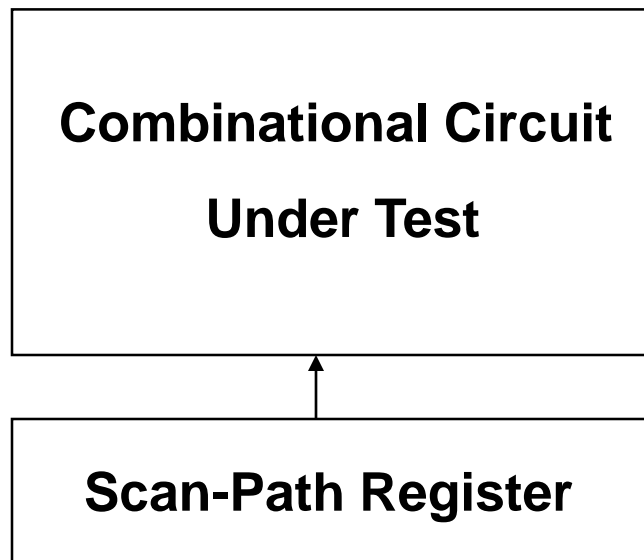
1100 **T** 1010 **T** 0101**T** 1001 **T**

Number of clocks = $(4 \times 4) + 4 = 20$

- Assumes existing scan architecture
- Drawback:
 - Long test application time

Built-In Self-Test

Test per Clock:



- **Initial test set:**

- T1: 1100
- T2: 1010
- T3: 0101
- T4: 1001

Assume, this is
the full test
sequence needed

- **Test application:**

- 1 100 1010 01 01 1001

T₁ T₄ T₃ T₂

- **Number of clocks = 8 < 20**

Pattern Generation

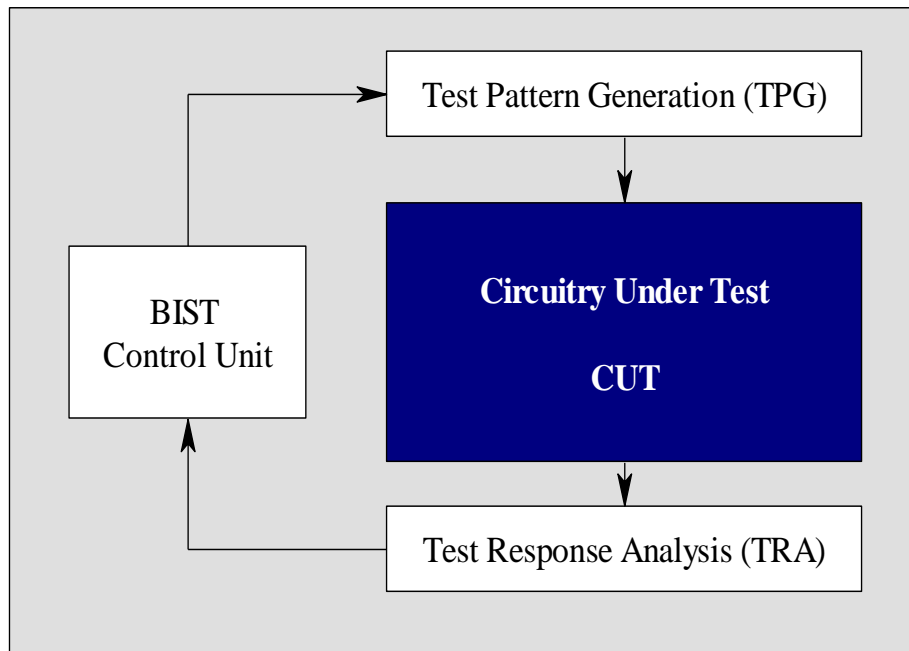
- Store in ROM – too expensive
- *Exhaustive* – too long
- *Pseudo-exhaustive* – preferred
- *Pseudo-random (LFSR)* – preferred
- Binary counters – use more hardware than LFSR
- Modified counters
- Test pattern *augmentation* (Hybrid BIST)
 - LFSR combined with a few patterns in ROM
 - LFSR with bit flipping
 - LFSR with bit fixing

LFSR Based Testing: Some Definitions

- ***Exhaustive testing*** – Apply all possible 2^n patterns to a circuit with n inputs
- ***Pseudo-exhaustive testing*** – Break circuit into small blocks (overlapping if needed) and test each exhaustively
- ***Pseudo-random testing*** – Algorithmic pattern generator that produces a subset of all possible tests with most of the properties of randomly-generated patterns
- **LFSR** – ***Linear feedback shift register***, hardware that generates pseudo-random pattern sequence
- **BILBO** – ***Built-in logic block observer***, extra hardware added to flip-flops so they can be **reconfigured** as an LFSR pattern generator or response compacter, a scan chain, or as flip-flops

Pattern Generation

Pseudorandom test generation by LFSR:

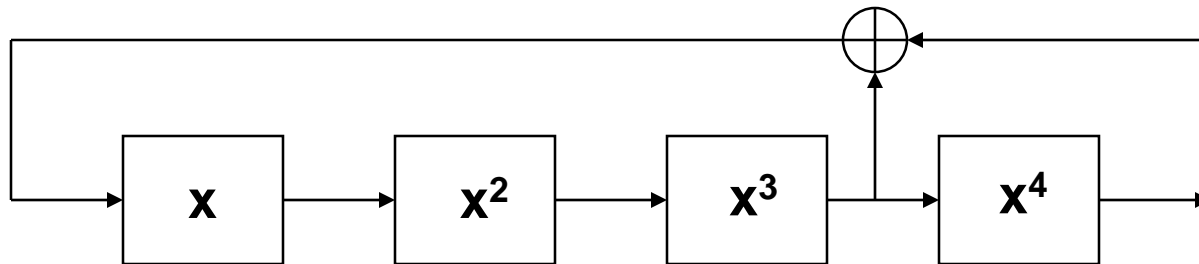


- **Using special LFSR registers**
 - Test pattern generator
 - Signature analyzer
- **Several proposals:**
 - BILBO
 - CSTP
- **Main characteristics of LFSR:**
 - polynomial
 - initial state
 - test length

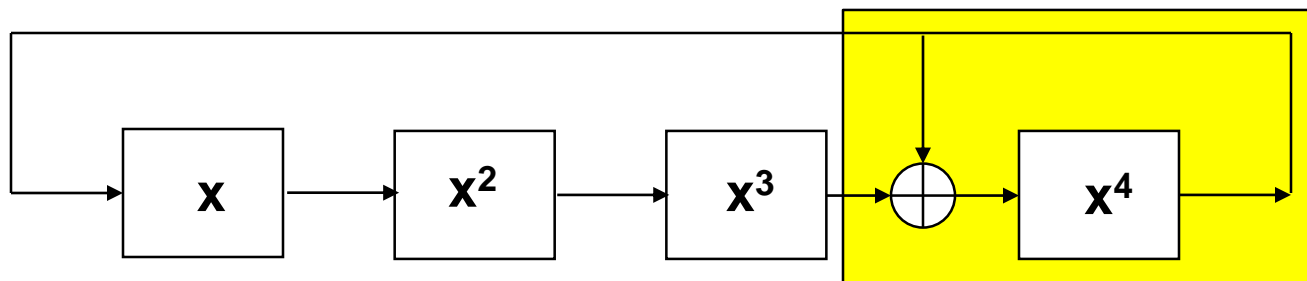
Pseudorandom Test Generation

LFSR – Linear Feedback Shift Register:

Standard LFSR



Modular LFSR

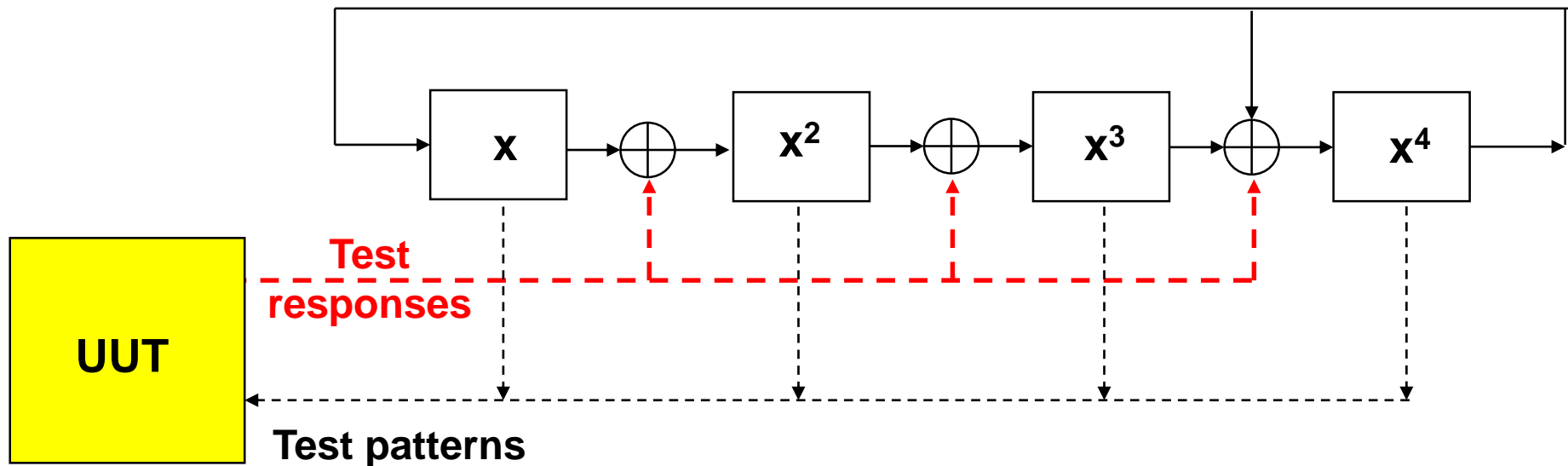


Polynomial: $P(x) = x^4 + x^3 + 1$

Pseudorandom Test Generation

LFSR – Linear Feedback Shift Register:

Why modular LFSR is useful for BIST?

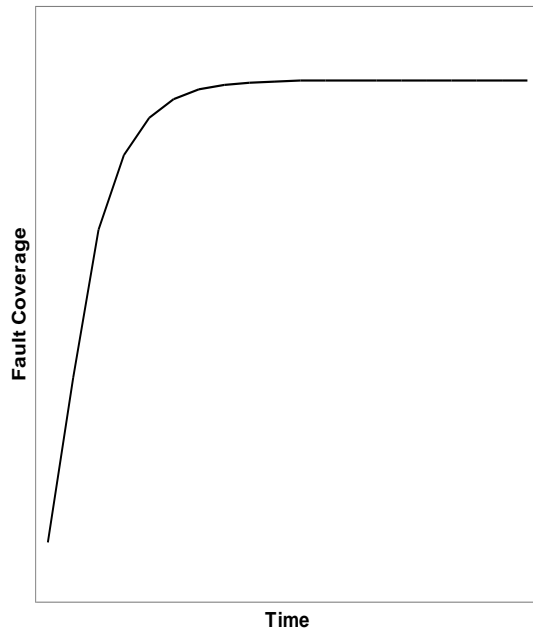


Polynomial: $P(x) = x^4 + x^3 + 1$

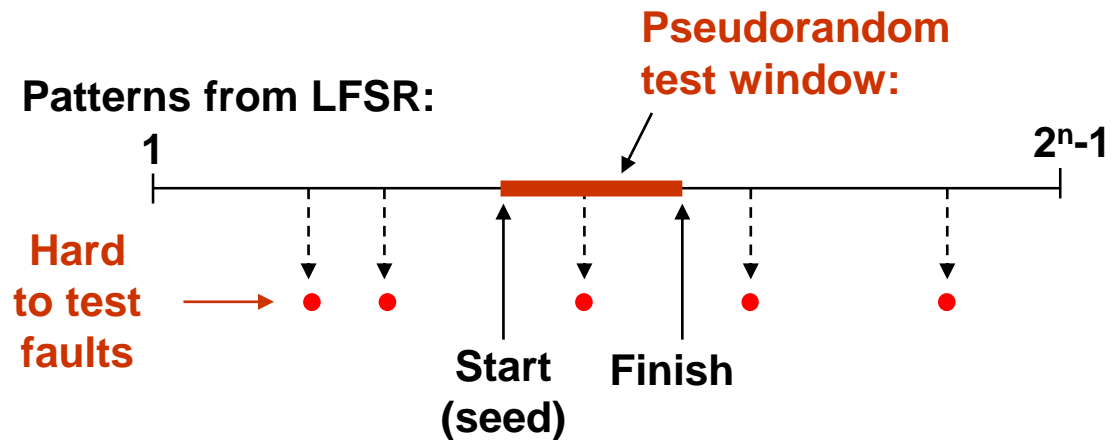
Problems with BIST: Hard to Test Faults

The main motivations of using random patterns are:

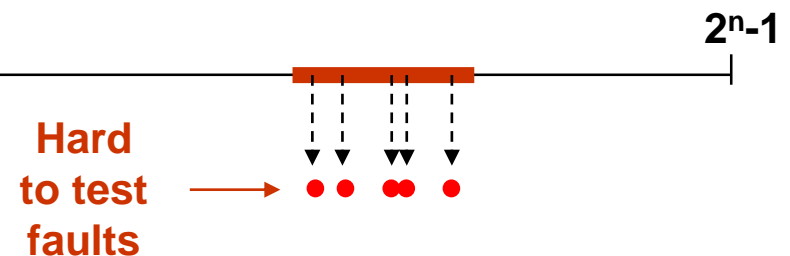
- low generation cost
- high initial efficiency



Problem: Low fault coverage

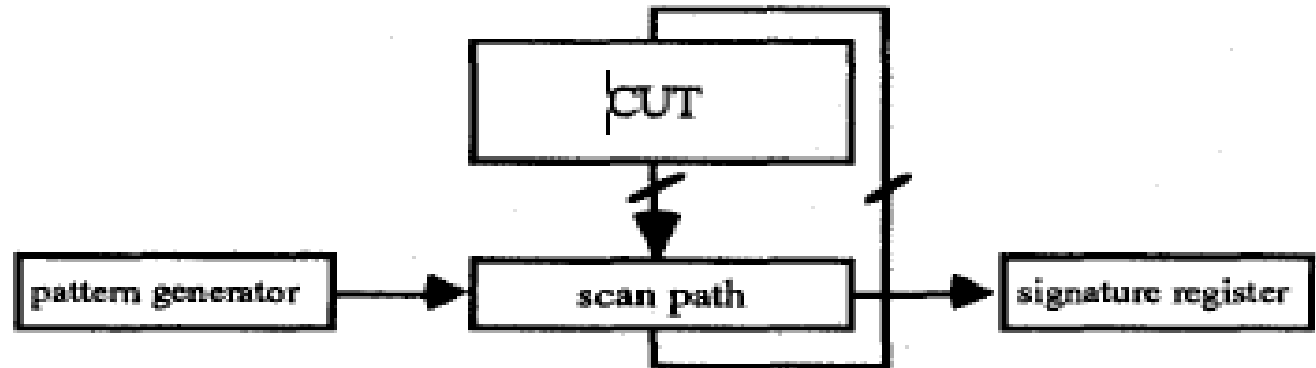


Dream solution: Find LFSR such that:

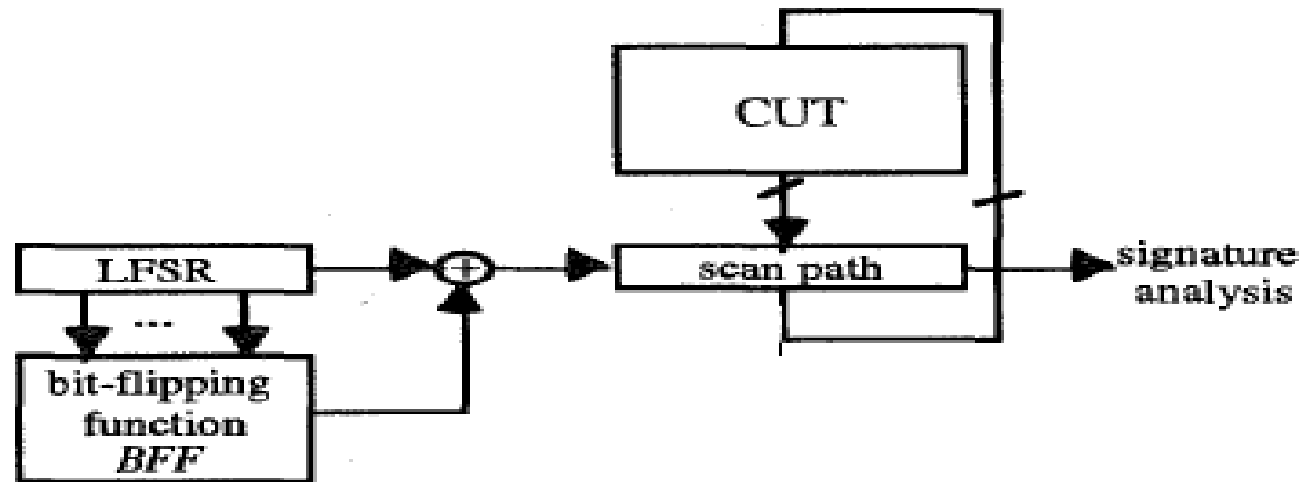


Pseudorandom Test Generation

Scan-based BIST

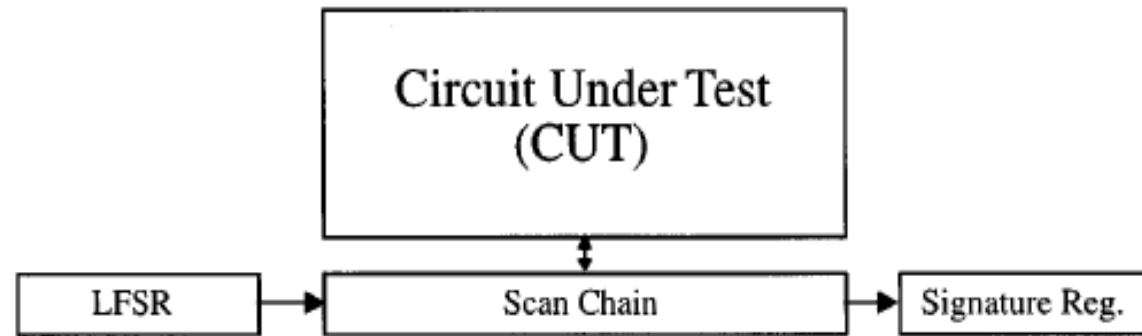


Bit-flipping
BIST



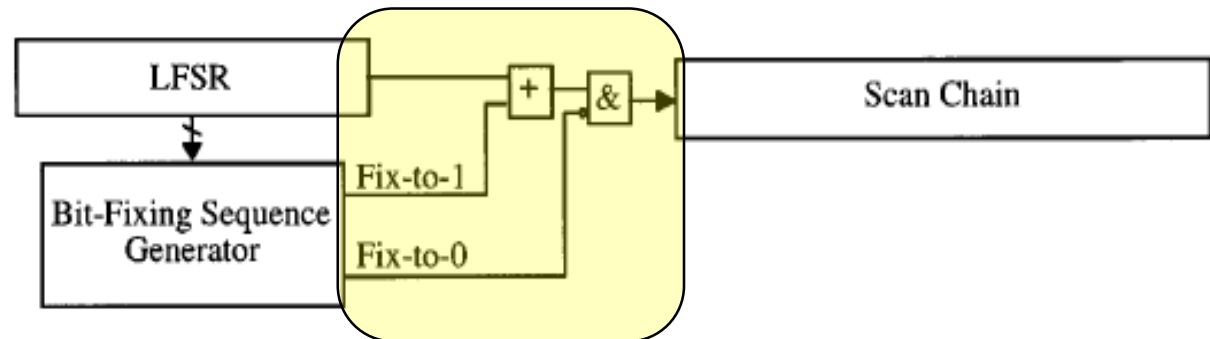
H.-J. Wunderlich, G. Kiefer. Bit flipping BIST. Proc. ICCAD, Nov. 1996, pp.337-343.

Pseudorandom Test Generation



Block diagram for a "test-per-scan" BIST scheme.

Bit-fixing BIST



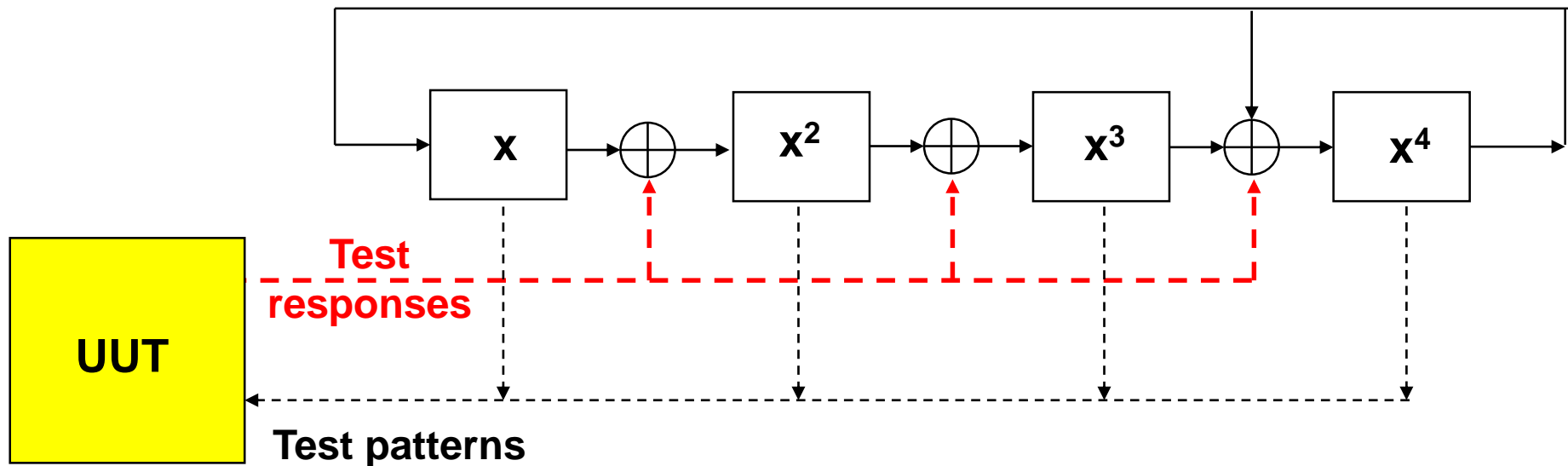
Logic for altering the pseudorandom bit sequence.

N.A. Toubas, E.J. McCluskey. Bit-fixing in pseudorandom sequences for scan BIST. IEEE Trans. on CAD of IC and Systems, Vol.20, No.4, Apr.2001.

Pseudorandom Test Generation

LFSR – Linear Feedback Shift Register:

Why modular LFSR is useful for BIST?



Polynomial: $P(x) = x^4 + x^3 + 1$

BILBO BIST Architecture

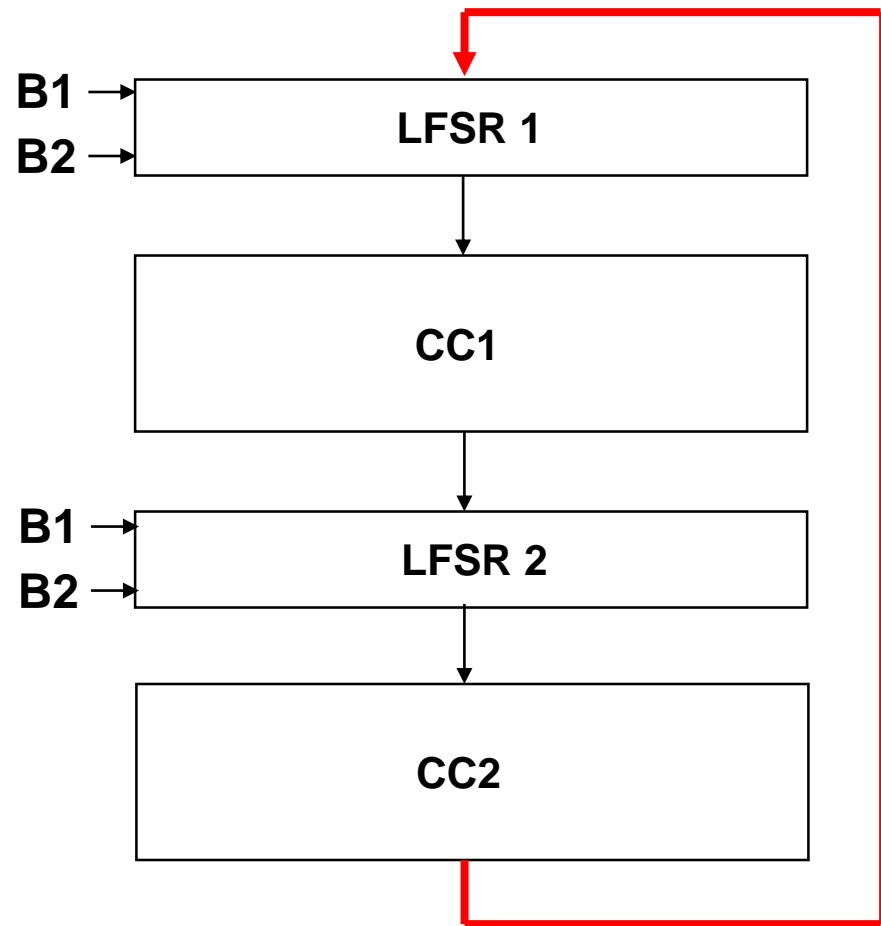
Working modes:

B1 B2

0	0	Normal mode
0	1	Reset
1	0	Test mode
1	1	Scan mode

Testing modes:

CC1:	LFSR 1 - TPG
	LFSR 2 - SA
CC2:	LFSR 2 - TPG
	LFSR 1 - SA



BILBO BIST Architecture

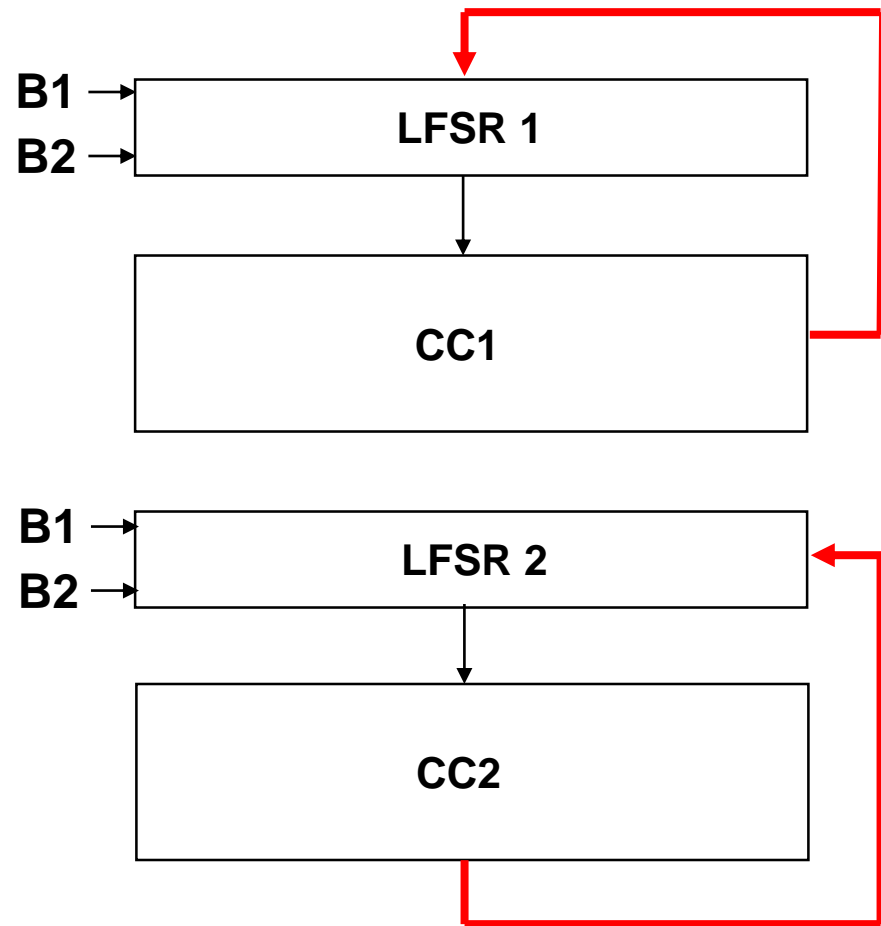
Working modes:

B1	B2	
0	0	Normal mode
0	1	Reset
1	0	Test mode
1	1	Scan mode

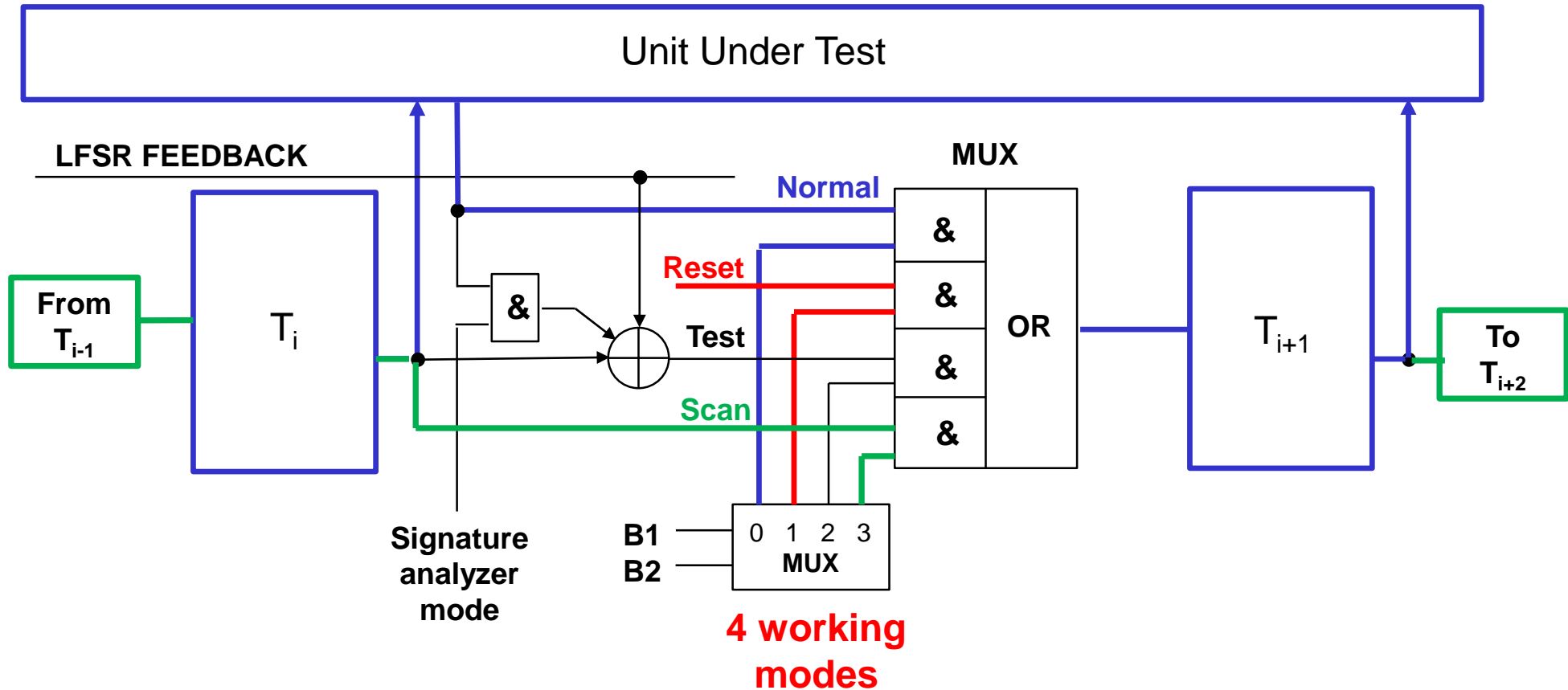
Testing modes:

CC1, CC2 Tested in parallel:

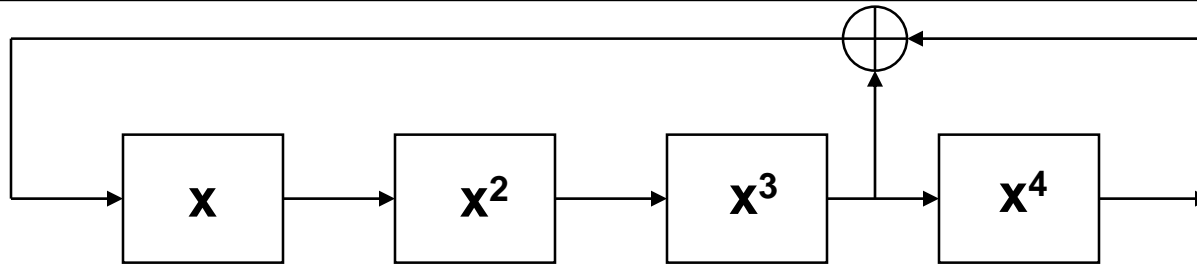
LFSR 1 }
LFSR 2 } **TPG + SA**



Reconfiguration of the LFSR



Pseudorandom Test Generation - LFSR



Two approaches to LFSR simulation:

Polynomial: $P(x) = x^4 + x^3 + 1$

Matrix calculation:

$$\begin{pmatrix} X_4(t+1) \\ X_3(t+1) \\ X_2(t+1) \\ X_1(t+1) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & h_3 & h_2 & h_1 \end{pmatrix} \begin{pmatrix} X_4(t) \\ X_3(t) \\ X_2(t) \\ X_1(t) \end{pmatrix} = \begin{pmatrix} X_3 \\ X_2 \\ X_1 \\ X_4 \oplus X_3 \end{pmatrix}$$

The matrix is annotated with "Shift" pointing to the first three rows and "Feedback" pointing to the last row. The feedback coefficients h_3, h_2, h_1 are shown as 1, 0, 0 with dashed arrows pointing to the matrix elements.

t	x	x ²	x ³	x ⁴	t	x	x ²	x ³	x ⁴
1	0	0	0	1	9	0	1	0	1
2	1	0	0	0	10	1	0	1	0
3	0	1	0	0	11	1	1	0	1
4	0	0	1	0	12	1	1	1	0
5	1	0	0	1	13	1	1	1	1
6	1	1	0	0	14	0	1	1	1
7	0	1	1	0	15	0	0	1	1
8	1	0	1	1	16	0	0	0	1

Theory of LFSR: Primitive Polynomials

Properties of Polynomials:

- **Irreducible polynomial** – cannot be factored, is divisible only by itself
- Any polynomial with all even exponents can be factored and hence is **reducible**
- Irreducible polynomial of degree n is characterized by:
 - An odd number of terms including 1 term $x^3 + x^2 + 1$
 - Divisibility into $x^k + 1$, where $k = 2^n - 1$ $x^7 + 1$
- An irreducible polynomial of degree n is **primitive** if it divides the polynomial $x^k + 1$ for $k = 2^n - 1$, but not for any smaller positive integer k

Theory of LFSR: Examples

Polynomials of degree $n=3$ (examples):

$$k = 2^n - 1 = 2^3 - 1 = 7$$

Primitive polynomials:

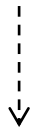
$$x^3 + x^2 + 1$$

$$x^3 + x + 1$$

The polynomials will divide evenly the polynomial $x^7 + 1$ but not any one of $k < 7$, hence, they are primitive

They are also **reciprocal**: coefficients are 1011 and 1101

Reducible polynomials (non-primitive):



$$x^3 + 1 = (x + 1)(x^2 + x + 1)$$

$$x^3 + x^2 + x + 1 = (x + 1)(x^2 + 1)$$

Primitive polynomial



Theory of LFSR: Examples

Is $x^4 + x^2 + 1$ a primitive polynomial?

Divisibility check:

Irreducible polynomial of degree n is characterized by:

- An odd number of terms including 1 term?

Yes, it includes 3 terms

- Divisibility into $1 + x^k$, where $k = 2^n - 1$

No, there is remainder

$x^4 + x^2 + 1$ is non-primitive?

$x^4 + x^2 + 1$	$x^{11} + x^9 + x^5 + x^3$ <hr style="border: 0.5px solid black;"/> $x^{15} + 1$ <hr style="border: 0.5px solid black;"/> $x^{15} + x^{13} + x^{11}$ <hr style="border: 0.5px solid black;"/> $x^{13} + x^{11} + 1$ <hr style="border: 0.5px solid black;"/> $x^{13} + x^{11} + x^9$ <hr style="border: 0.5px solid black;"/> $x^9 + 1$ <hr style="border: 0.5px solid black;"/> $x^9 + x^7 + x^5$ <hr style="border: 0.5px solid black;"/> $x^7 + x^5 + 1$ <hr style="border: 0.5px solid black;"/> $x^7 + x^5 + x^3$ <hr style="border: 0.5px solid black;"/> $x^3 + 1$
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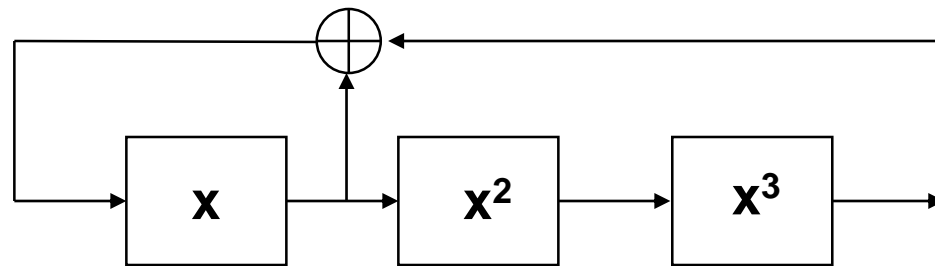
Theory of LFSR: Examples

Simulation of the behaviour of LFSR by polynomial:

Primitive polynomials

$$x^3 + x + 1$$

100
110
111
011
101
010
001
100



Theory of LFSR: Examples

Comparison of test sequences generated:

Primitive polynomials

$$x^3 + x + 1$$



100

110

111

011

101

010

001

100

$$x^3 + x^2 + 1$$

100

010

101

110

111

011

001

100

Non-primitive polynomials

$$x^3 + 1$$

100

010

001

100

010

001

100

010

$$x^3 + x^2 + x + 1$$

100

110

011

001

100

110

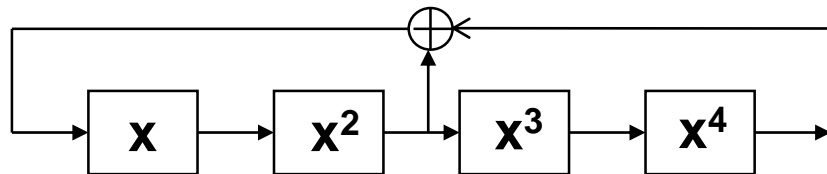
011

001

Theory of LFSR: Examples

Non-primitive polynomial

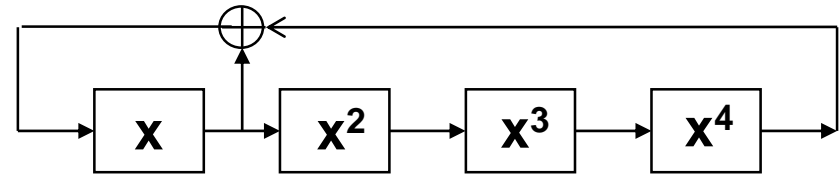
$$x^4 + x^2 + 1$$



0001	1001	0110
1000	1100	1011
0100	1110	1101
1010	1111	0110
0101	0111	
0010	0011	
0001	1001	

Primitive polynomial

$$x^4 + x + 1$$

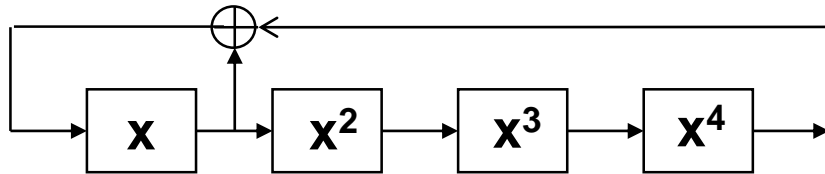


0001	1011	1001
1000	0101	0100
1100	1010	0010
1110	1101	0001
1111	0110	
0111	0011	

Theory of LFSR: Examples

Primitive polynomial

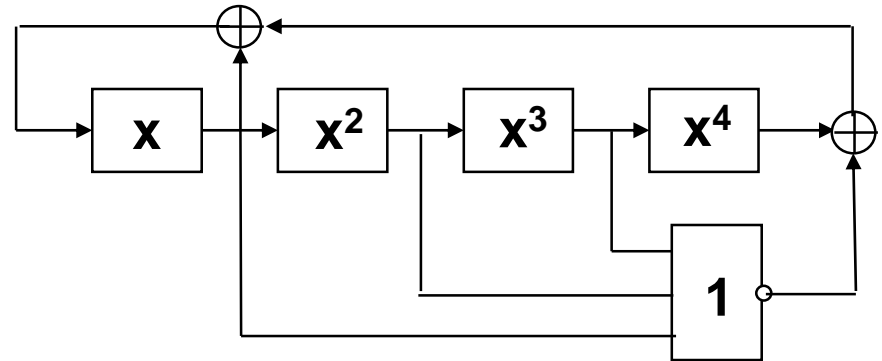
$$x^4 + x + 1$$



0001	1011	1001
1000	0101	0100
1100	1010	0010
1110	1101	0001
1111	0110	
0111	0011	

The code **0000** is missing

Zero generation:



0000	1011	1001
1000	0101	0100
1100	1010	0010
1110	1101	0001
1111	0110	
0111	0011	0000

Pseudorandom Testing with LFSR

Primitive polynomial

$$x^4 + x + 1$$

Red patterns are test patterns

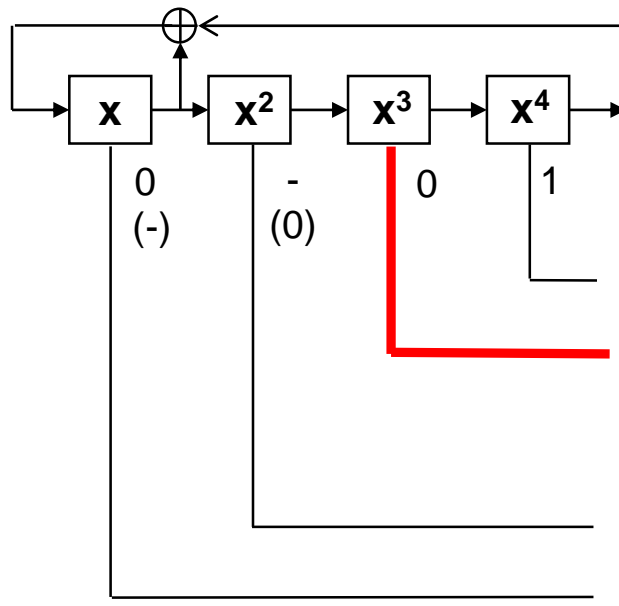


0001	1011	1001
1000	0101	0100
1100	1010	0010
1110	1101	0001
1111	0110	
0111	0011	



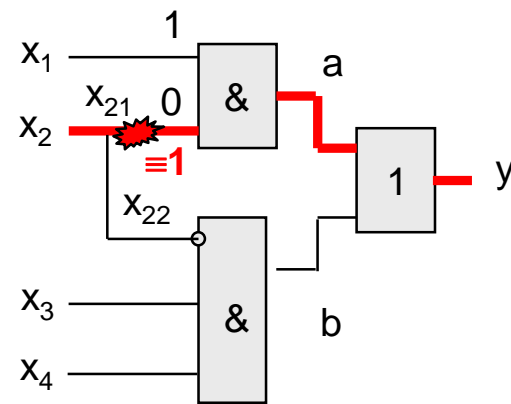
No match in the blue sequence

LFSR



Circuit Under test

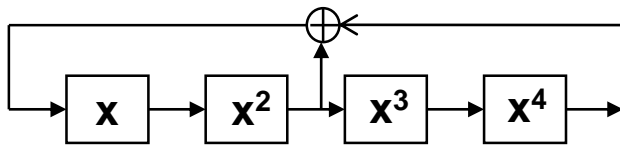
For testing the fault $x_{21} \equiv 1$ the test patterns **0001**, **0101** and **1001** can be used



Pseudorandom Testing with LFSR

Non-primitive polynomial

$$x^4 + x^2 + 1$$



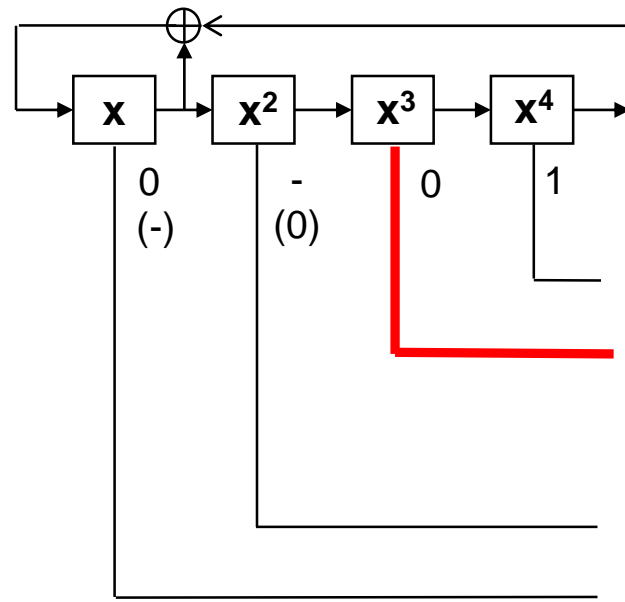
0001	1001	0110
1000	1100	1011
0100	1110	1101
1010	1111	0110
0101	0111	
0010	0011	
0001	1001	



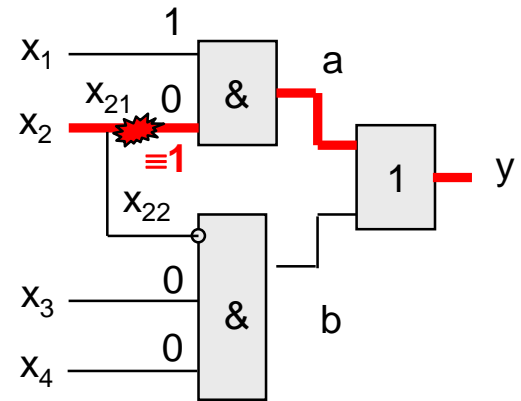
Blue patterns are not testing the fault



Be careful: no proper patterns can be generated using the seed 0110



For testing the fault $x_{21} \equiv 1$
the test patterns **0001**, **0101**
and **1001** can be used



Theory of LFSR: Primitive Polynomials

Number of primitive polynomials of degree N

N	No
1	1
2	1
4	2
8	16
16	2048
32	67108864

Table of primitive polynomials up to degree 31

N	Primitive Polynomials
1,2,3,4,6,7,15,22	$1 + X + X^n$
5,11, 21, 29	$1 + X^2 + X^n$
10,17,20,25,28,31	$1 + X^3 + X^n$
9	$1 + X^4 + X^n$
23	$1 + X^5 + X^n$
18	$1 + X^7 + X^n$
8	$1 + X^2 + X^3 + X^4 + X^n$
12	$1 + X + X^3 + X^4 + X^n$
13	$1 + X + X^4 + X^6 + X^n$
14, 16	$1 + X + X^3 + X^4 + X^n$

Theory of LFSR: Primitive Polynomials

Examples of PP (exponents of terms):

$$x^3 + x + 1$$

n	other				n	other			
1	0				9	4	0		
2	1	0			10	3	0		
3	1	0			11	2	0		
4	1	0			12	7	4	3	0
5	2	0			13	4	3	1	0
6	1	0			14	12	11	1	0
7	1	0			15	1	0		
8	6	5	1	0	16	5	3	2	0

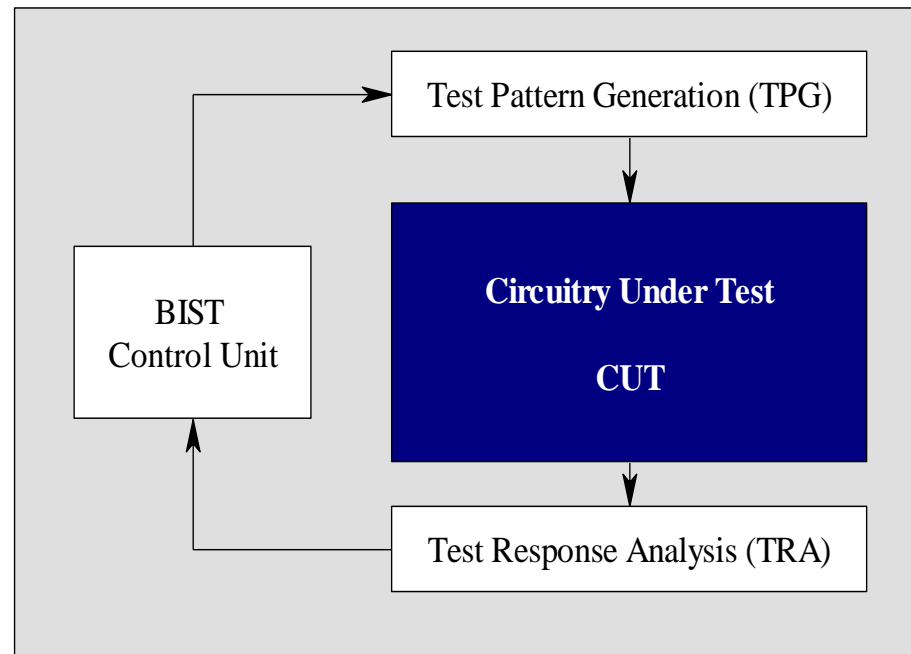
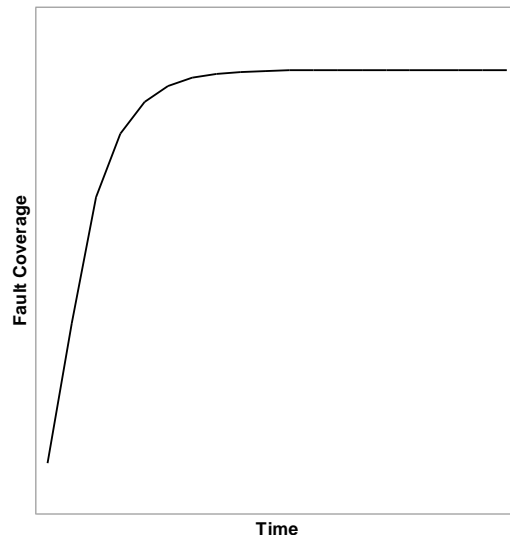
$$x^{13} + x^4 + x^3 + x + 1$$

BIST: Fault Coverage

Pseudorandom Test generation by LFSR:

Motivation for LFSR:

- low generation cost
- high initial efficiency



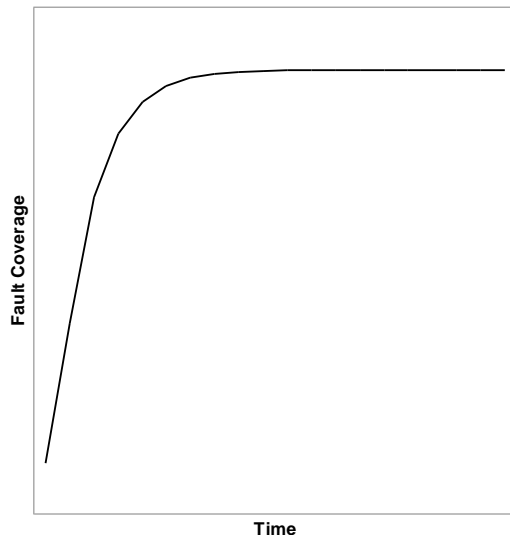
Drawback: 100% fault coverage is difficult to achieve

BIST: Fault Coverage

Pseudorandom Test generation by LFSR:

Motivation for LFSR:

- low generation cost
- high initial efficiency



Reasons of the high initial efficiency:

A circuit may implement 2^{2^n} functions

A test vector partitions the functions into 2 equal sized equivalence classes (correct circuit in one of them)

The second vector partitions into 4 classes etc.

After m patterns the fraction of functions distinguished from the correct function is

$$\frac{1}{2^{2^n} - 1} \sum_{i=1}^m 2^{2^n - i}, \quad 1 \leq m \leq 2^n$$

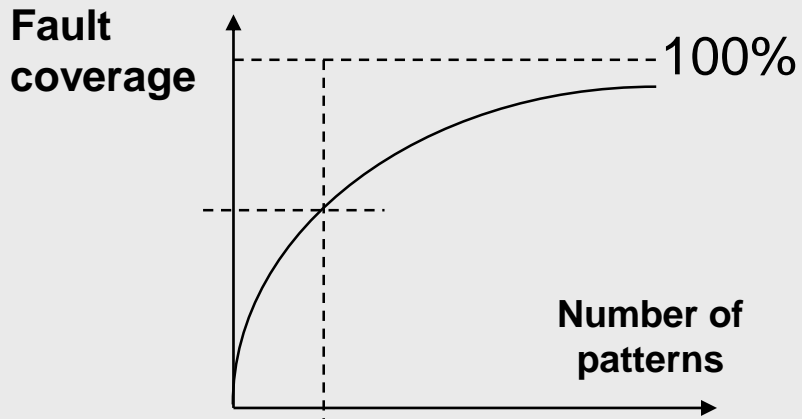
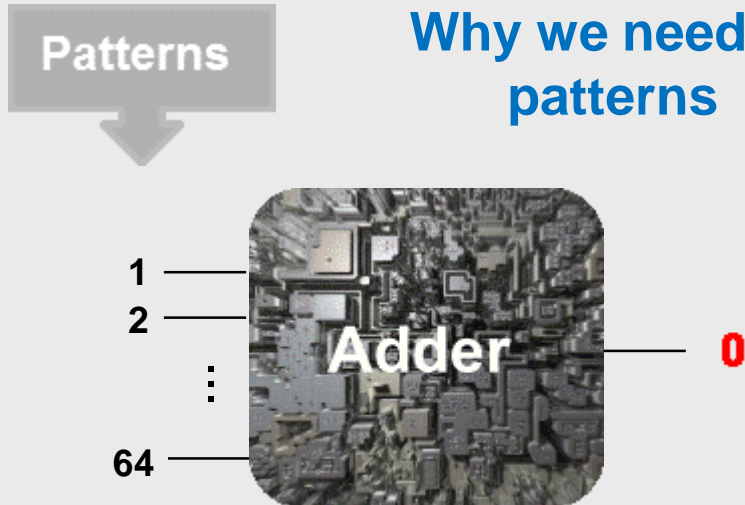
Fault Coverage: Functional View

Truth table for adder:

Patterns	Possible functions
00...000	01 0 1 0 1...101
00...001	00 1 1 1 0...011
00...010	00 1 0 0 1...101
...	...
11...111	00 0 0 0 0...111

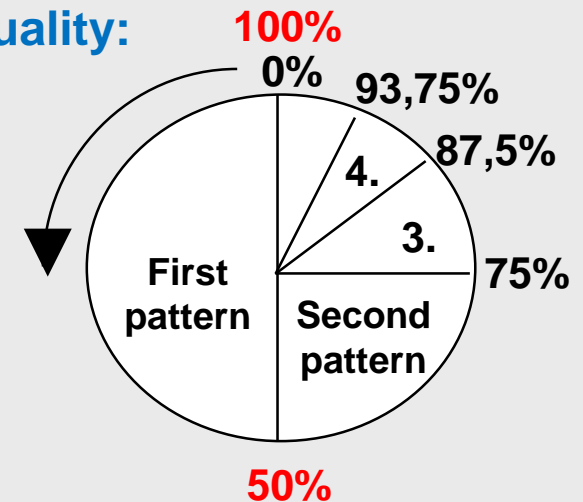
2⁶⁴ (bracketed next to the first three rows)
First pattern (arrow pointing to the yellow row)
Correct function (arrow pointing to the blue column)
50% tested (arrow pointing from the blue column)

Why we need 2⁶⁴ patterns



Test quality:

All columns in truth table can be removed where for yellow pattern the result is **1**



BIST: Fault Coverage

Explanation of the formula of fault coverage:

1) General case:

$$\frac{1}{2^{2^n} - 1} \sum_{i=1}^m 2^{2^n - i}, \quad 1 \leq m \leq 2^n$$

tested functions

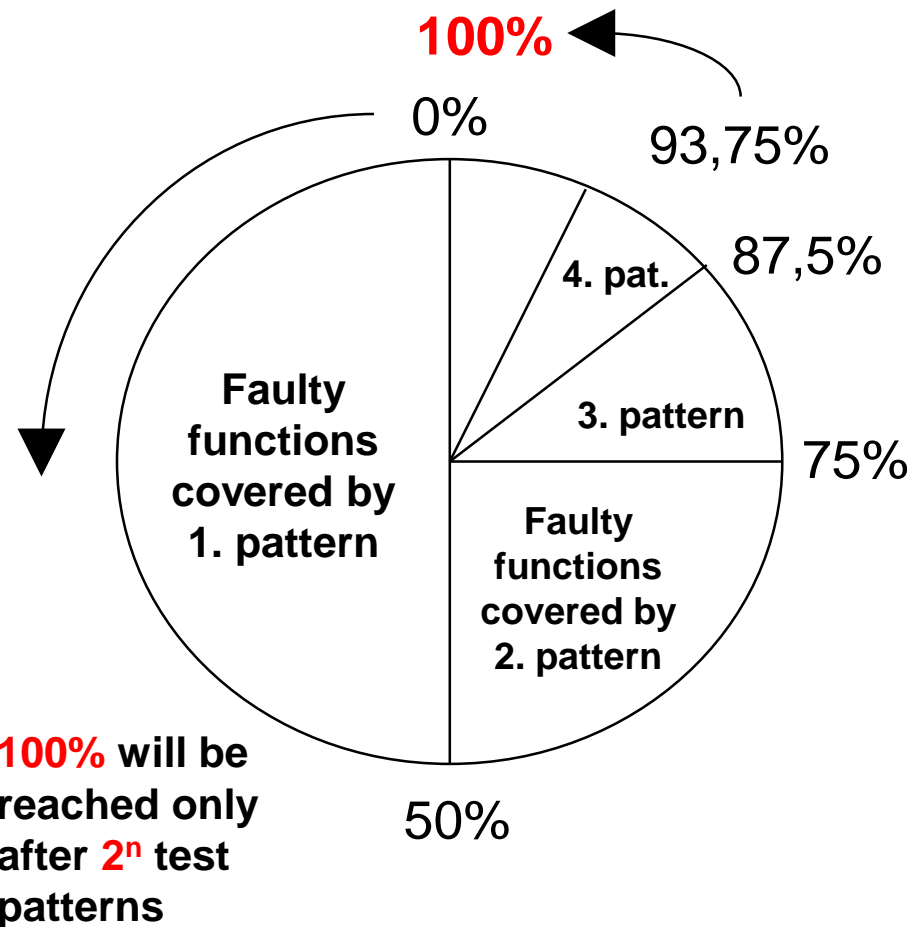
2) Example:

$n = 2, m = 1, i = 1$:

all functions

$$\frac{1}{2^{2^2} - 1} \sum_{i=1}^1 2^{2^2 - i} = \frac{2^3}{15} = \frac{8}{15}$$

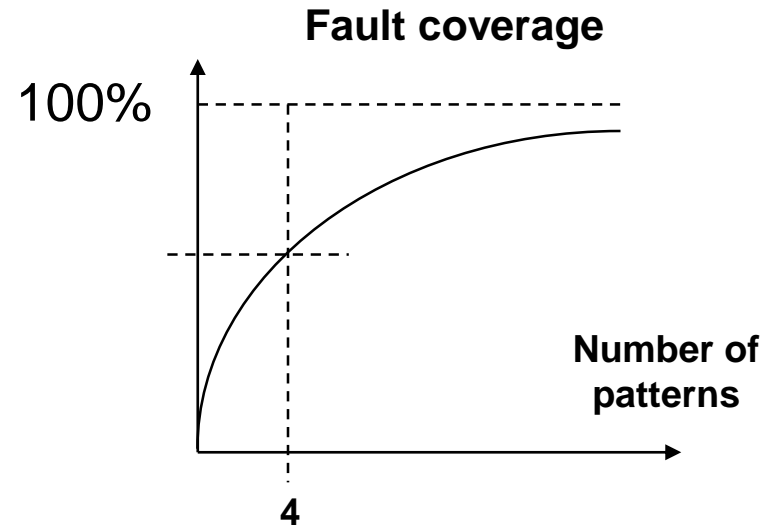
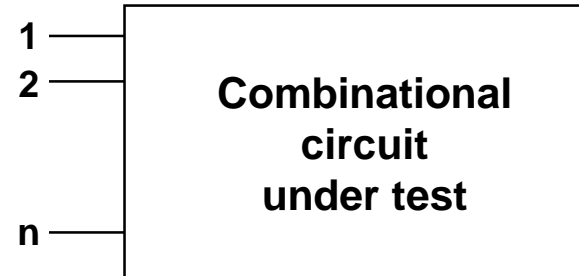
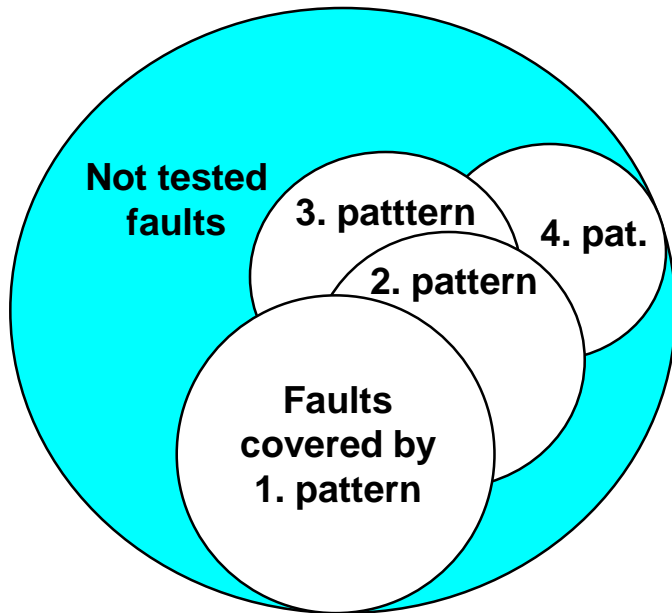
n – number of inputs,
 m – number of test patterns,
 i – share of each pattern



BIST: Structural Approach to Test

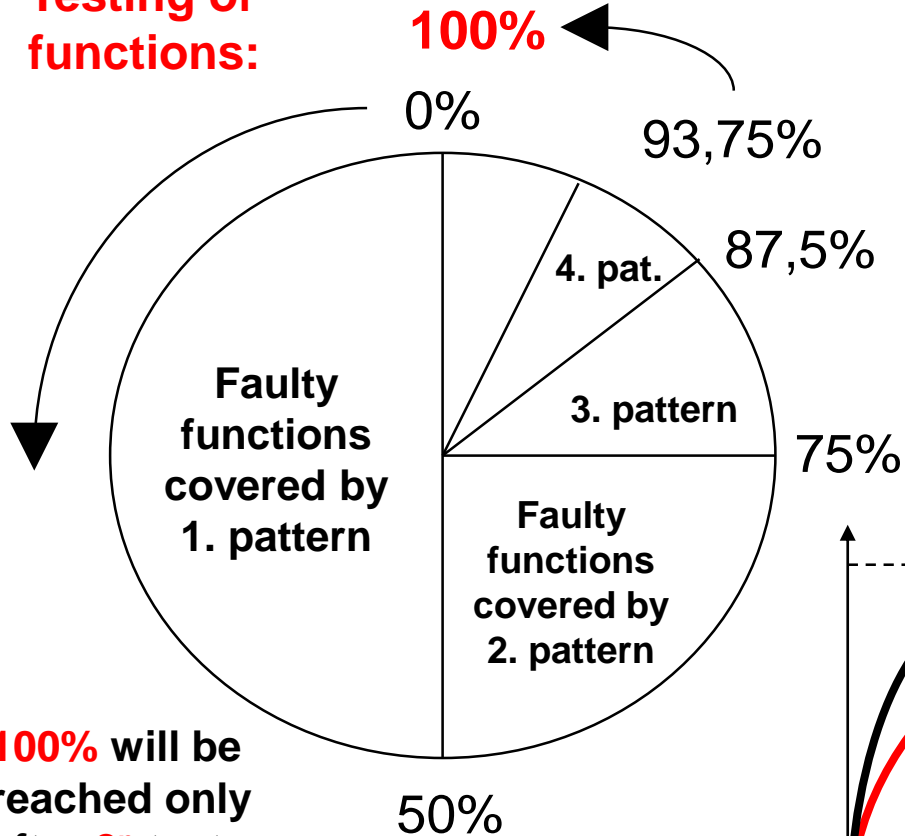
Deterministic test approach:

Testing of structural faults:



BIST: Two Approaches to Test

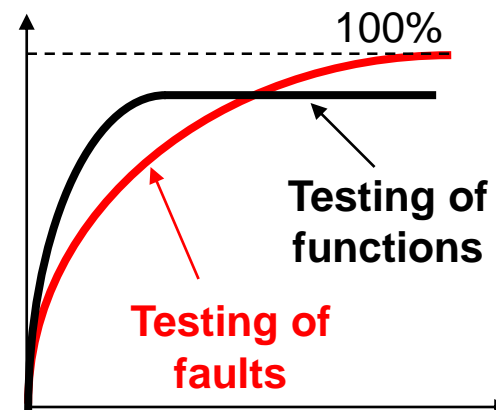
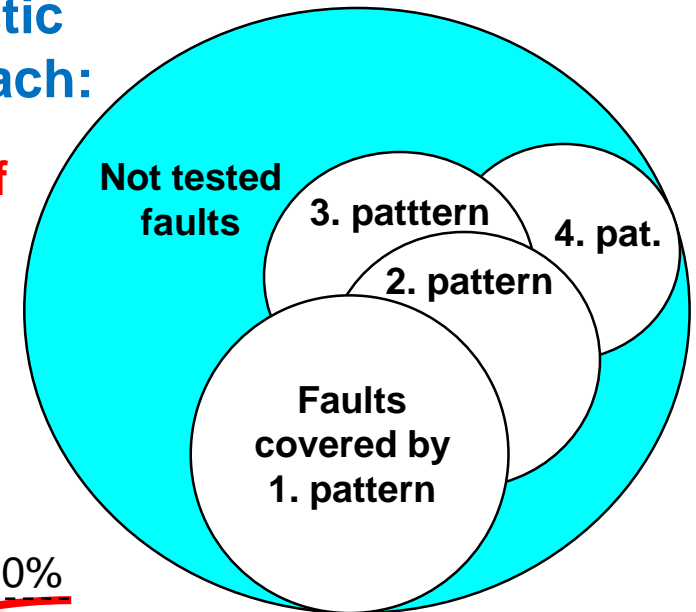
Testing of functions:



100% will be reached only after 2ⁿ test patterns

Deterministic test approach:

Testing of faults:

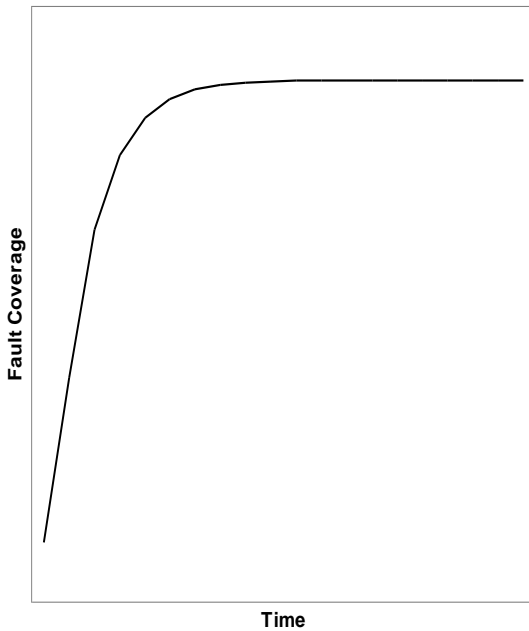


100% will be reached when all faults from the fault list are covered

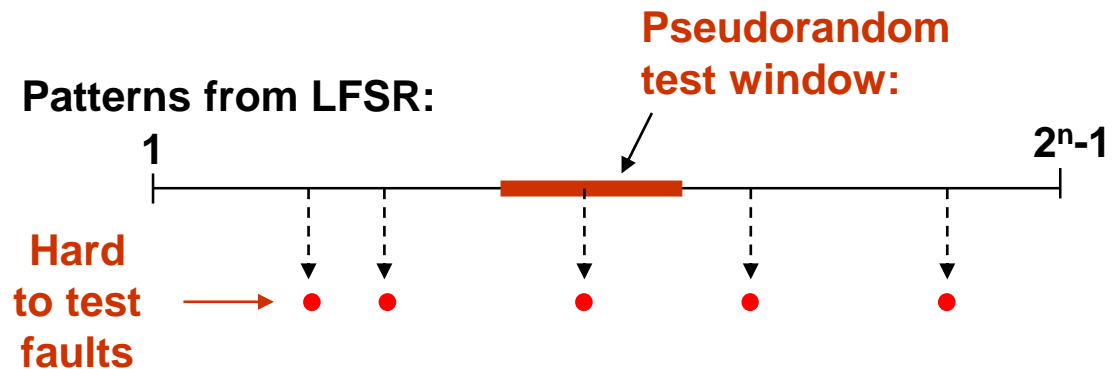
Problems with BIST: Hard to Test Faults

The main motivations of using random patterns are:

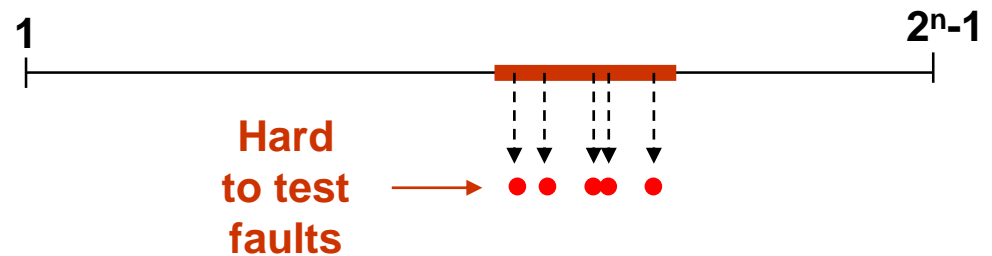
- low generation cost
- high initial efficiency



Problem: **Low fault coverage**

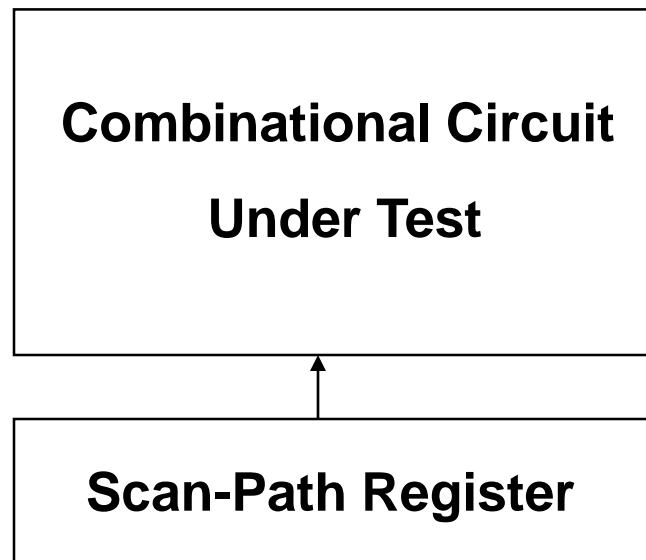


Dream solution: Find LFSR such that:



Deterministic Scan-Path Test

Test per Clock:

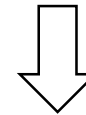


- **Initial test set:**

- T1: 1100
- T2: 1010
- T3: 0101
- T4: 1001



How to generate
the shortest
sequence by
LFSR



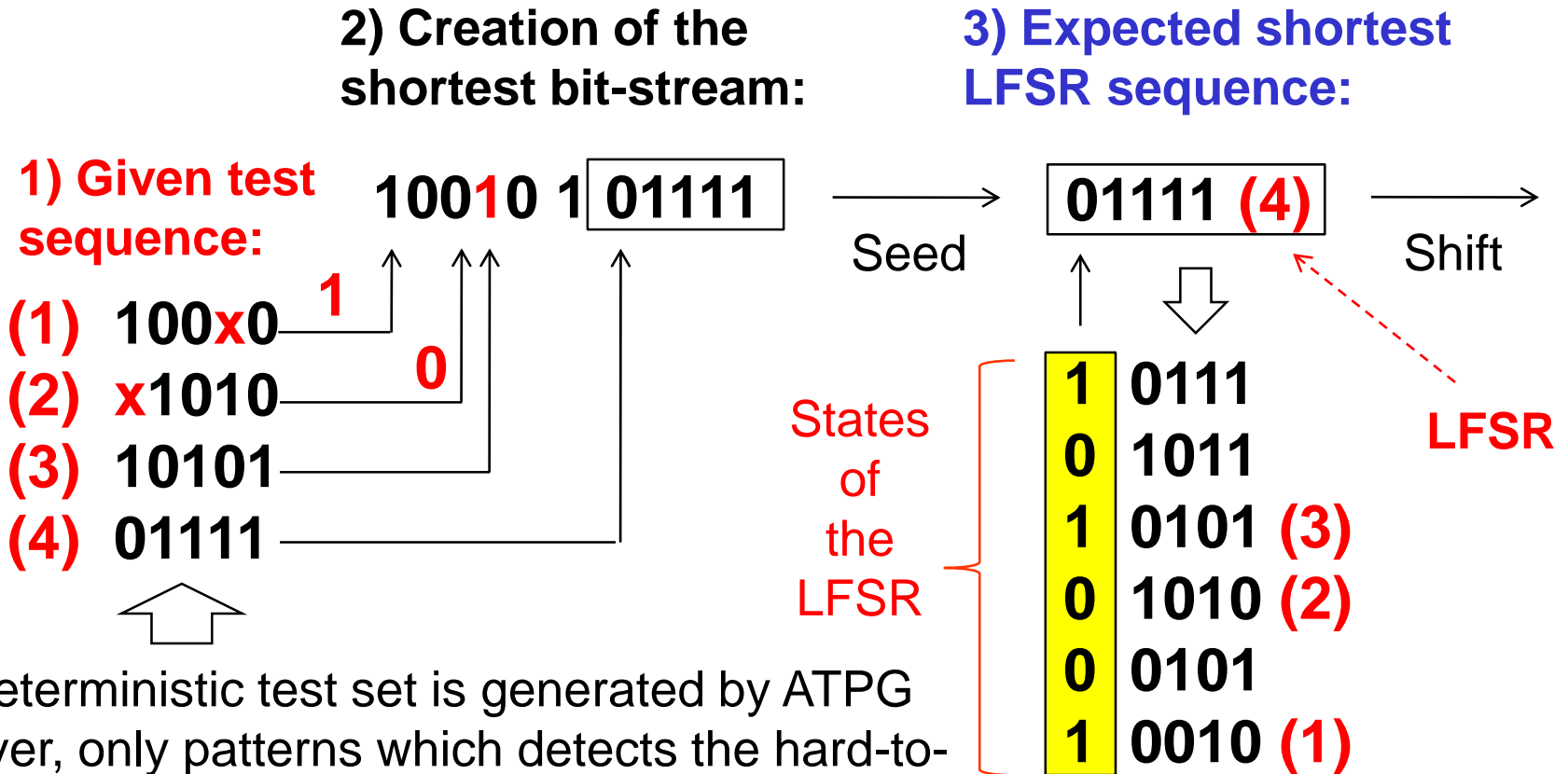
- **Test application:**

- **1 100 1010** 01 01 1001
-
- T₁ T₄ T₃ T₂

- **Number of clocks = 8 < 20**

Deterministic Synthesis of LFSR

Generation of the polynomial and seed for the given test sequence



This deterministic test set is generated by ATPG
 However, only patterns which detects the hard-to-test faults can be chosen

Deterministic Synthesis of LFSR

Generation of the polynomial and seed for the given test sequence

System of linear equations: $a_k x_1 \oplus b_k x_2 \oplus c_k x_3 \oplus d_k x_4 \oplus e_k x_5 = f_k$

Expected shortest LFSR sequence:

01111 (4)

1 0111

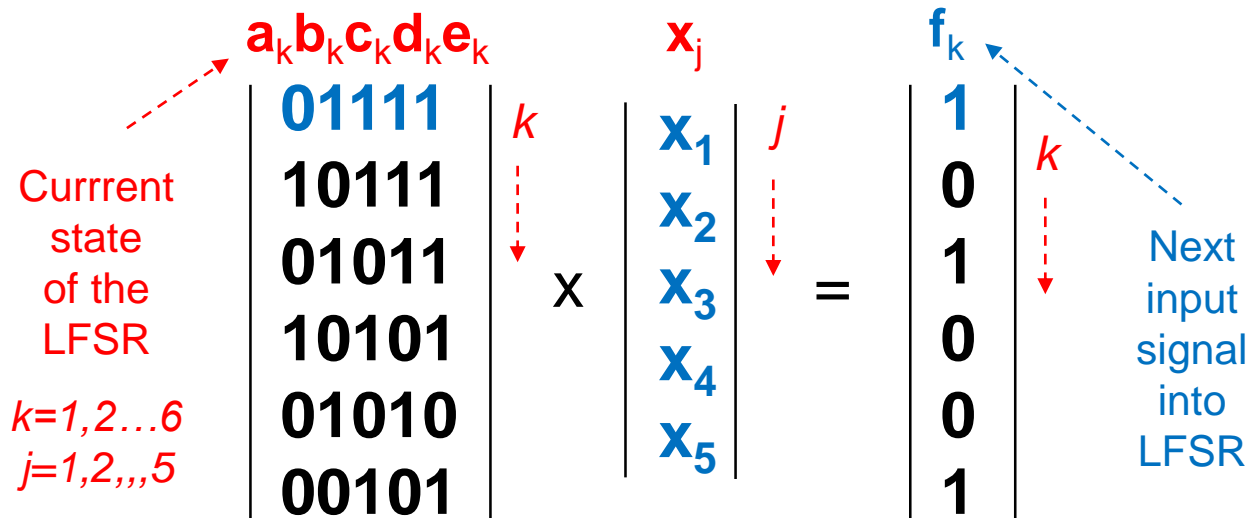
0 1011

1 0101 (3)

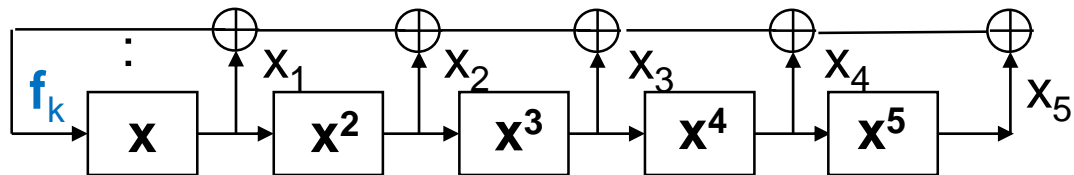
0 1010 (2)

0 0101

1 0010 (1)



We are looking for the values of x_i



Deterministic Synthesis of LFSR

Generation of the polynomial and seed for the given test sequence

System of linear equations:

$$\begin{array}{l}
 1 \\
 2 \\
 3 \\
 4 \\
 5 \\
 6
 \end{array}
 \begin{array}{|c|}
 \hline
 \mathbf{01111} \\
 \hline
 \mathbf{10111} \\
 \hline
 \mathbf{01011} \\
 \hline
 \mathbf{10101} \\
 \hline
 \mathbf{01010} \\
 \hline
 \mathbf{00101} \\
 \hline
 \end{array}
 \times
 \begin{array}{|c|}
 \hline
 \mathbf{x}_1 \\
 \hline
 \mathbf{x}_2 \\
 \hline
 \mathbf{x}_3 \\
 \hline
 \mathbf{x}_4 \\
 \hline
 \mathbf{x}_5 \\
 \hline
 \end{array}
 =
 \begin{array}{|c|}
 \hline
 \mathbf{1} \\
 \hline
 \mathbf{0} \\
 \hline
 \mathbf{1} \\
 \hline
 \mathbf{0} \\
 \hline
 \mathbf{0} \\
 \hline
 \mathbf{1} \\
 \hline
 \end{array}$$

Solving the equation by Gaussian elimination with swapping of rows

Rows:

1,2,4,6
4,6
1,3
2,4
1,3,6

Results:

$$\begin{array}{l}
 \mathbf{01000} \\
 \mathbf{10000} \\
 \mathbf{00100} \\
 \mathbf{00010} \\
 \mathbf{00001} \\
 \mathbf{00001}
 \end{array}
 \times
 \begin{array}{|c|}
 \hline
 \mathbf{x}_1 \\
 \hline
 \mathbf{x}_2 \\
 \hline
 \mathbf{x}_3 \\
 \hline
 \mathbf{x}_4 \\
 \hline
 \mathbf{x}_5 \\
 \hline
 \end{array}
 =
 \begin{array}{|c|}
 \hline
 \mathbf{0} \\
 \hline
 \mathbf{1} \\
 \hline
 \mathbf{0} \\
 \hline
 \mathbf{0} \\
 \hline
 \mathbf{1} \\
 \hline
 \mathbf{1} \\
 \hline
 \end{array}
 \begin{array}{l}
 \mathbf{f}_k \\
 \mathbf{f}_2 \\
 \mathbf{f}_3
 \end{array}$$

$$a_k x_1 \oplus b_k x_2 \oplus c_k x_3 \oplus d_k x_4 \oplus e_k x_5 = f_k, k = 1, 2, \dots, 6$$

Examples: (4) 10101 0 (1) 01111 1

(6) 00101 1 (3) 01011 1

(4 ⊕ 6) 10000 1 ⇒ k=2 (1 ⊕ 3) 00100 0 ⇒ k=3

4) Solution: $x_1 x_2 x_3 x_4 x_5$
1 0 0 0 1

Deterministic Synthesis of LFSR

Generation of the polynomial and seed for the given test sequence

Solving the equation by Gaussian elimination with swapping of rows

$$a_k x_1 \oplus b_k x_2 \oplus c_k x_3 \oplus d_k x_4 \oplus e_k x_5 = f_k, k=1,2,\dots,6$$

$$\begin{aligned} \wedge_{x_1 x_2 x_3 x_4 x_5} 01000 &= 0 \Rightarrow x_2 = 0 \\ \wedge_{x_1 x_2 x_3 x_4 x_5} 10000 &= 1 \Rightarrow x_1 = 1 \\ \wedge_{x_1 x_2 x_3 x_4 x_5} 00100 &= 0 \Rightarrow x_3 = 0 \\ \wedge_{x_1 x_2 x_3 x_4 x_5} 00010 &= 0 \Rightarrow x_4 = 0 \\ \wedge_{x_1 x_2 x_3 x_4 x_5} 00001 &= 1 \Rightarrow x_5 = 1 \end{aligned}$$

$$\begin{array}{c|c|c|c} 01000 & & x_1 & 0 \\ 10000 & & x_2 & 1 \\ 00100 & \times & x_3 & 0 \\ 00010 & & x_4 & 0 \\ 00001 & & x_5 & 1 \\ 00001 & & & 1 \end{array} = \begin{array}{c} f_k \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \end{array}$$

4) Solution: $x_1 x_2 x_3 x_4 x_5$
1 0 0 0 1

Deterministic Synthesis of LFSR

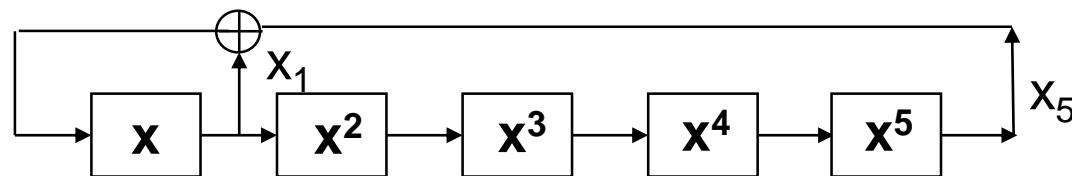
Embedding deterministic test patterns into LFSR sequence:

4) Solution: $x_1 x_2 x_3 x_4 x_5$
 1 0 0 0 1



5) Polynomial: $x^5 + x + 1$ Seed: 01111

LFSR sequence:



Given
deterministic
test
sequence:

(1) 100x0
 (2) x1010
 (3) 10101
 (4) 01111

(1) 01111 (4)

(2) 10111

(3) 01011

(4) 10101 (3)

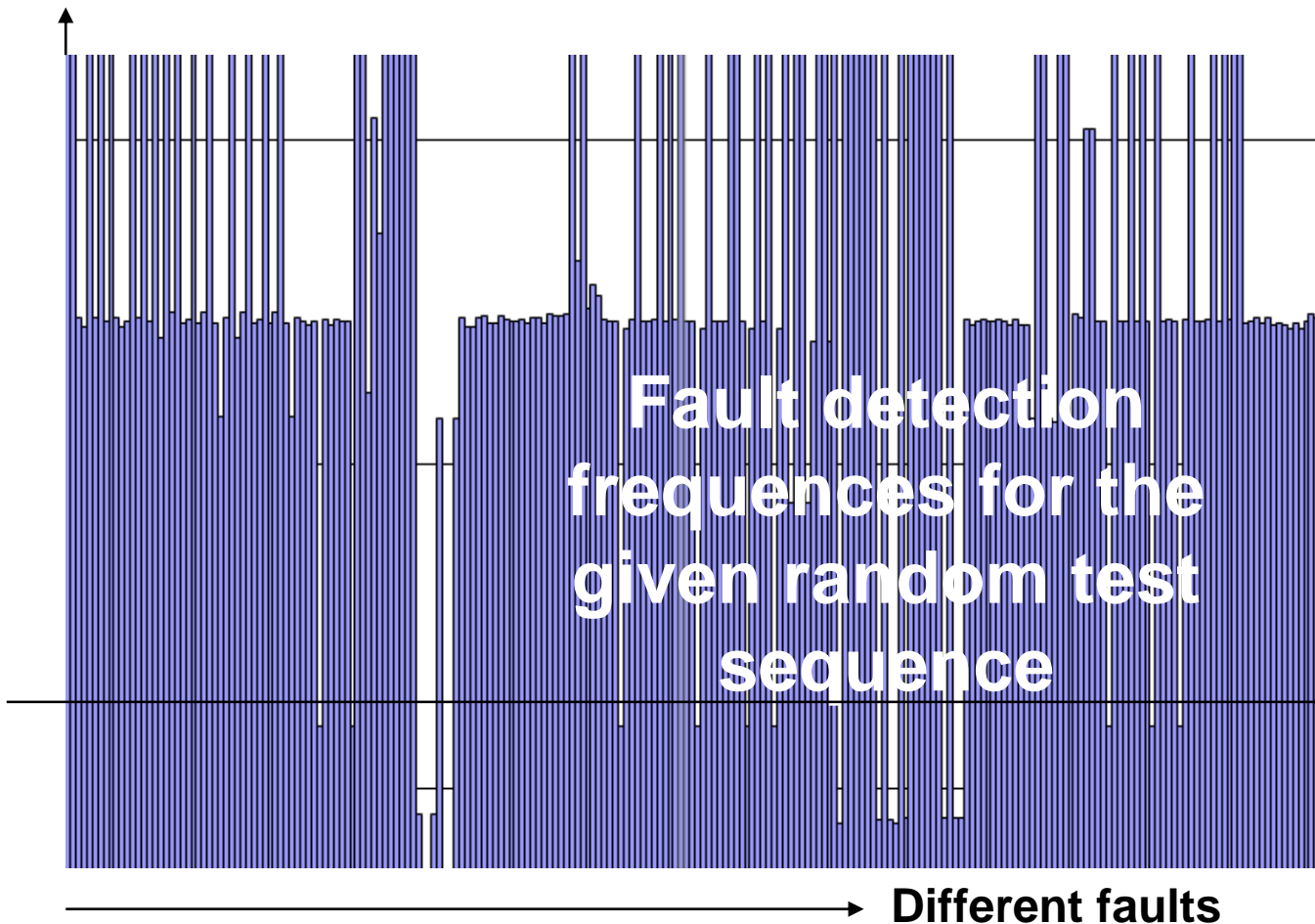
(5) 01010 (2)

(6) 00101

(7) 10010 (1)

Which Test Patterns to Select for as **HTF**?

Frequencies of fault detection



For deterministic LFSR based BIST, **only the patterns** which detects **HTFs** can be chosen for the synthesis process

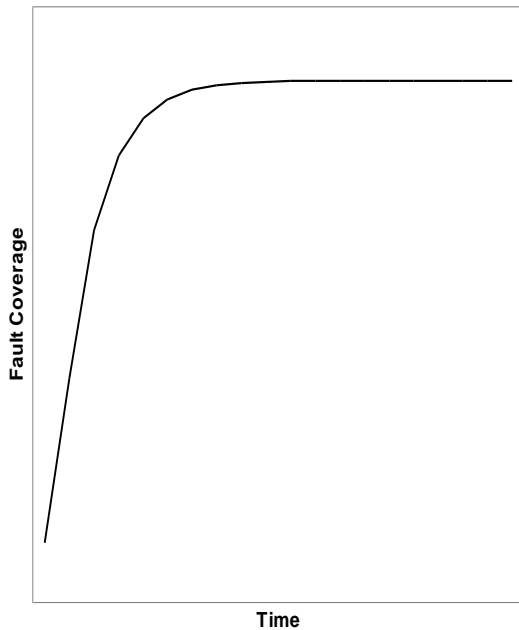
Easily detectable faults

Hard-to-test faults (**HTF**)

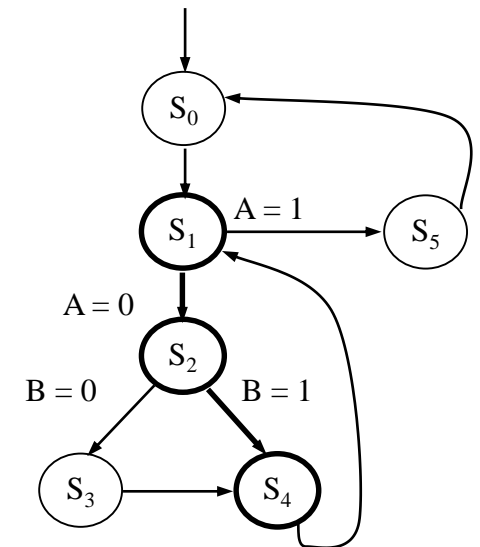
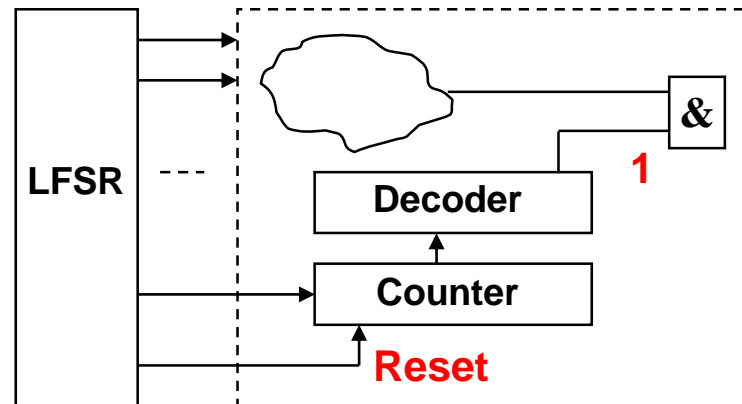
Other Problems with Pseudorandom Test

The main motivations of using random patterns are:

- low generation cost
- high initial efficiency



Problem: **low fault coverage**

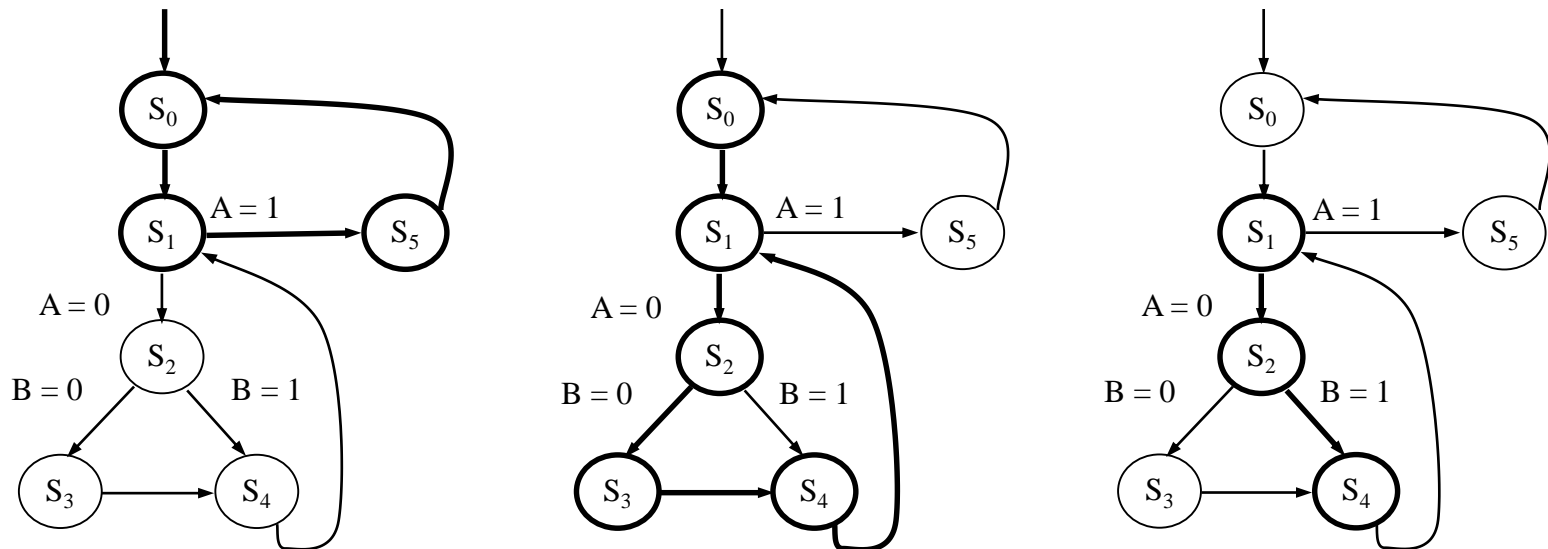


If **Reset = 1** signal has probability 0,5 then counter will not work and 1 for AND gate may never be produced

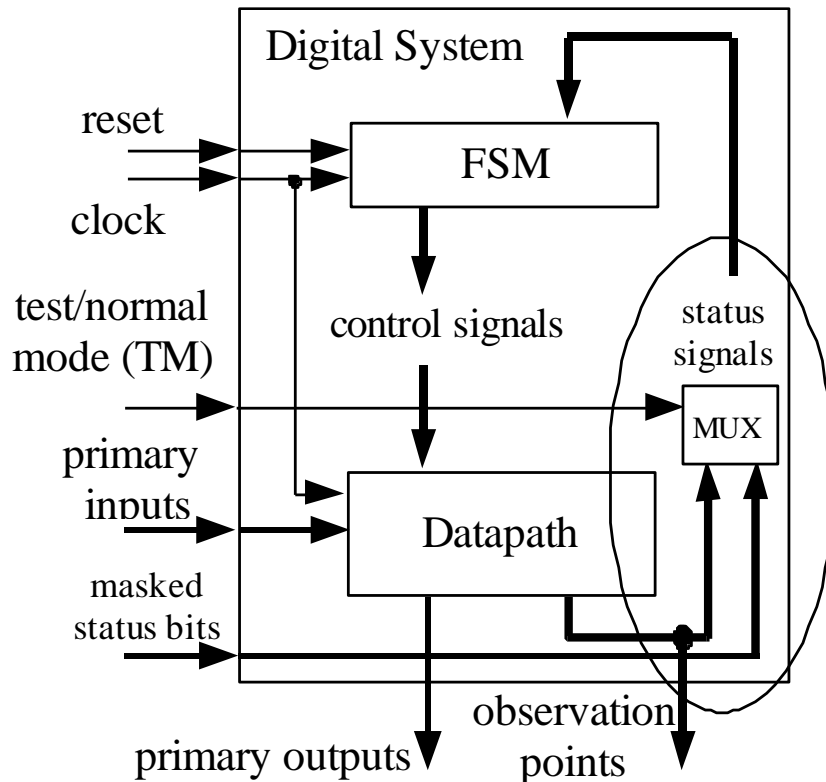
Sequential BIST

A DFT technique of BIST for sequential circuits is proposed

The approach proposed is based on **all-branches coverage metrics** which is known to be more powerful than all-statement coverage

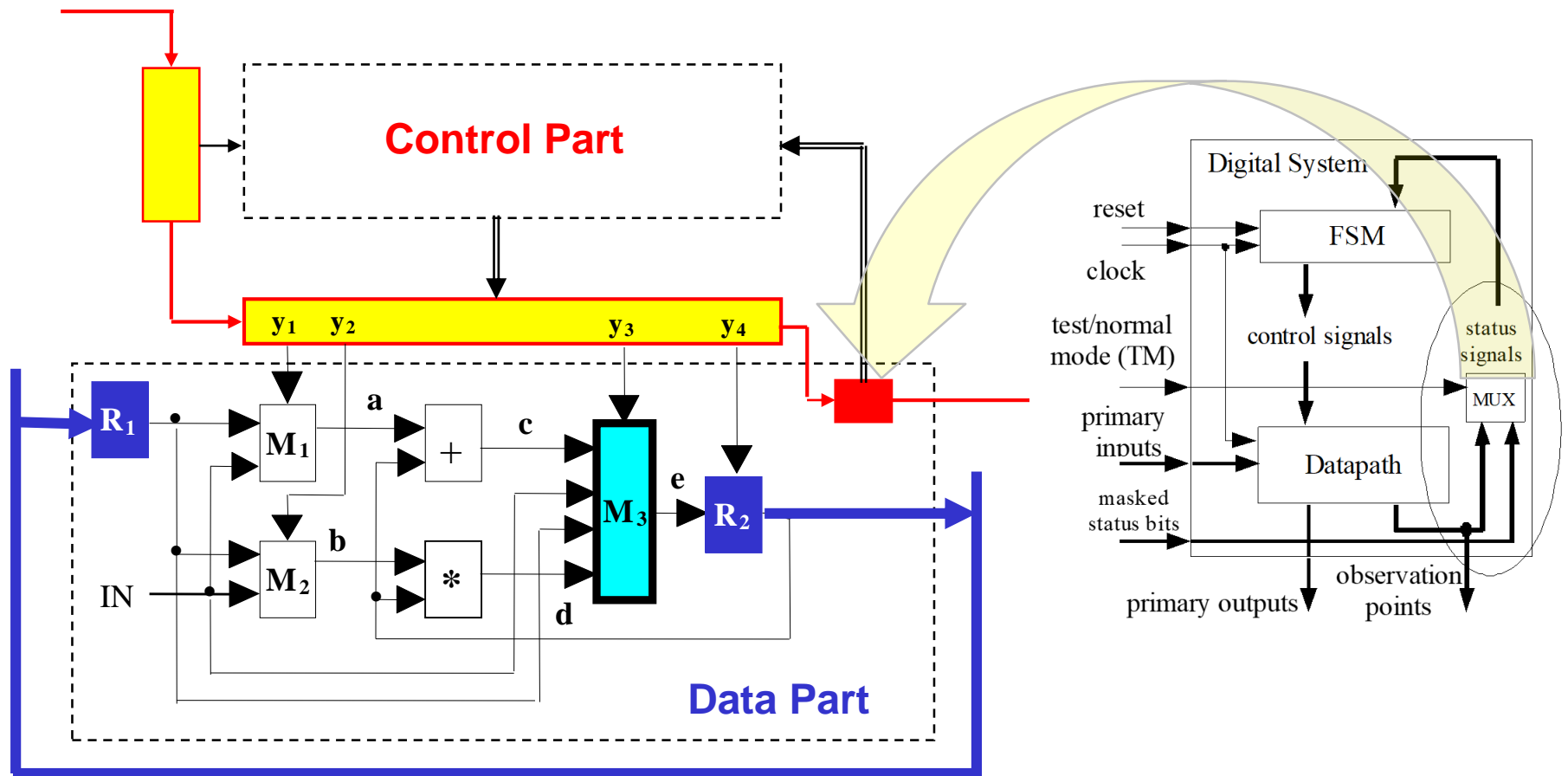


Sequential BIST



- **Status signals** entering the control part are made **controllable**
- In the test mode we can force the UUT to traverse all the branches in the FSM state transition graph
- The proposed idea of architecture requires **small** device area **overhead** since a simple controller can be implemented to manipulate the control signals

Example for Sequential BIST



BIST: Different Techniques

Pseudorandom Test generation by LFSR:

Full identification is achieved only after 2^n input combinations have been tried out (**exhaustive test**)

$$\frac{1}{2^{2^n} - 1} \sum_{i=1}^m 2^{2^n - i},$$
$$1 \leq m \leq 2^n$$

A better fault model (**stuck-at-0/1**) may limit the number of partitions necessary

Pseudorandom testing of sequential circuits:

The following rules suggested:

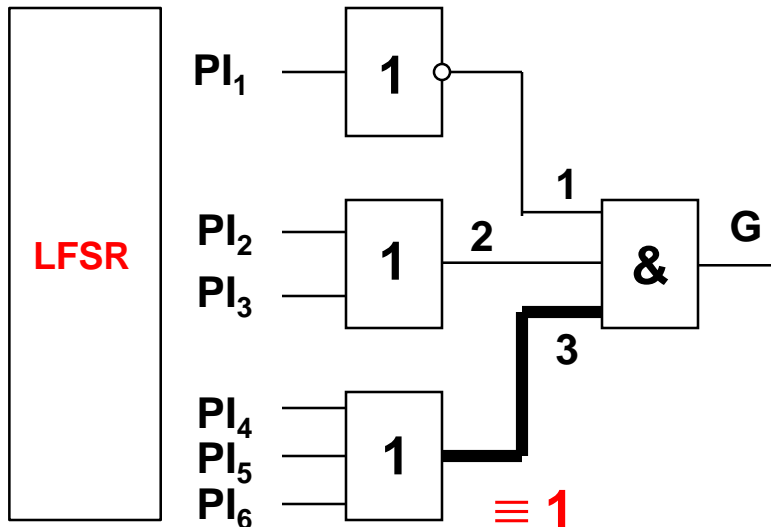
- clock-signals should not be random
- control signals such as reset, should be activated with low probability
- data signals are chosen randomly

Microprocessor testing

- A test generator picks randomly an instruction and generates random data patterns
- By repeating this sequence a specified number of times it will produce a test program which will test the microprocessor by randomly exercising its logic

BIST: Weighted pseudorandom test

Calculation of signal probabilities:



For PI_1 : $P = 0.15$

For PI_2 and PI_3 : $P = 0.6$

For $PI_4 - PI_6$: $P = 0.4$

Probability of detecting **the fault** $\equiv 1$
at the input 3 of the gate G:

1) equal probabilities ($p = 0.5$):

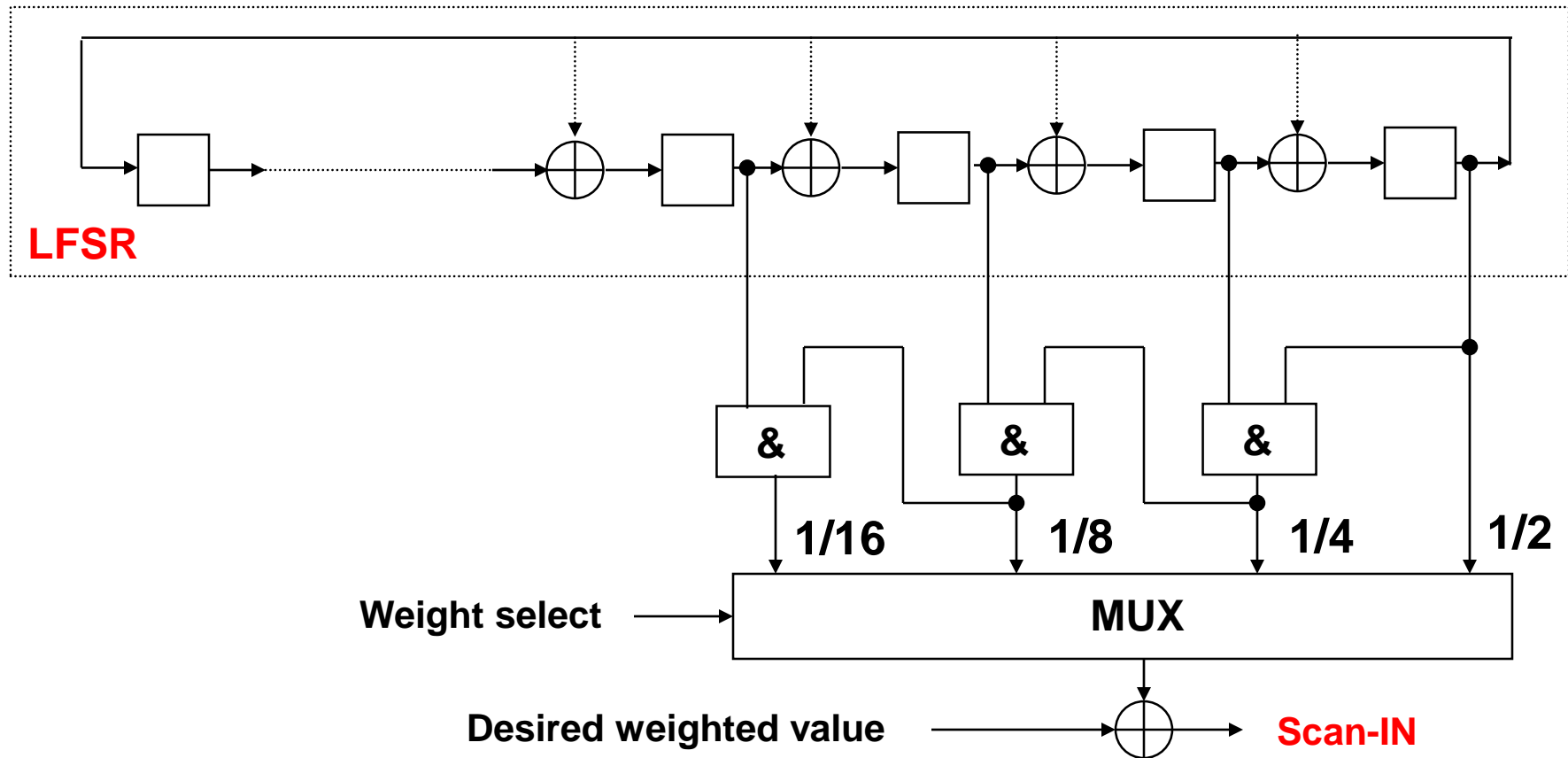
$$\begin{aligned}
 P &= 0.5 * (0.25 + 0.25 + 0.25) * 0.5^3 = \\
 &= 0.5 * 0.75 * 0.125 = \\
 &= \mathbf{0.046}
 \end{aligned}$$

2) weighted probabilities:

$$\begin{aligned}
 P &= 0.85 * \\
 &* (0.6 * 0.4 + 0.4 * 0.6 + 0.6^2) * \\
 &* 0.6^3 = \\
 &= 0.85 * 0.84 * 0.22 = \\
 &= \mathbf{0.16}
 \end{aligned}$$

BIST: Weighted pseudorandom test

Hardware implementation of weight generator

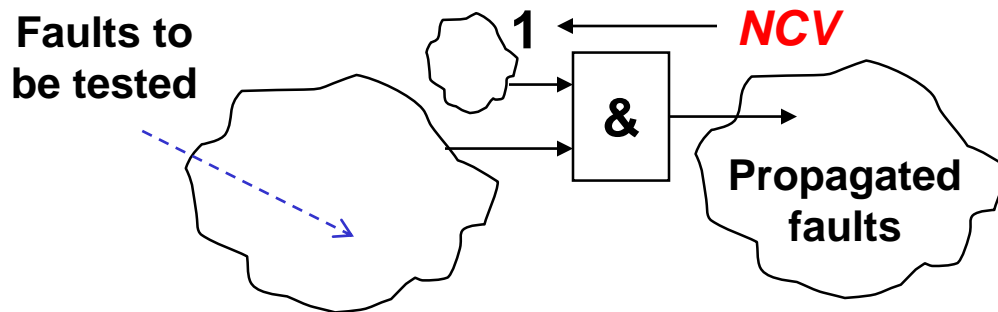


BIST: Weighted pseudorandom test

Problem: random-pattern-resistant faults

Solution: weighted pseudorandom testing

The probabilities of pseudorandom signals are weighted, **the weights are determined by circuit analysis**

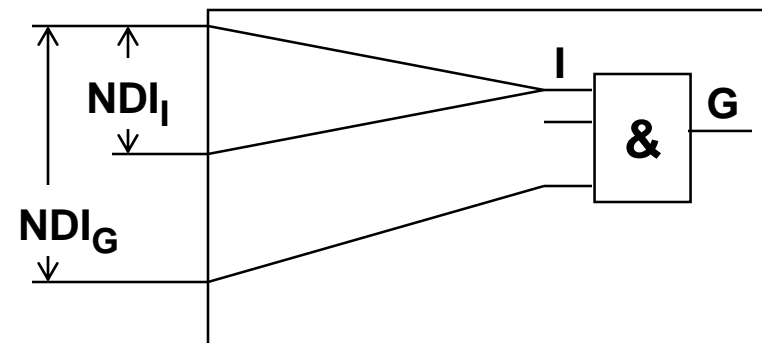


NCV – **non-controlling value**

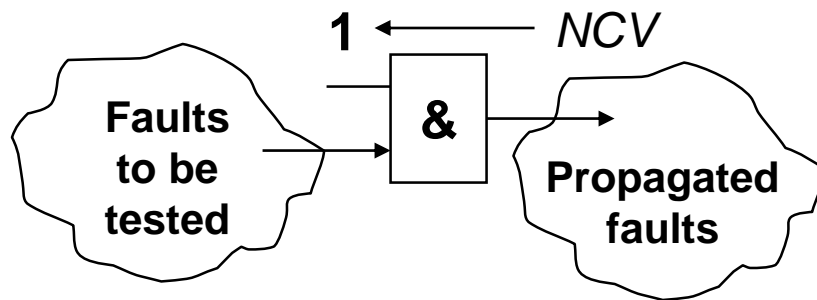
The more faults that must be tested through a gate input, the more the other inputs should be weighted to NCV

NDI - number of primary inputs for each gate determined by the back-trace cone

NDI - relative **measure of the number of faults** to be detected through the gate



BIST: Weighted pseudorandom test

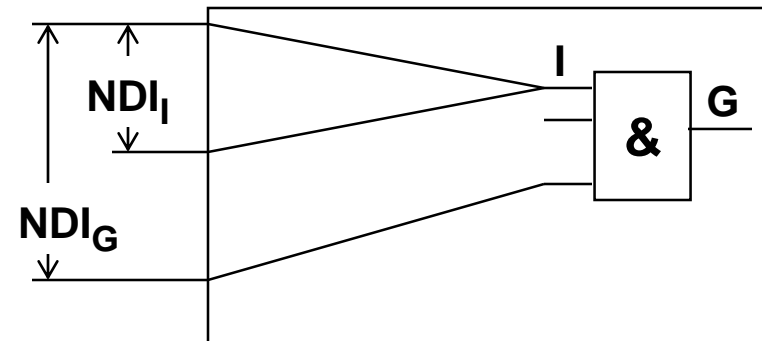


$$R_I = NDI_G / NDI_I$$

R_I - the **desired ratio of the NCV (1) to the controlling value (0)** for each gate input

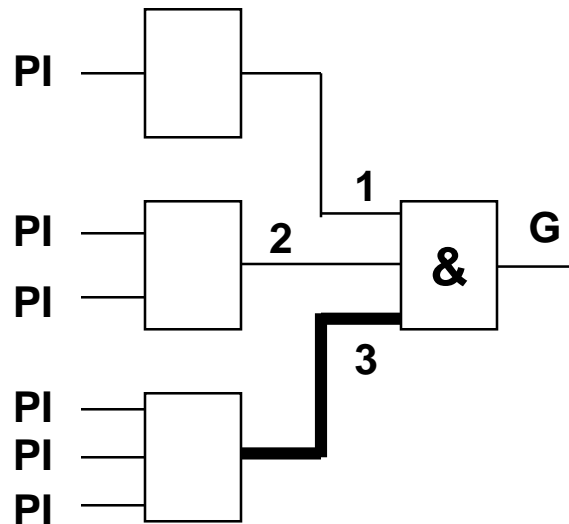
NCV - noncontrolling value

The more faults that must be tested through a gate input, the more the other inputs should be weighted to NCV



BIST: Weighted pseudorandom test

Example:



$$R_1 = \text{NDI}_G / \text{NDI}_1 = 6/1 = 6$$

$$R_2 = \text{NDI}_G / \text{NDI}_2 = 6/2 = 3$$

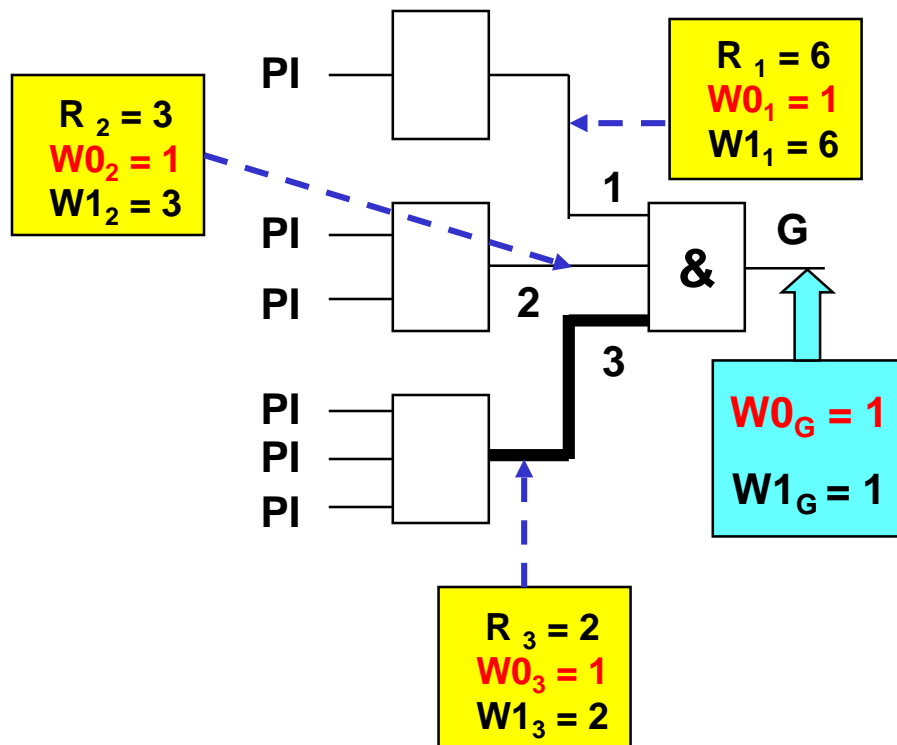
$$R_3 = \text{NDI}_G / \text{NDI}_3 = 6/3 = 2$$

More faults must be detected through the **third input** than through others

This results in the other inputs being weighted more heavily towards NCV

BIST: Weighted pseudorandom test

Calculation of signal weights:



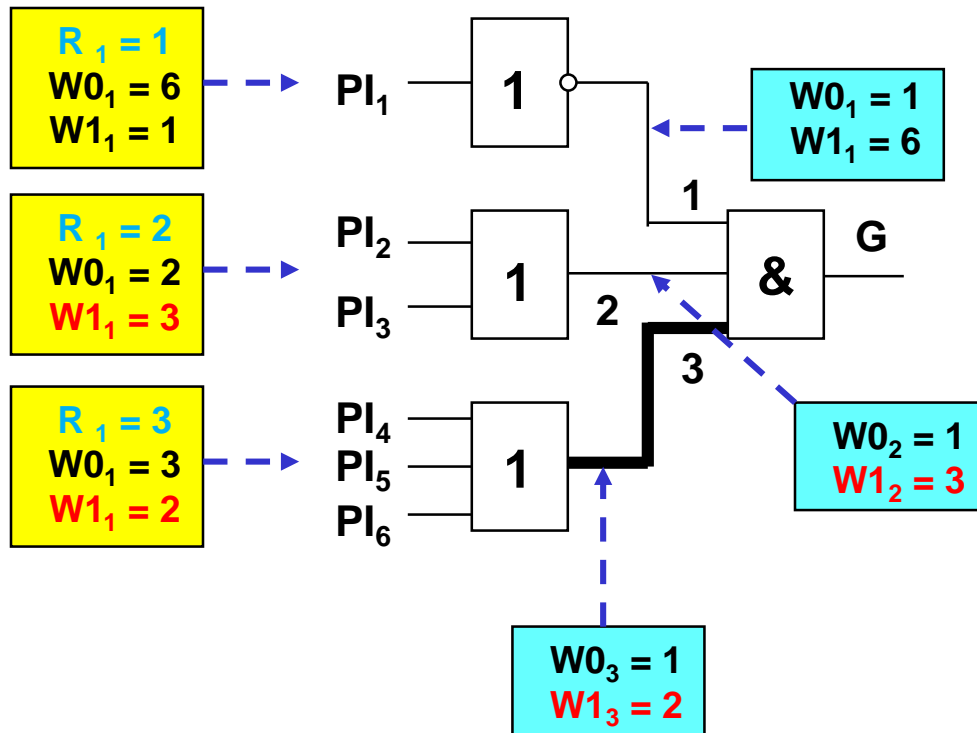
$W0$, $W1$ - **weights** of the signals
are calculated by backtracking

Calculation of $W0$, $W1$ for inputs

Function	$W0_{IN}$	$W1_{IN}$
AND	$W0_G$	$R_I * W1_G$
NAND	$W1_G$	$R_I * W0_G$
OR	$R_I * W0_G$	$W1_G$
NOR	$R_I * W1_G$	$W0_G$

BIST: Weighted pseudorandom test

Calculation of signal weights:



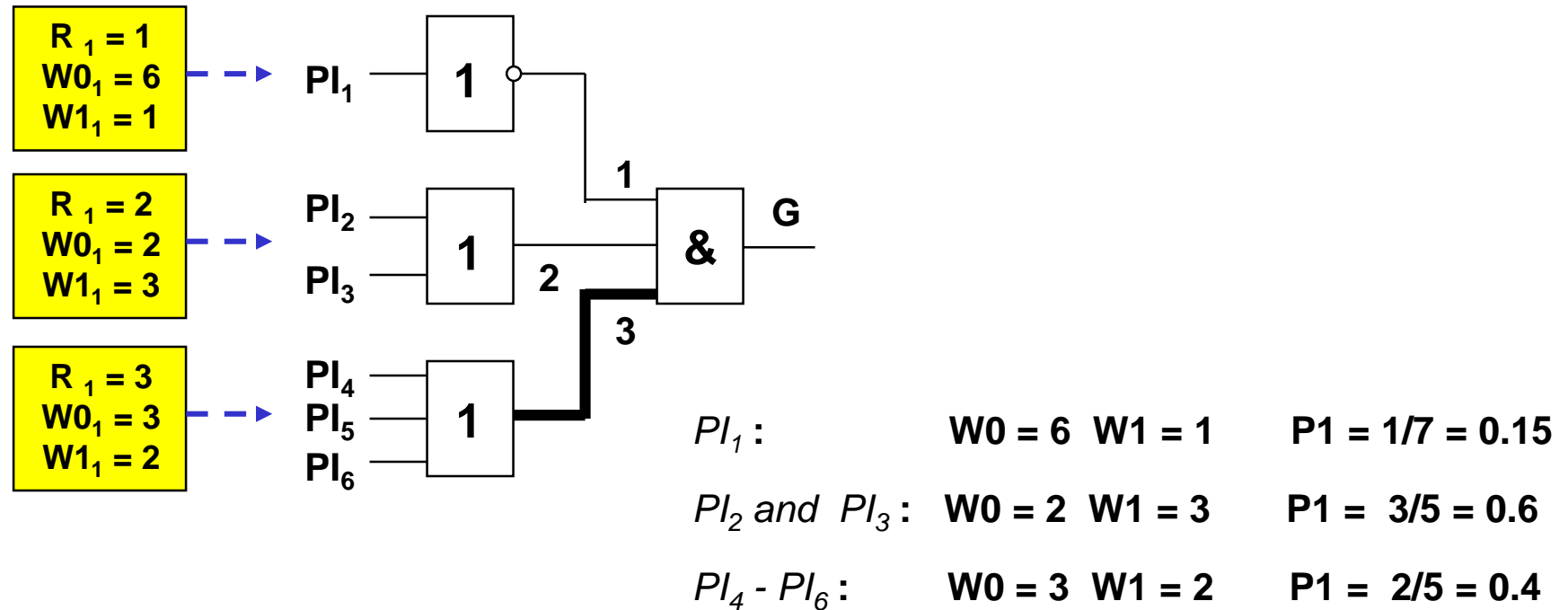
Backtracing from all the outputs to all the inputs of the given cone

Weights are calculated for all gates and inputs

Function	$W0_i$	$W1_i$
OR	$R_i * W0_G$	$W1_G$
NOR	$R_i * W1_G$	$W0_G$

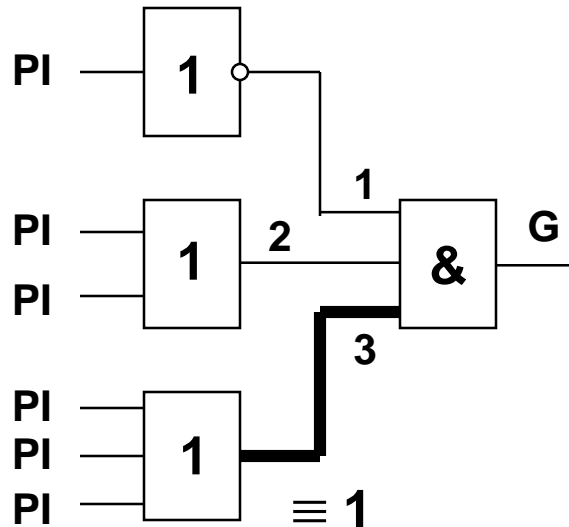
BIST: Weighted pseudorandom test

Calculation of signal probabilities:



BIST: Weighted pseudorandom test

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Probability of detecting the fault $\equiv 1$
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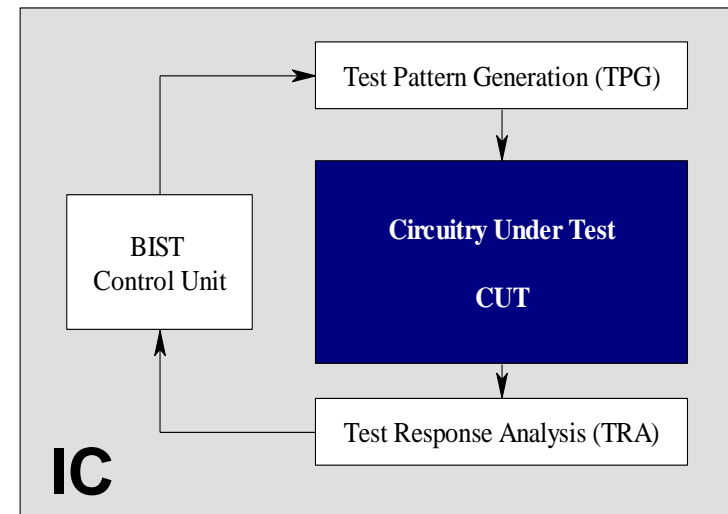
$$\begin{aligned} P &= 0.5 * (0.25 + 0.25 + 0.25) * 0.5^3 = \\ &= 0.5 * 0.75 * 0.125 = \\ &= \mathbf{0.046} \end{aligned}$$

2) weighted probabilities:

$$\begin{aligned} P &= 0.85 * \\ &* (0.6 * 0.4 + 0.4 * 0.6 + 0.6^2) * \\ &* 0.6^3 = \\ &= 0.85 * 0.84 * 0.22 = \\ &= \mathbf{0.16} \end{aligned}$$

The Main BIST Problems

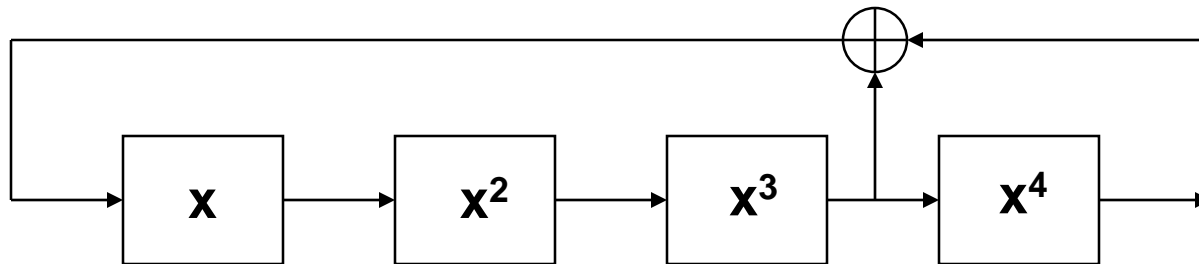
- **On circuit**
 - Test pattern generation
 - Response verification
- **Random pattern generation,**
Very long tests
Hard-to-test faults
- **Response compression**
Aliasing of results



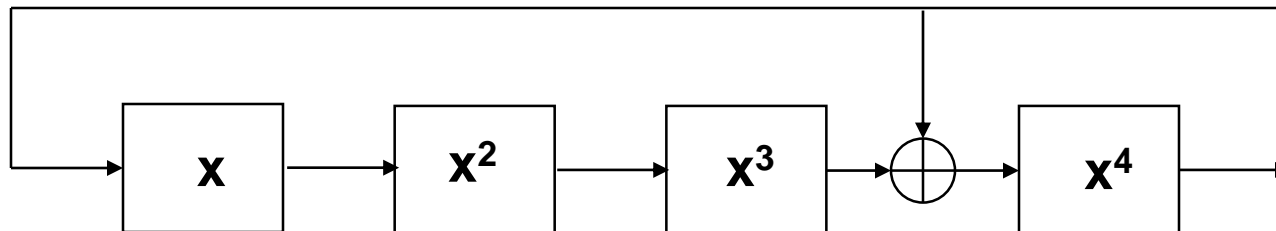
Pseudorandom Test Generation

LFSR – Linear Feedback Shift Register:

Standard LFSR



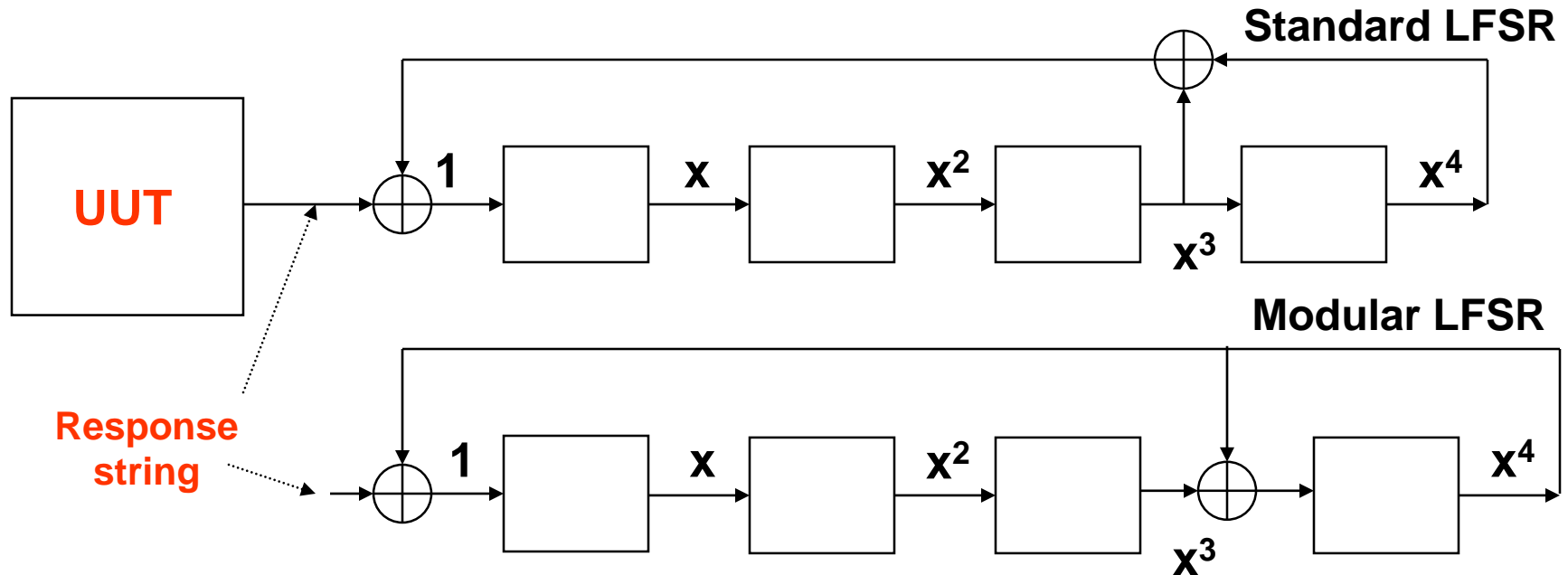
Modular LFSR



Polynomial: $P(x) = x^4 + x^3 + 1$

BIST: Signature Analysis

Signature analyzer:



Response is compacted
by LFSR

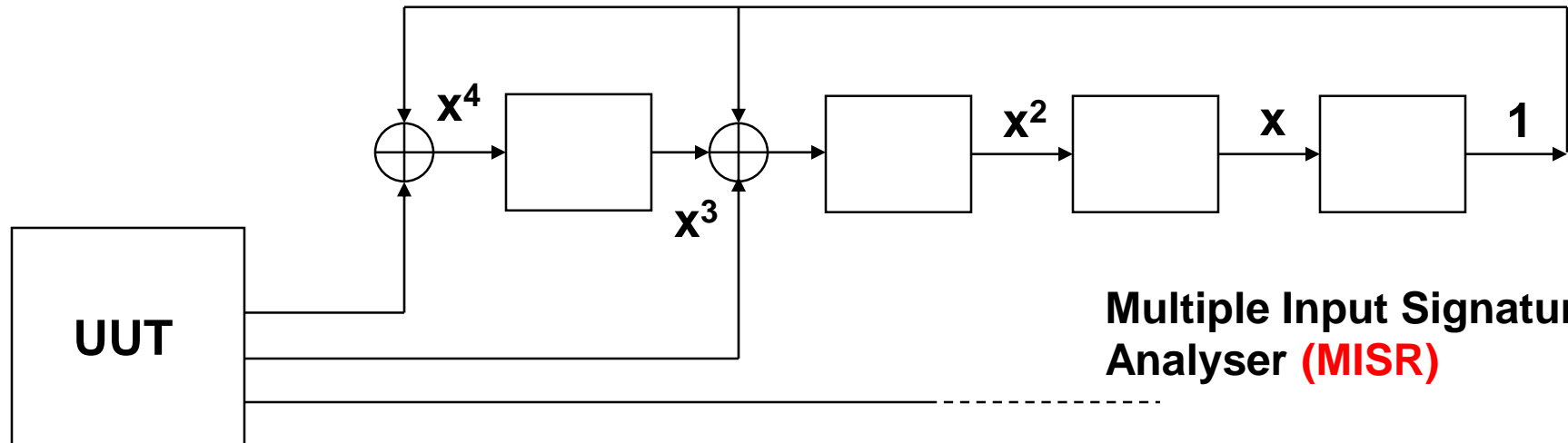
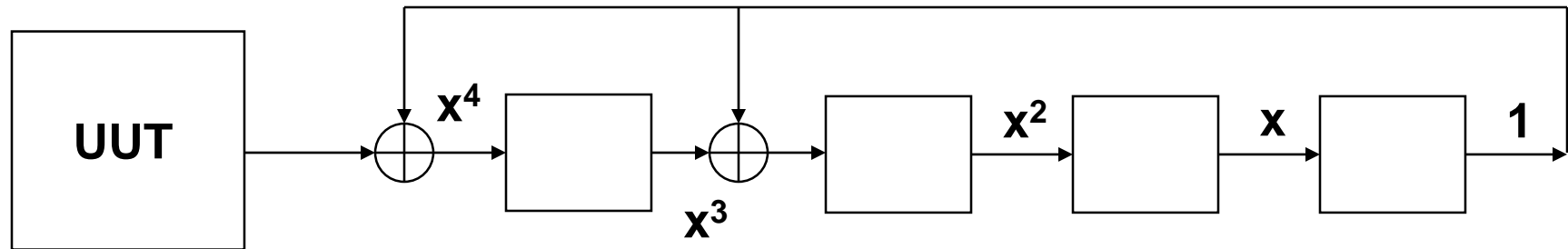
The content of LFSR after
test is called signature

$$\text{Polynomial: } P(x) = x^4 + x^3 + 1$$

BIST: Signature Analysis

Parallel Signature Analyzer:

Single Input Signature Analyser

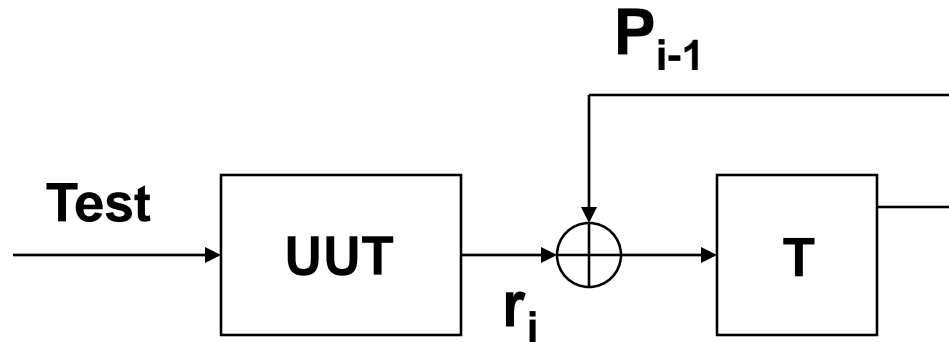


Multiple Input Signature Analyser (MISR)

Special Cases of Response Compression

1. Parity checking

$$P(R) = \left(\sum_{i=1}^m r_i \right) \bmod 2$$

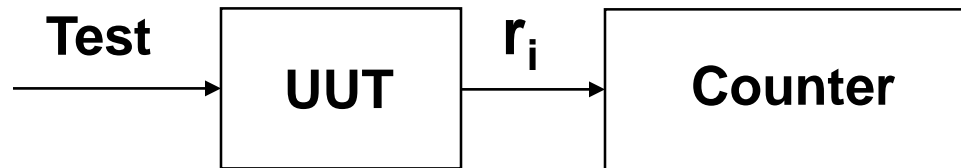


2. One counting

$$P(R) = \sum_{i=1}^m r_i$$

3. Zero counting

$$P(R) = \sum_{i=1}^m \bar{r}_i$$

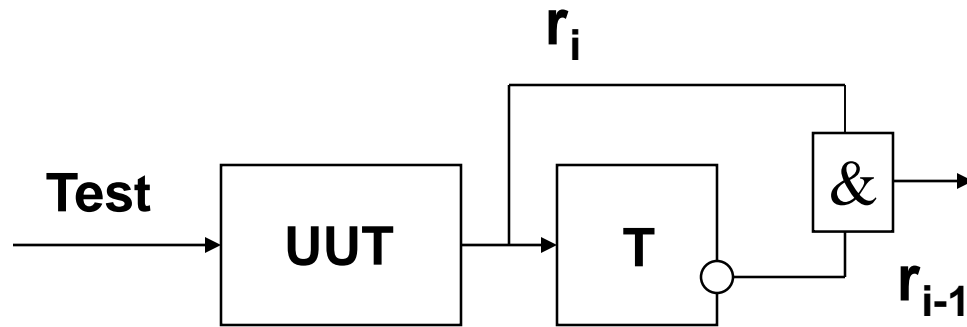


Special Cases of Response Compression

4. Transition counting

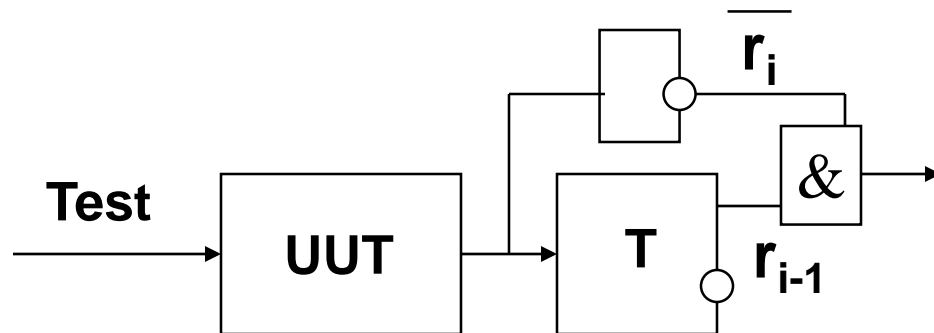
a) Transition 0→1

$$P(R) = \sum_{i=2}^m (\overline{r_{i-1}} r_i)$$



b) Transition 1→0

$$P(R) = \sum_{i=2}^m (r_{i-1} \overline{r_i})$$



5. Signature analysis

Theory of LFSR

The principles of CRC (**Cyclic Redundancy Coding**) are used in LFSR based test response compaction

Coding theory treats **binary strings as polynomials**:

$R = r_{m-1} r_{m-2} \dots r_1 r_0$ - m-bit binary sequence (binary string)

$R(x) = r_{m-1} x^{m-1} + r_{m-2} x^{m-2} + \dots + r_1 x + r_0$ - polynomial in x

Example:

$$11001 \rightarrow R(x) = x^4 + x^3 + 1$$

Only the coefficients are of interest, not the actual value of x

However, for $x = 2$, $R(x)$ is the decimal value of the bit string

Theory of LFSR

Arithmetic of coefficients:

- linear algebra over the field of 0 and 1: all integers mapped into either 0 or 1
- mapping: representation of **any integer** n by **remainder** r resulting from the division of n by 2:

$$n = 2m + r, r \in \{0,1\} \quad \text{or} \quad r = n \pmod{2}$$

Linear - refers to the arithmetic unit (modulo-2 adder), used in CRC generator (linear, since **each bit has equal weight** upon the output)

Examples (addition, multiplication):

$$\begin{array}{r} x^4 + x^3 \quad + x + 1 \\ + x^4 \quad + x^2 + x \\ \hline x^3 + x^2 \quad + 1 \end{array}$$

$$\begin{array}{r} x^4 + x^3 \quad + x + 1 \\ * x + 1 \\ \hline x^5 + x^4 \quad + x^2 + x \\ \quad x^4 + x^3 \quad + x + 1 \\ \hline x^5 \quad + x^3 + x^2 \quad + 1 \end{array}$$

BIST: Signature Analysis

Division of one polynomial $P(x)$ by another $G(x)$ produces a **quotient** polynomial $Q(x)$, and if the division is not exact, a **remainder** polynomial $R(x)$

$$\frac{P(x)}{G(x)} = Q(x) + \frac{R(x)}{G(x)}$$

Example:

$$\frac{P(x)}{G(x)} = \frac{x^7 + x^3 + x}{x^5 + x^3 + x + 1} = x^3 + x^2 + 1 + \frac{x^2 + 1}{x^5 + x^3 + x + 1}$$

Remainder $R(x)$ is used as a **check word** in **data transmission**

The transmitted code consists of the **message $P(x)$** followed by the **check word $R(x)$**

Upon receipt, the reverse process occurs: the message $P(x)$ is divided by known $G(x)$, and a mismatch between $R(x)$ and the remainder from the division indicates an error

BIST: Signature Analysis

In signature testing we mean the use of CRC encoding as the **data compressor G(x)** and the use of the **remainder R(x)** as the signature of the test response string P(x) from the UUT

$$\frac{P(x)}{G(x)} = Q(x) + \frac{R(x)}{G(x)}$$

Signature is the CRC code word

Example:

G(x)

1 0 1 0 1 1

1 0 1

= Q(x) = x² + 1

1 0 0 0 1 0 1 0

P(x)

1 0 1 0 1 1

$$\frac{P(x)}{G(x)} = \frac{x^7 + x^3 + x}{x^5 + x^3 + x + 1}$$

0 0 1 0 0 1 1 0

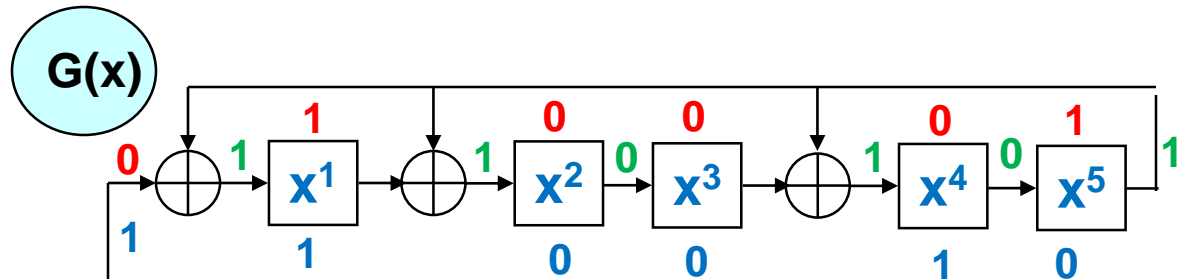
1 0 1 0 1 1

0 0 1 1 0 1

= R(x) = x³ + x² + 1

Signature

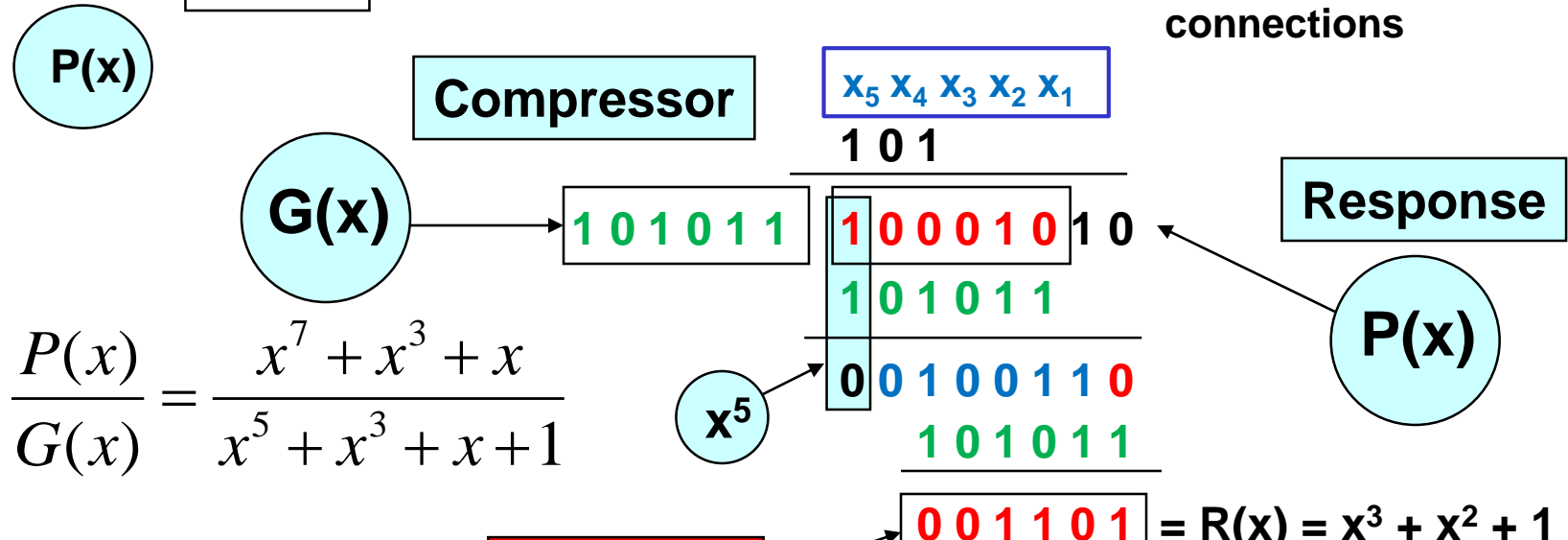
BIST: Hardware for Signature Analysis



Division process can be mechanized using LFSR

Divisor polynomial $G(x)$ is defined by the feedback connections

IN: 01 **010001** → Shifted into LFSR

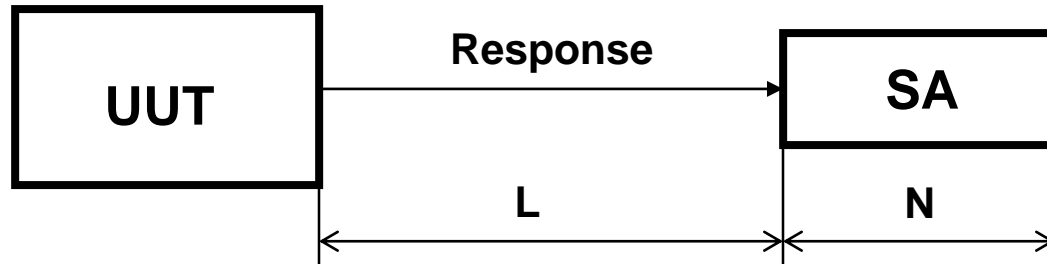


$$\frac{P(x)}{G(x)} = \frac{x^7 + x^3 + x}{x^5 + x^3 + x + 1}$$

Signature $001101 = R(x) = x^3 + x^2 + 1$

BIST: Signature Analysis

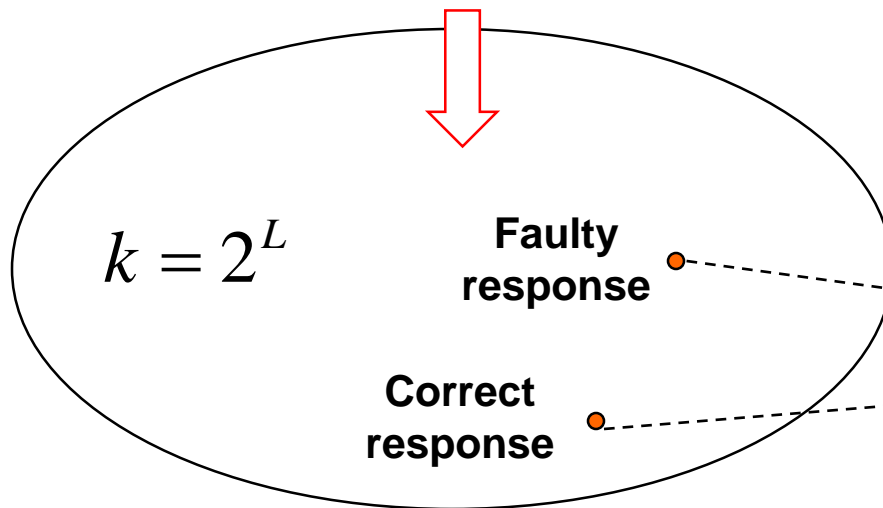
Aliasing:



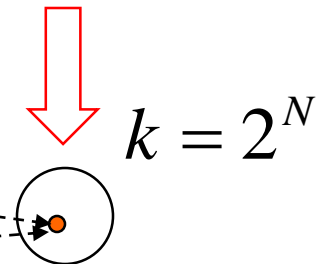
L - test length

N - number of stages in Signature Analyzer

All possible responses



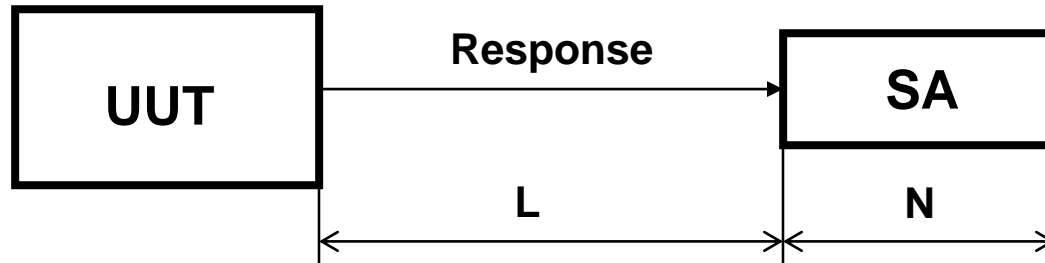
All possible signatures



$$N \ll L$$

BIST: Signature Analysis

Aliasing:



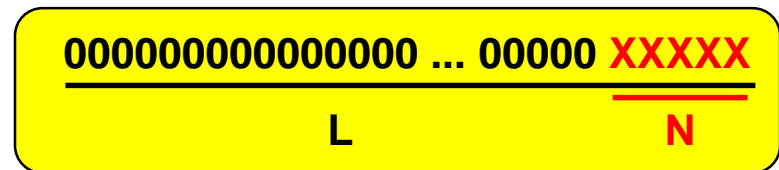
L - test length

N - number of stages in Signature Analyzer

$k = 2^L$ - number of different possible responses

No aliasing is possible for those strings with $L - N$ leading zeros since they are represented by polynomials of degree $N - 1$ that are not divisible by characteristic polynomial of LFSR

$2^{L-N} - 1$ ---- **Aliasing is possible**

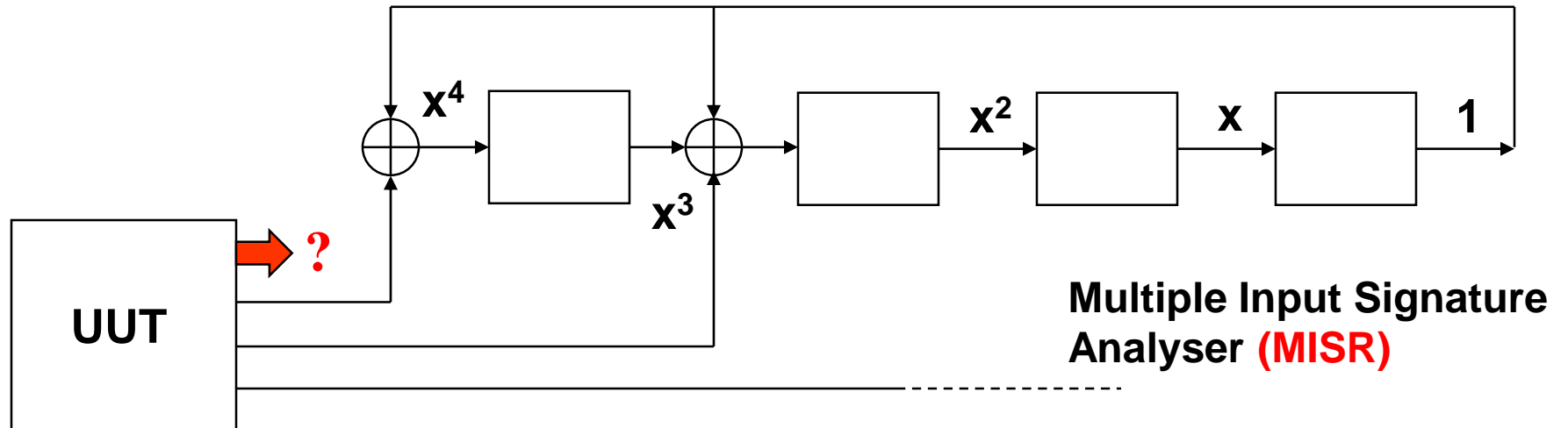
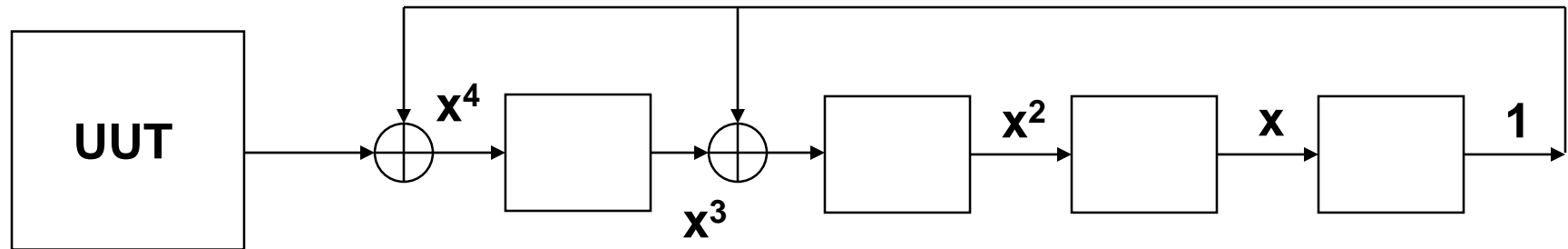


Probability of aliasing: $P = \frac{2^{L-N} - 1}{2^L - 1} \xrightarrow{L \gg 1} P = \frac{1}{2^N}$

BIST: Signature Analysis

Parallel Signature Analyzer:

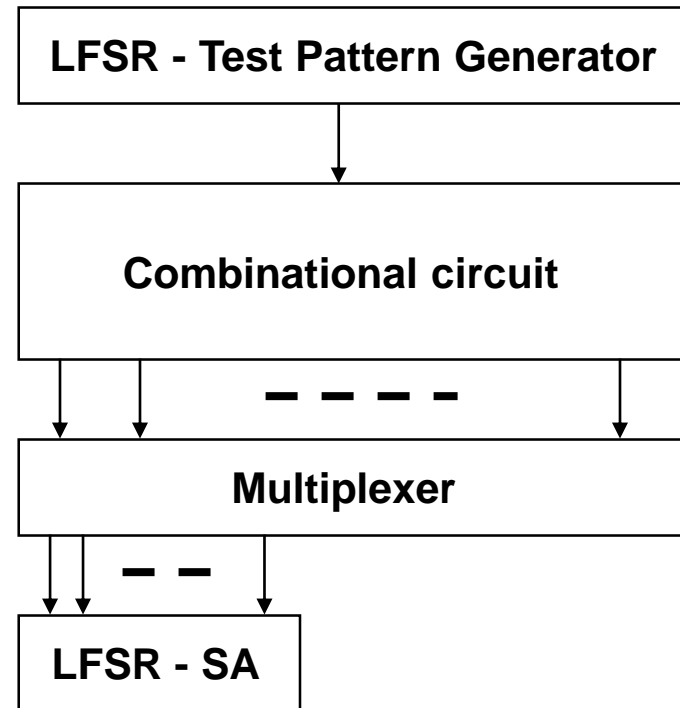
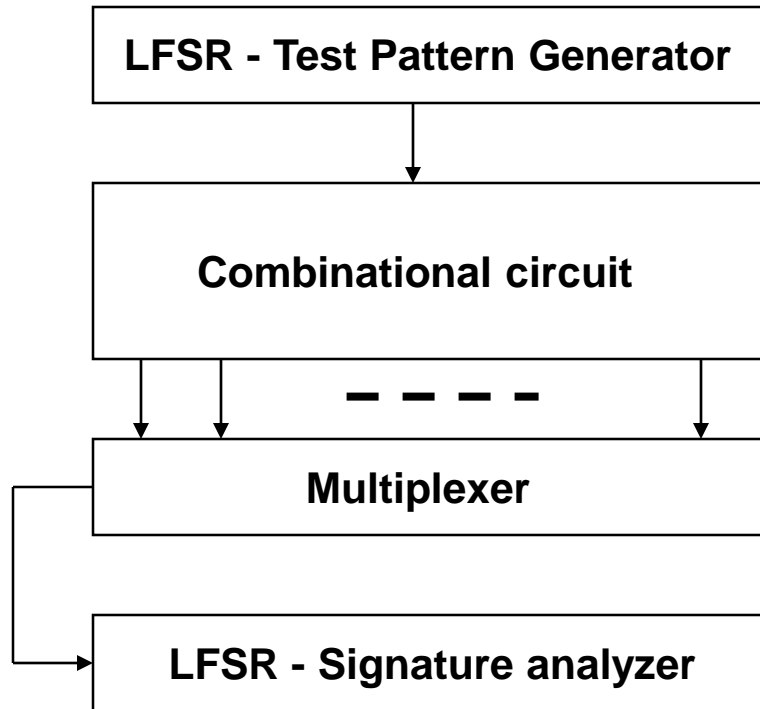
Single Input Signature Analyser



Multiple Input Signature Analyser (MISR)

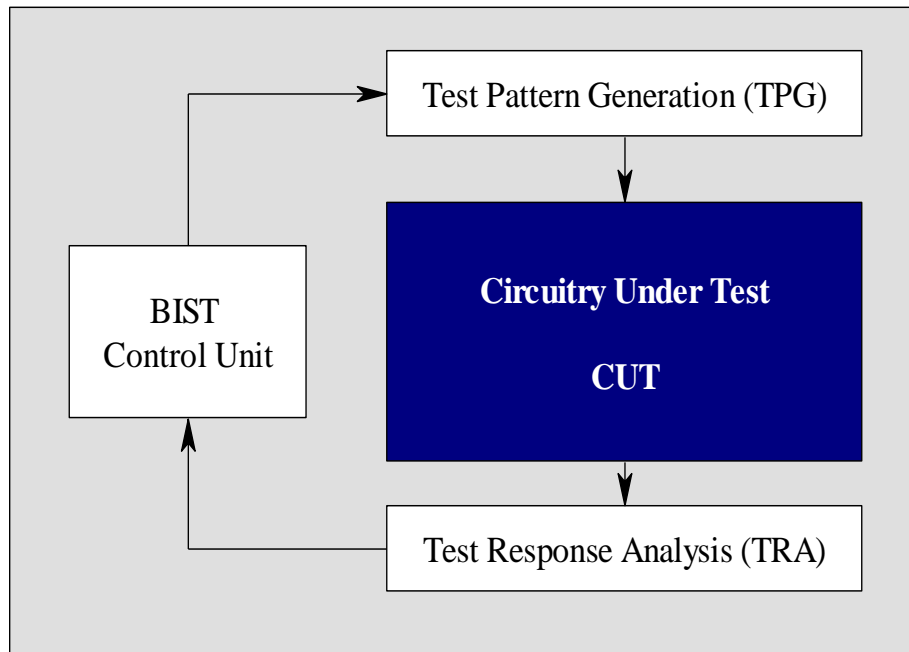
BIST: Signature Analysis

Signature calculating for multiple outputs:



BIST Architectures

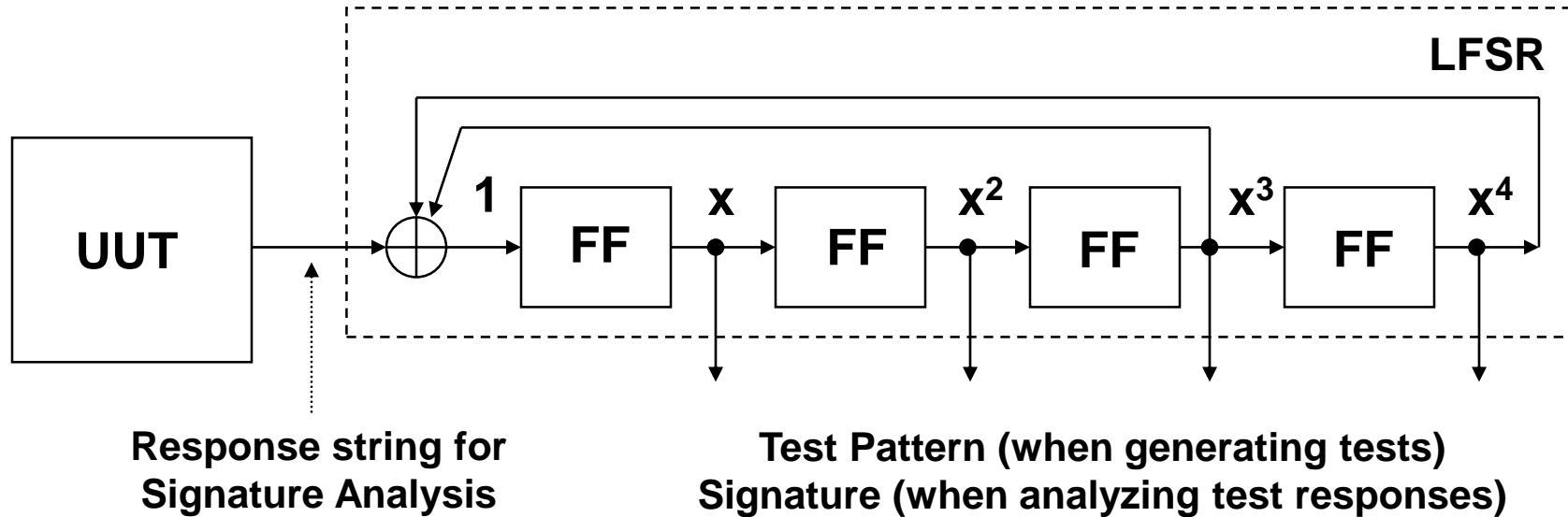
General Architecture of BIST



- **BIST components:**
 - Test pattern generator (TPG)
 - Test response analyzer (TRA)
 - BIST controller
- **A part of a system (hardcore) must be operational to execute a self-test**
- **At minimum the hardcore usually includes power, ground, and clock circuitry**
- **Hardcore should be tested by**
 - external test equipment or
 - it should be designed self-testable by using various forms of redundancy

BIST: Joining TPG and SA

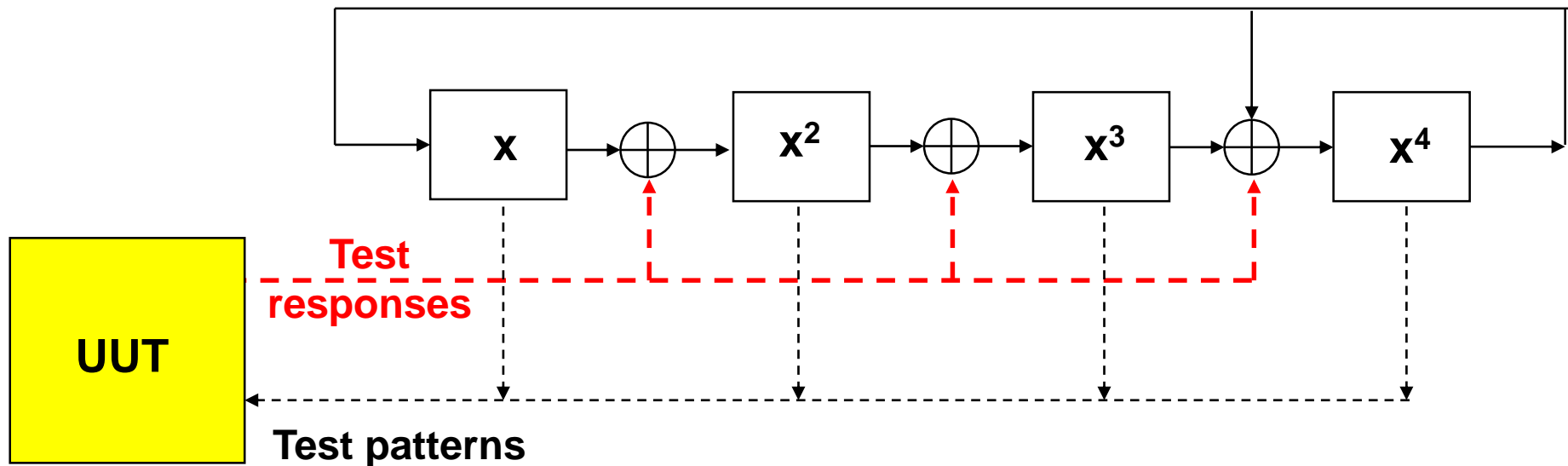
Two functionalities of LFSR:



Pseudorandom Test Generation

LFSR – Linear Feedback Shift Register:

Why modular LFSR is useful for BIST?



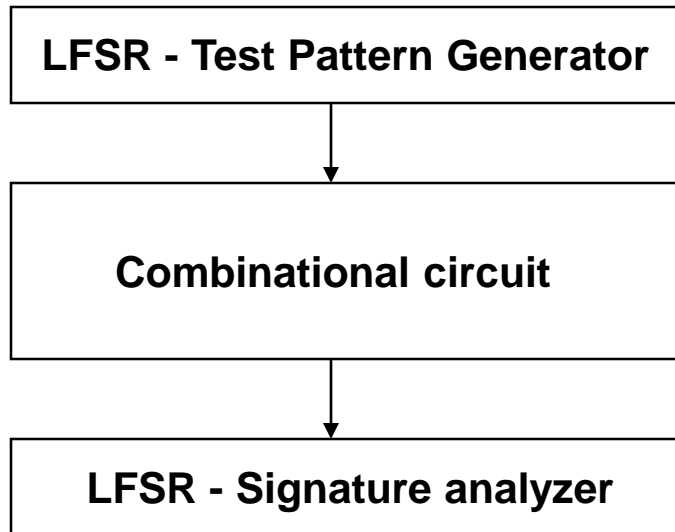
Polynomial: $P(x) = x^4 + x^3 + 1$

Instead of **BILBO** we have now **CSTP** architecture

BIST Architectures

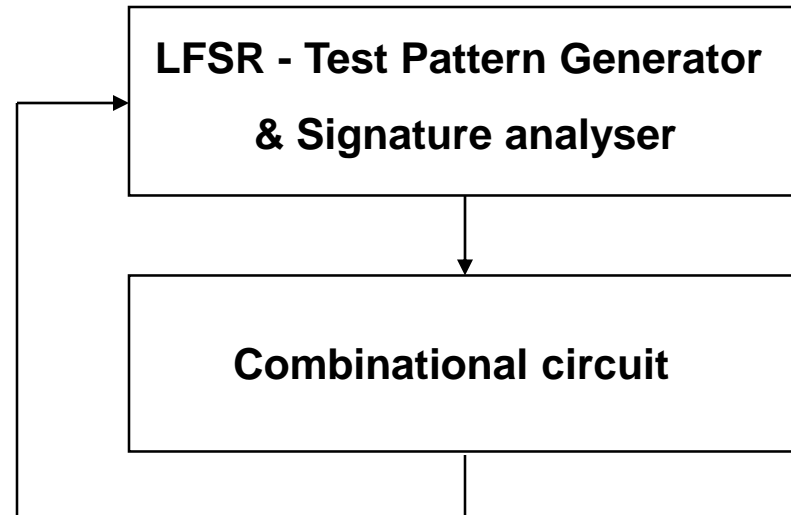
Test per Clock:

Disjoint TPG and SA: BILBO

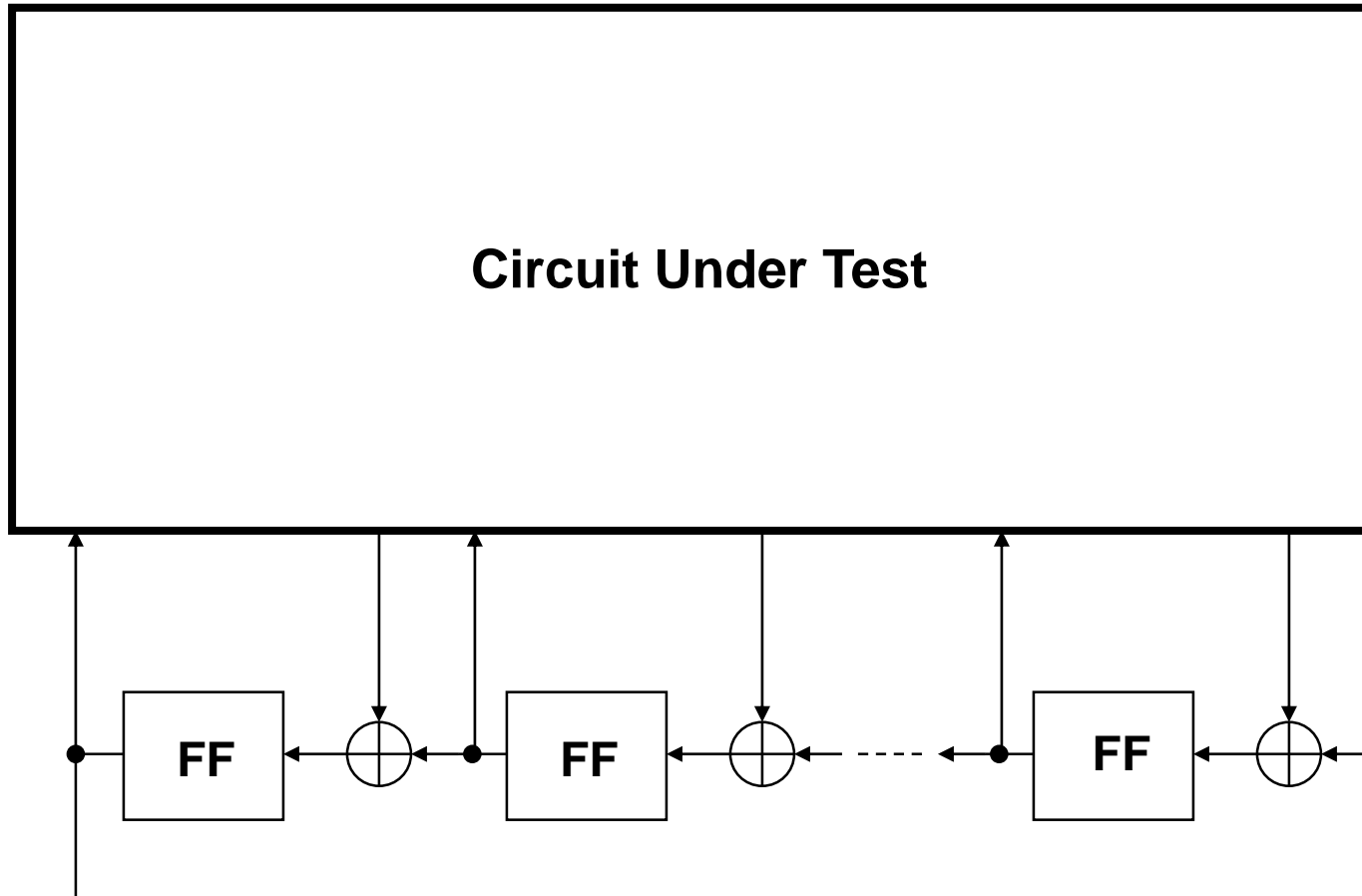


Joint TPG and SA:

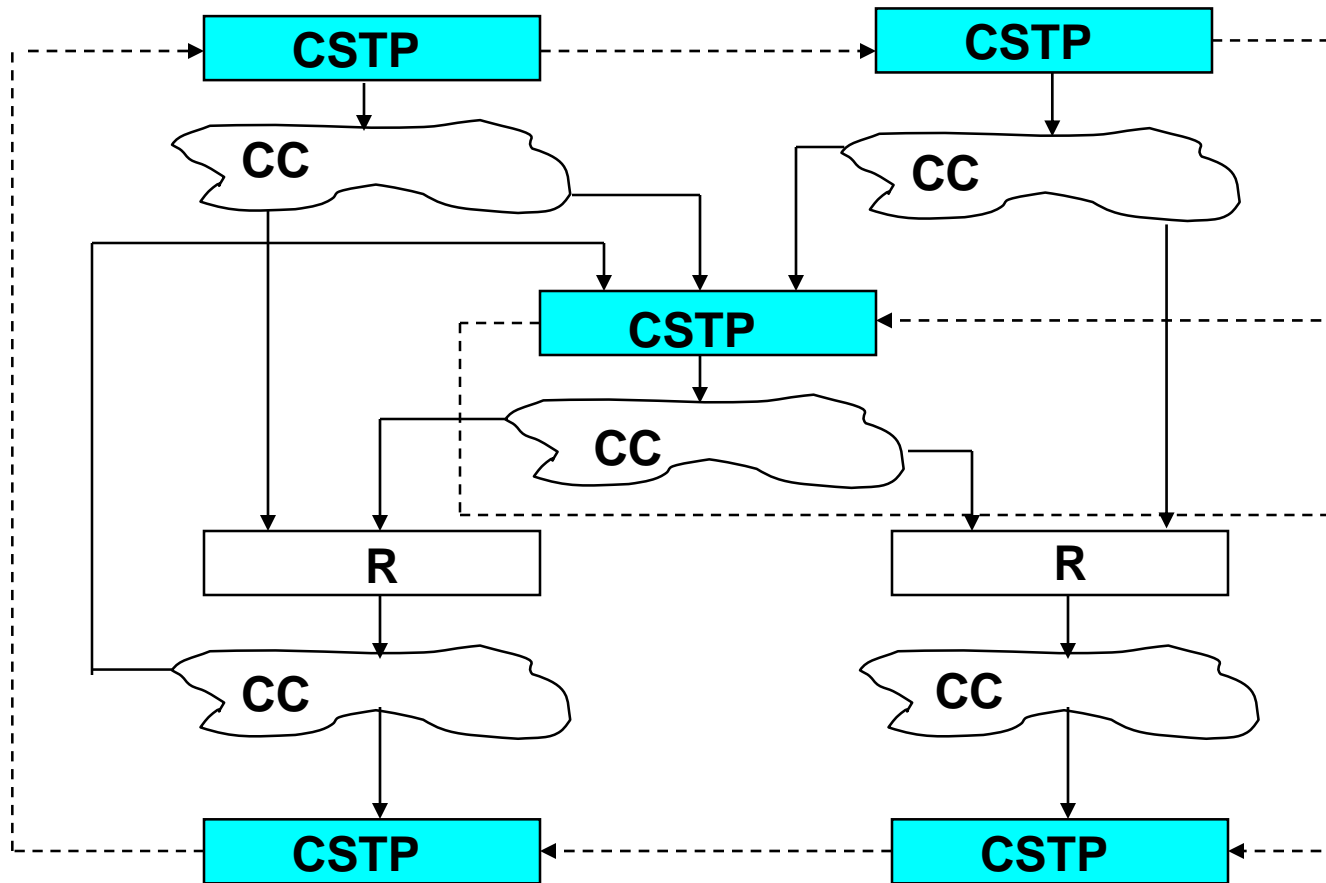
CSTP - Circular Self-Test Path:



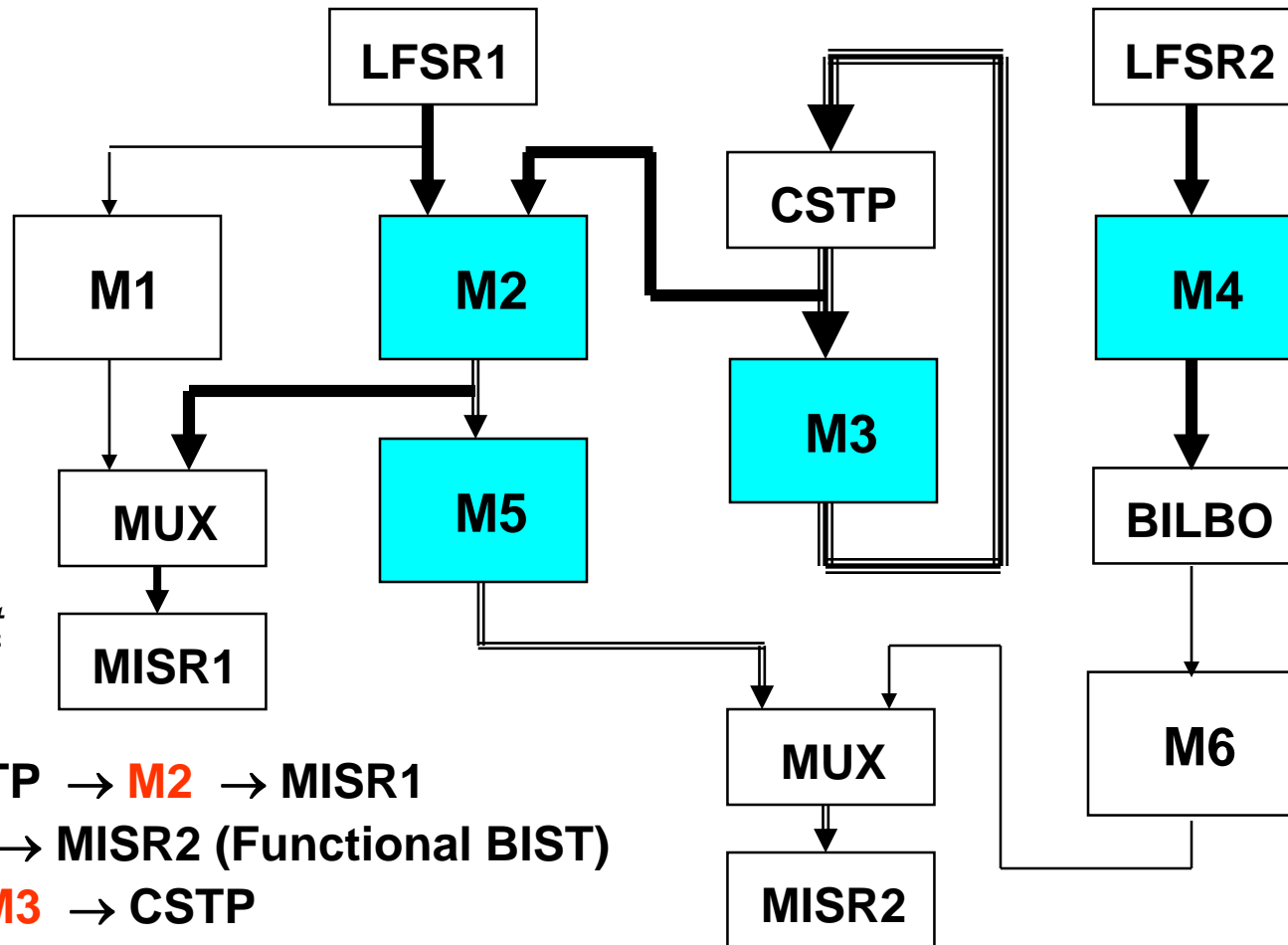
BIST: Circular Self-Test Architecture



BIST: Circular Self-Test Path



BIST Embedding Example



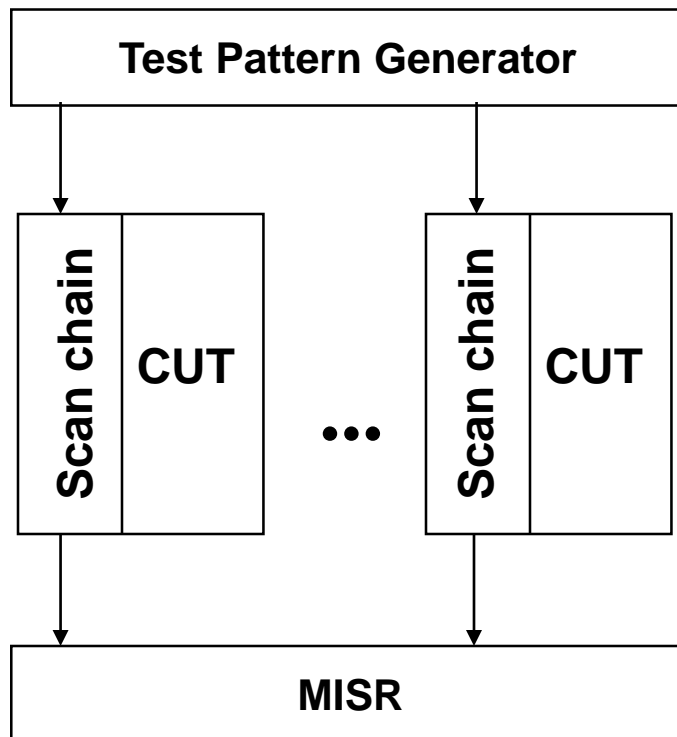
Concurrent testing:

LFSR, CSTP → **M2** → MISR1
M2 → **M5** → MISR2 (Functional BIST)
CSTP → **M3** → CSTP
LFSR2 → **M4** → BILBO

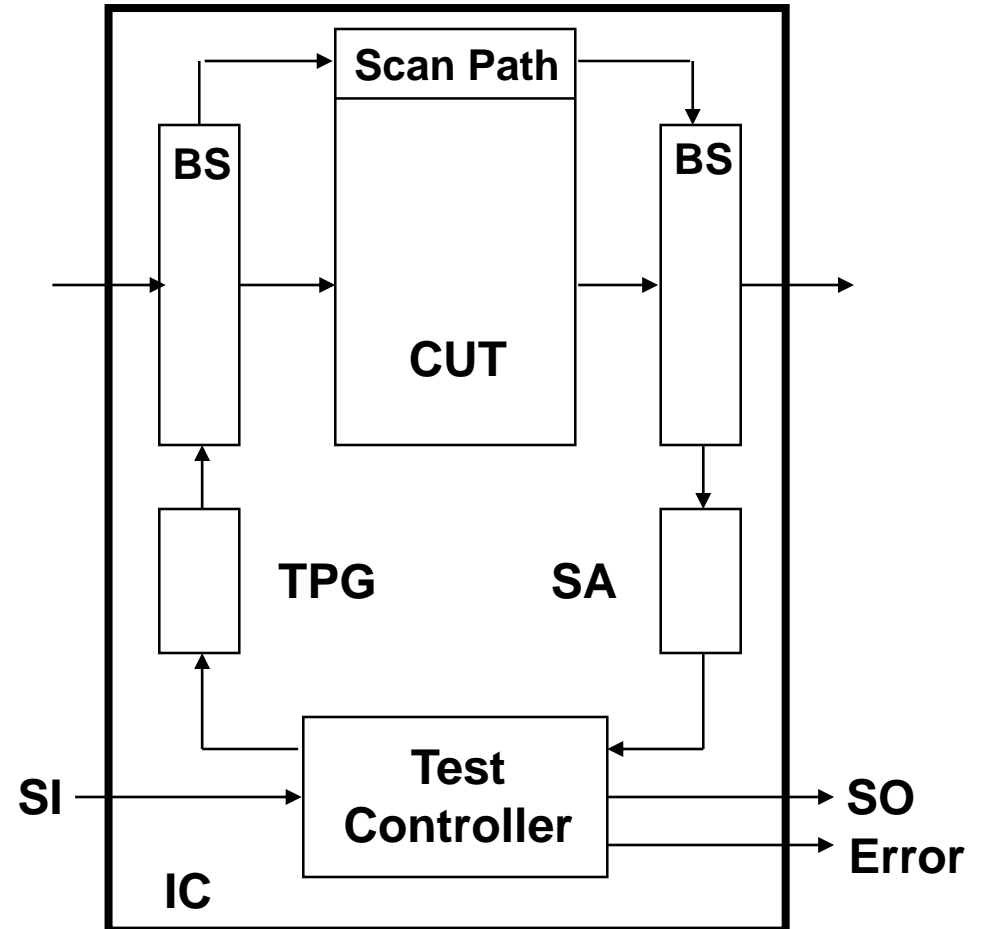
BIST Architectures

STUMPS:

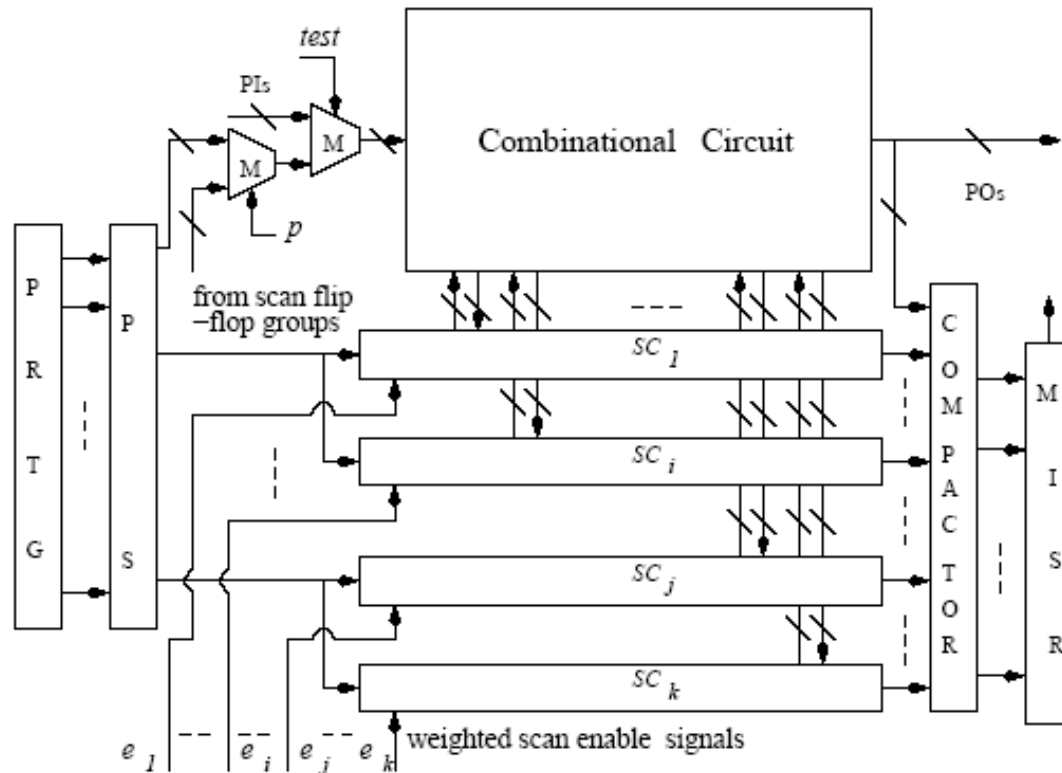
Self-Testing Unit Using MISR and Parallel Shift Register Sequence Generator



LOCST: LSSD On-Chip Self-Test



Scan-Based BIST Architecture



- PS – Phase shifter**
- Scan-Forest**
- Scan-Trees**
- Scan-Segments (SC)**
- Weighted scan-enables for SS**
- Compactor - EXORs**

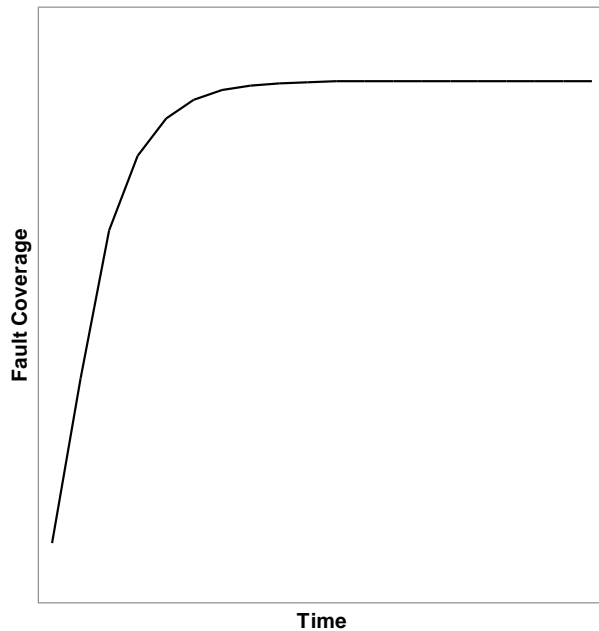
Figure 1: Scan-based BIST for n -detection with weighted scan-enable signals and scan forest.

Copyright: D.Xiang 2003

Problems with BIST

The main motivations of using random patterns are:

- low generation cost
- high initial efficiency

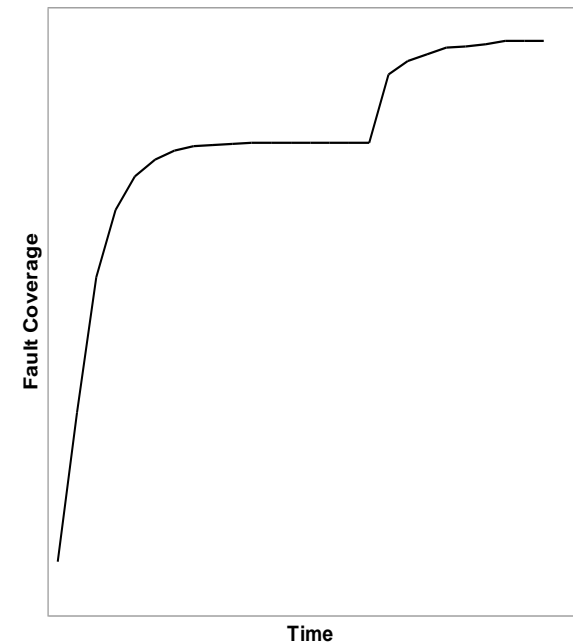


Problems:

- **Very long test application time**
- **Low fault coverage**
- Area overhead
- Additional delay

Possible solutions

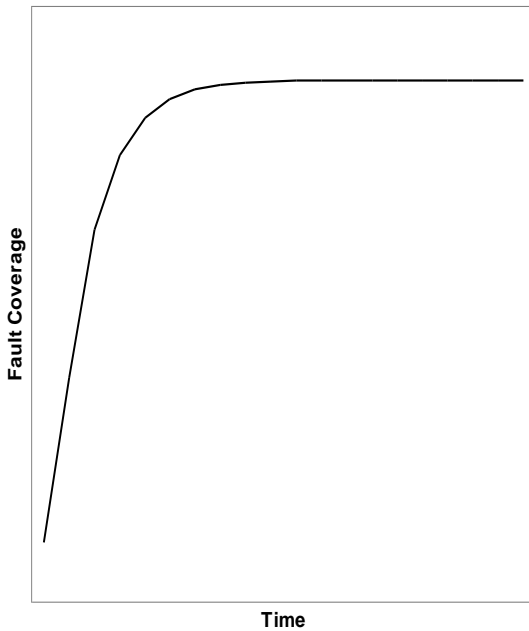
- **Weighted pseudorandom test**
- **Combining pseudorandom test with deterministic data**
 - Multiple seed
 - Bit flipping
- **Hybrid BIST**



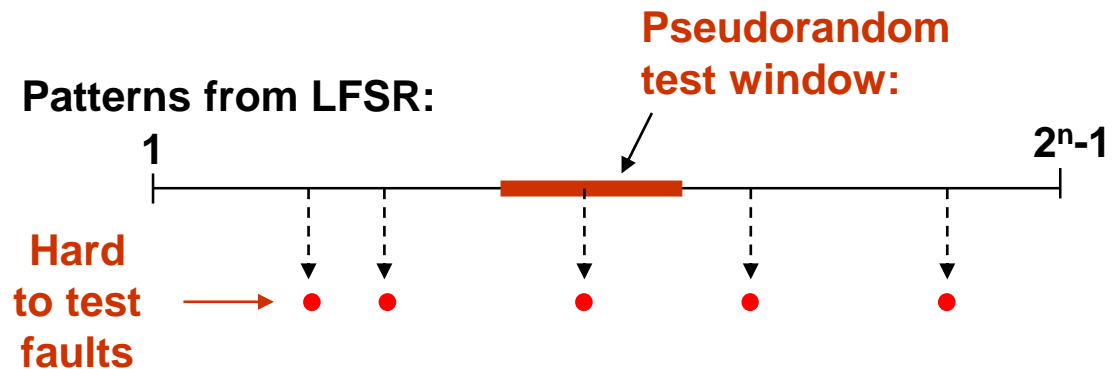
Problems with BIST: Hard to Test Faults

The main motivations of using random patterns are:

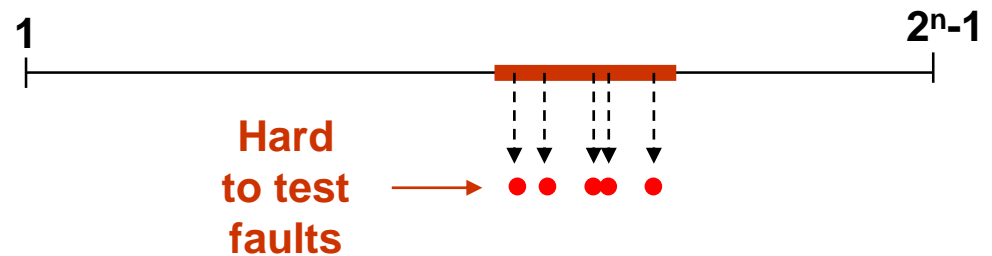
- low generation cost
- high initial efficiency



Problem: **Low fault coverage**

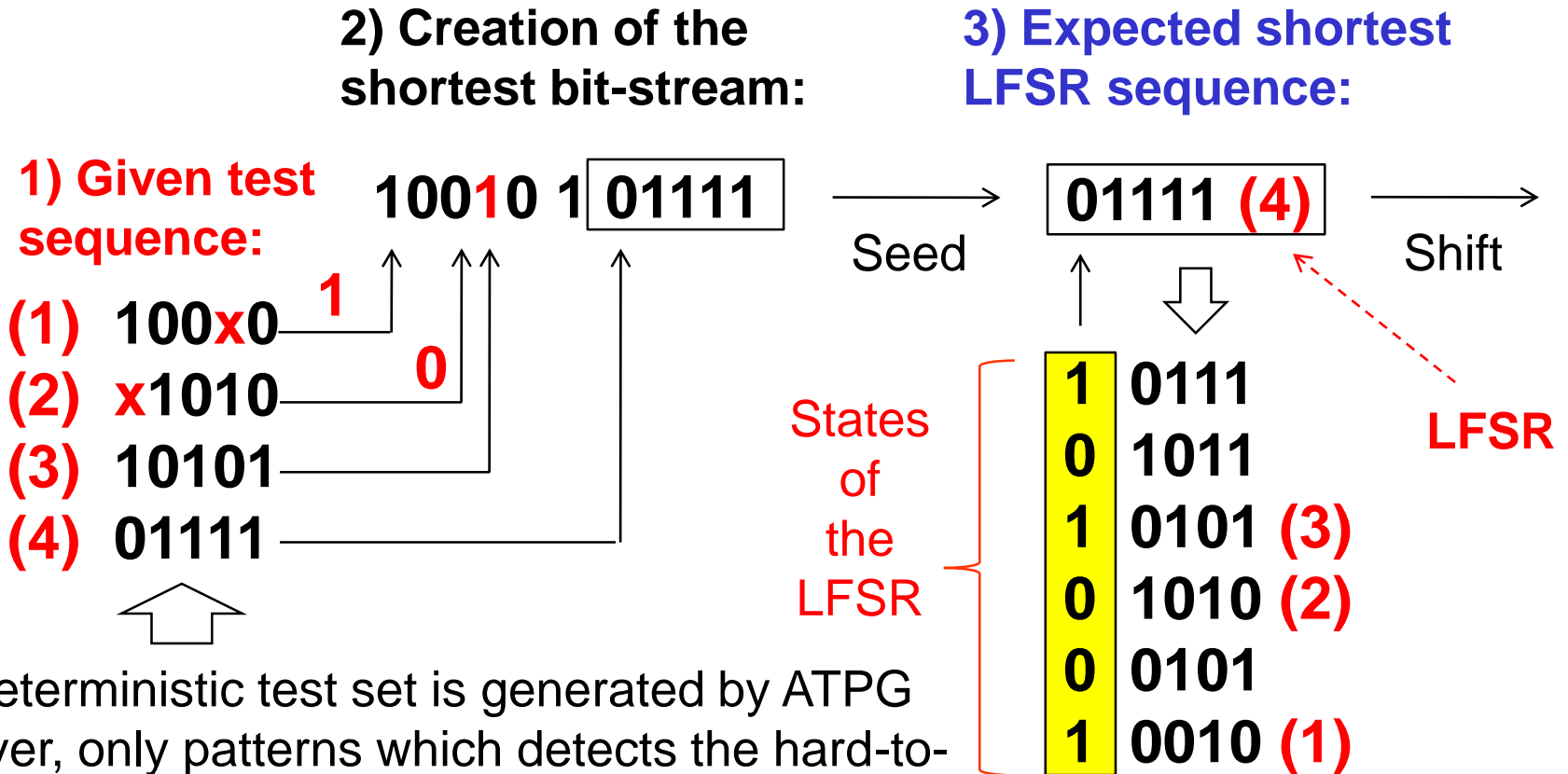


Dream solution: Find LFSR such that:



Deterministic Synthesis of LFSR

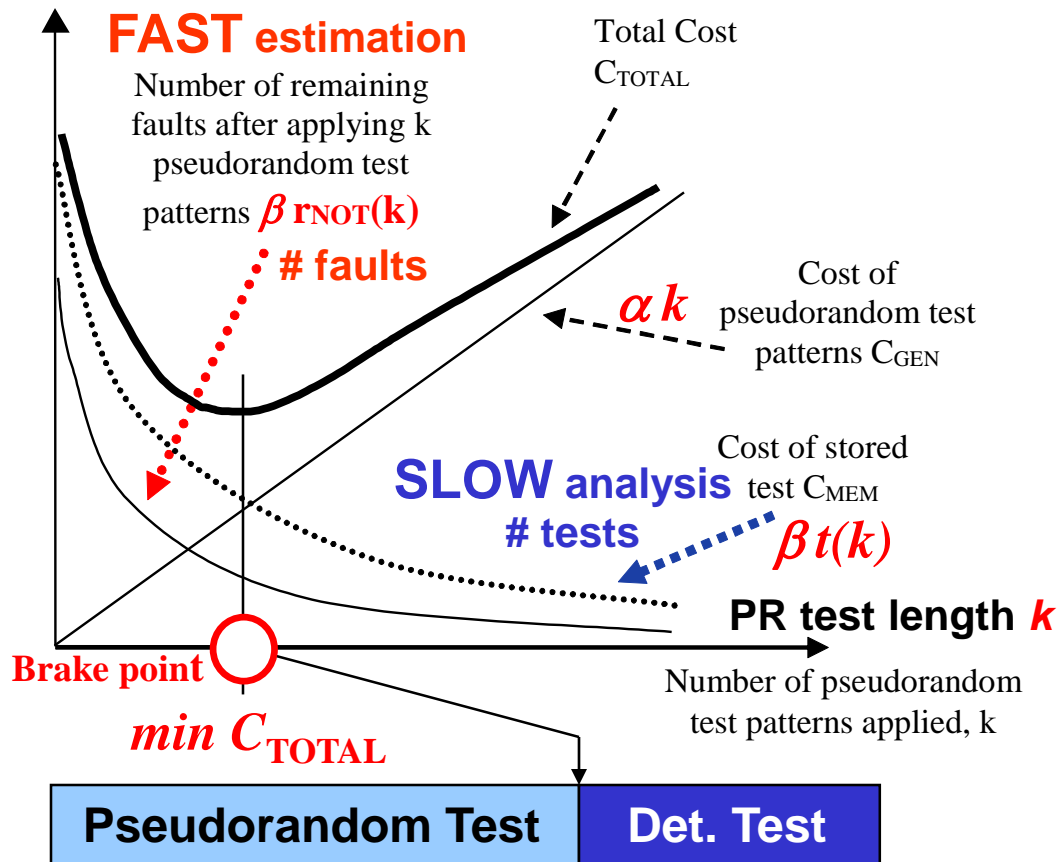
Generation of the polynomial and seed for the given test sequence



This deterministic test set is generated by ATPG
 However, only patterns which detects the hard-to-test faults can be chosen

Optimization of Hybrid BIST

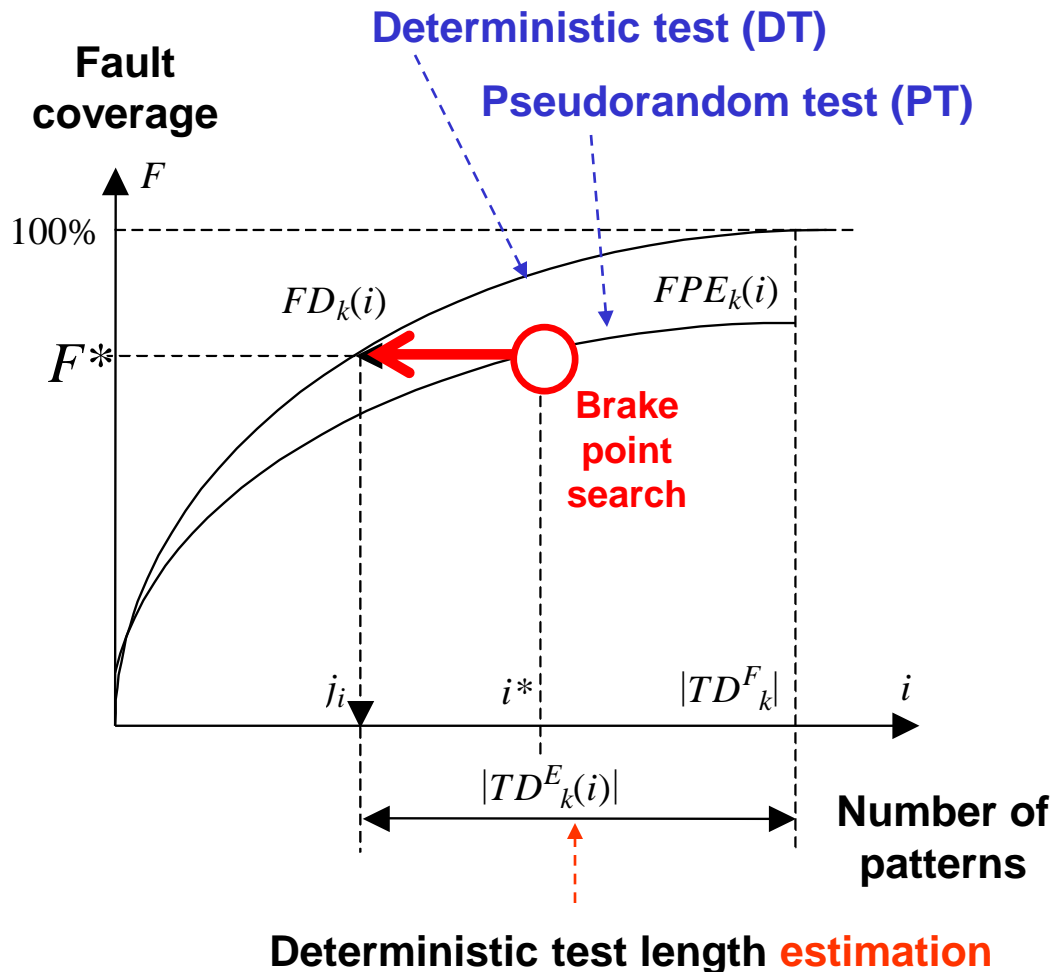
Cost of BIST: $C_{TOTAL} = \alpha k + \beta t(k)$



PR test length	# faults not detected (fast analysis)		# tests needed (slow analysis)	
k	$r_{DET}(k)$	$r_{NOT}(k)$	$FC(k)$	$t(k)$
1	155	839	15.6%	104
2	76	763	23.2%	104
3	65	698	29.8%	100
4	90	608	38.8%	101
5	44	564	43.3%	99
10	104	421	57.6%	95
20	44	311	68.7%	87
50	51	218	78.1%	74
100	16	145	85.4%	52
200	18	114	88.5%	41
411	31	70	93.0%	26
954	18	28	97.2%	12
1560	8	16	98.4%	7
2153	11	5	99.5%	3
3449	2	3	99.7%	2
4519	2	1	99.9%	1
4520	1	0	100.0%	0

How to convert #faults to #tests

Deterministic Test Length Estimation



Fast estimation for the length of deterministic test:

For each PT length i^* we determine

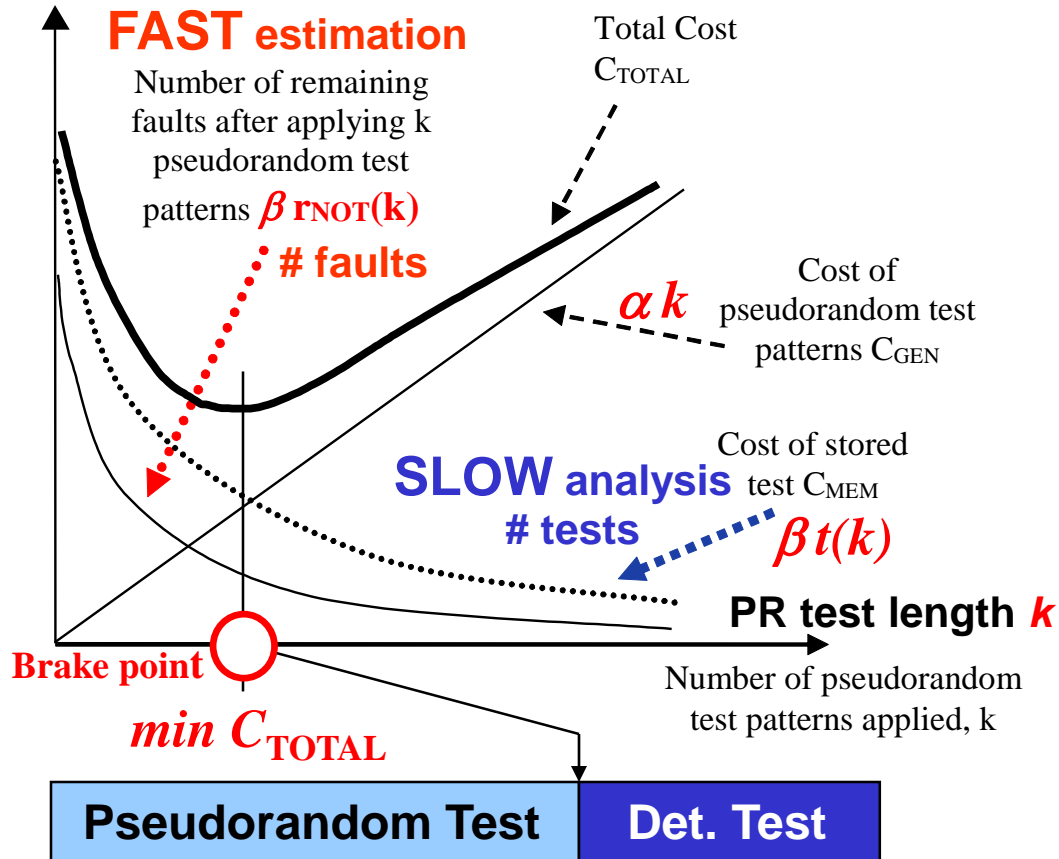
- PT fault coverage F^* , and
- the imaginable part of DT $FD_k(i)$ to be needed for the same fault coverage

Then the remaining part of DT $TD_k^E(i)$ will be the **estimation** of the DT length

Second idea for estimation: estimating number of patterns

Deterministic Test Length Estimation

Cost of BIST: $C_{TOTAL} = \alpha k + \beta t(k)$



PR test length	$r_{DET}(k)$	# faults not detected (fast analysis) $r_{NOT}(k)$	$FC(k)$	# tests needed (slow analysis) $t(k)$
k	$r_{DET}(k)$	$r_{NOT}(k)$	$FC(k)$	$t(k)$
1	155	839	15.6%	104
2	76	763	23.2%	104
3	65	698	29.8%	100
4	90	608	38.8%	101
5	44	564	43.3%	99
10	104	421	57.6%	95
20	44	311	68.7%	87
50	51	218	78.1%	74
100	16	145	85.4%	52
200	18	114	88.5%	41
411	31	70	93.0%	26
954	18	28	97.2%	12
1560	8	16	98.4%	7
2153	11	5	99.5%	3
3449	2	3	99.7%	2
4519	2	1	99.9%	1
4520	1	0	100.0%	0

How to convert #faults to #tests

Calculation of the Deterministic Test Cost

Two possibilities to find the length of deterministic data for each possible breakpoint in the pseudorandom test sequence:

ATPG based approach

For each breakpoint of P-sequence, ATPG is used

Fault table based approach

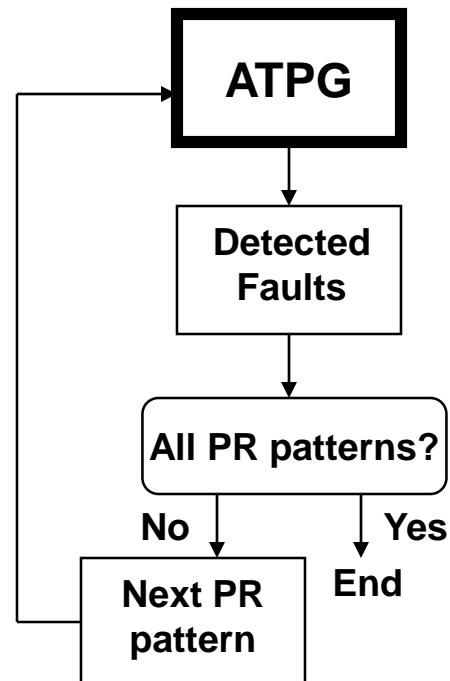
A deterministic test set with fault table is calculated

For each breakpoint of P-sequence, the fault table is updated for not yet detected faults

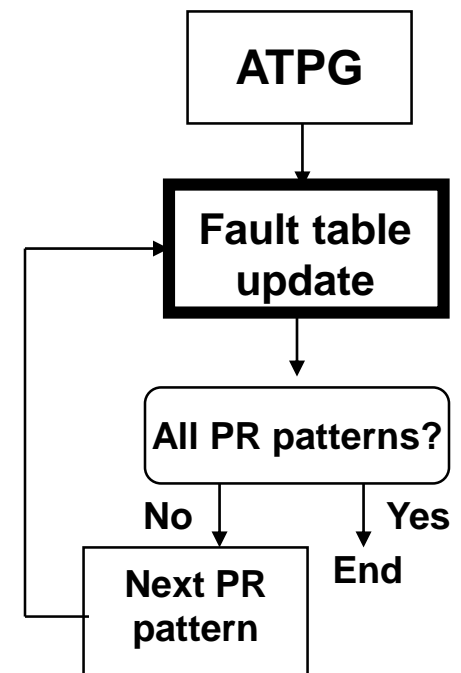
FAST estimation

Only fault coverage is calculated

ATPG based:



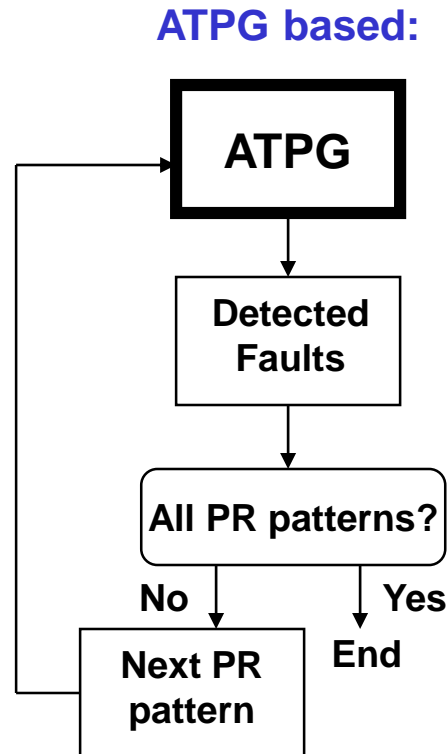
Fault table based:



Calculation of the Deterministic Test Cost

ATPG based approach

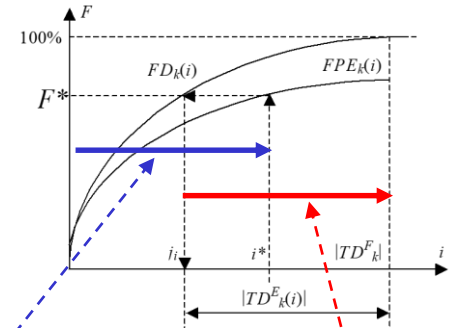
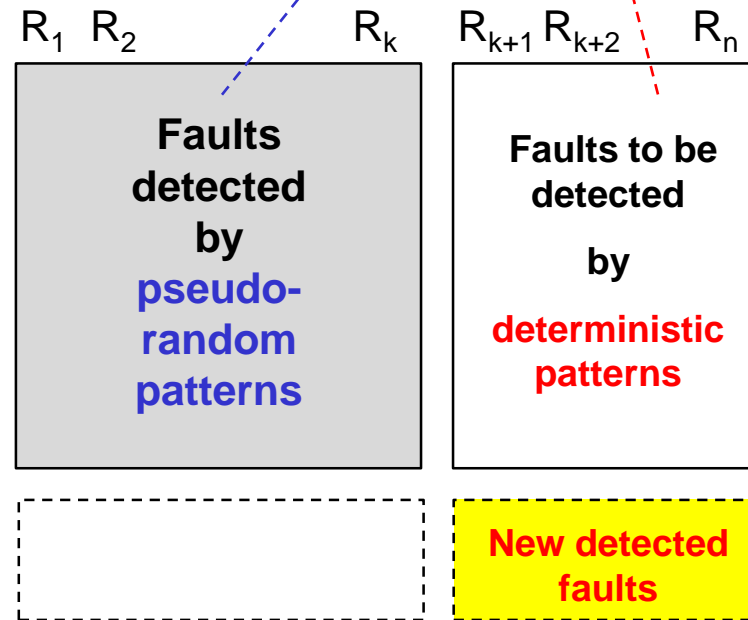
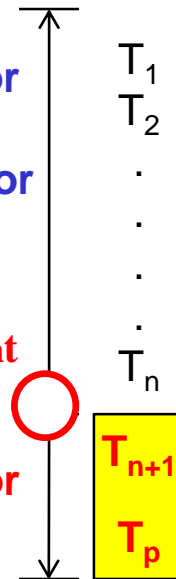
For each breakpoint of P-sequence, ATPG is used



Task for
fault
simulator

Brake point

Task for
ATPG



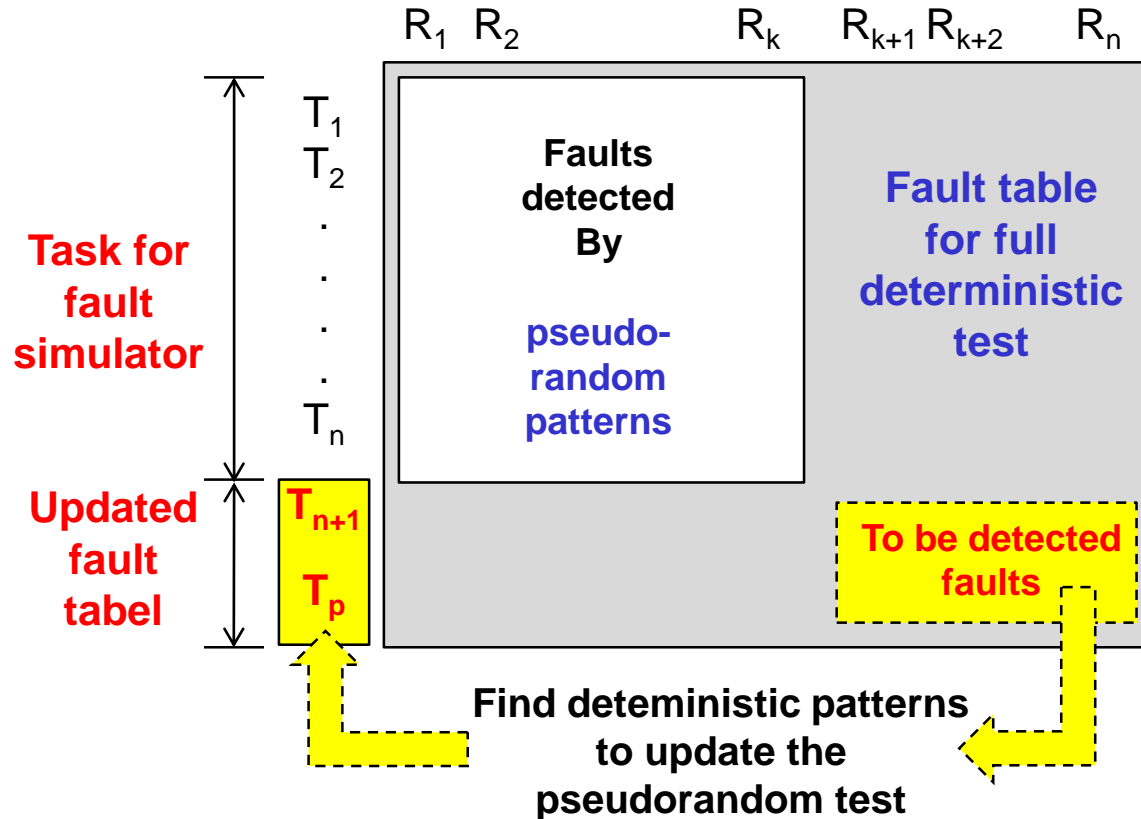
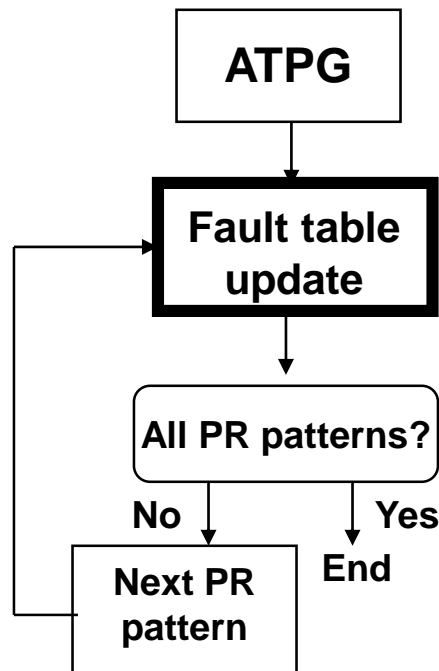
Calculation of the Deterministic Test Cost

Fault table based approach

A deterministic test set with fault table is calculated

For each breakpoint of P-sequence, the fault table is updated and remaining det. patterns are determined

Fault table based:



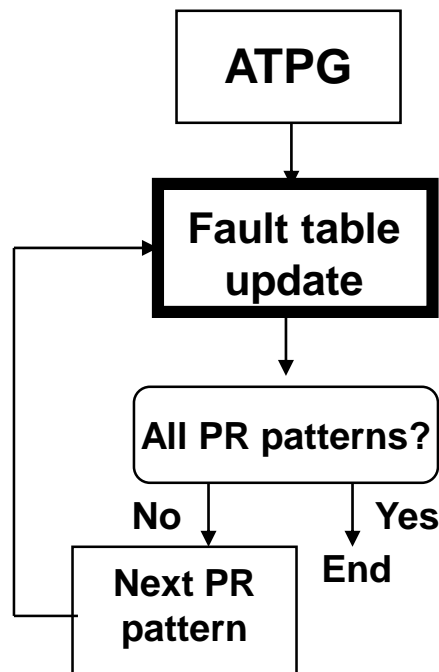
Calculation of the Deterministic Test Cost

Fault table based approach

A deterministic test set with fault table is calculated

For each breakpoint of P-sequence, the fault table is updated and remaining det. patterns are determined

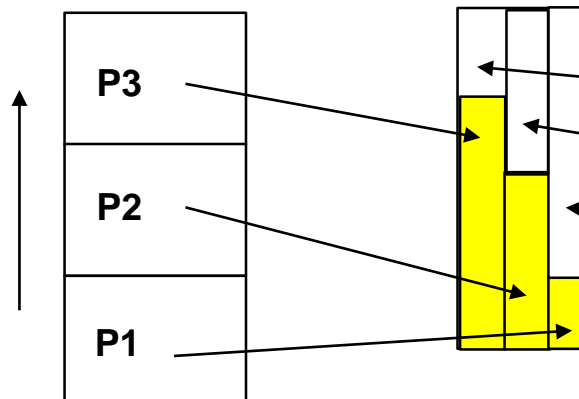
Fault table based:



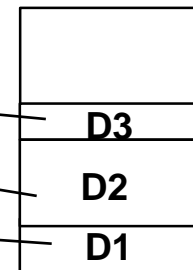
Pseudo-random patterns



Fault table coverage



Deterministic patterns



Experimental Data: HybBIST Optimization

Finding optimal brakepoint in the pseudorandom sequence:



Optimized hybrid test process:

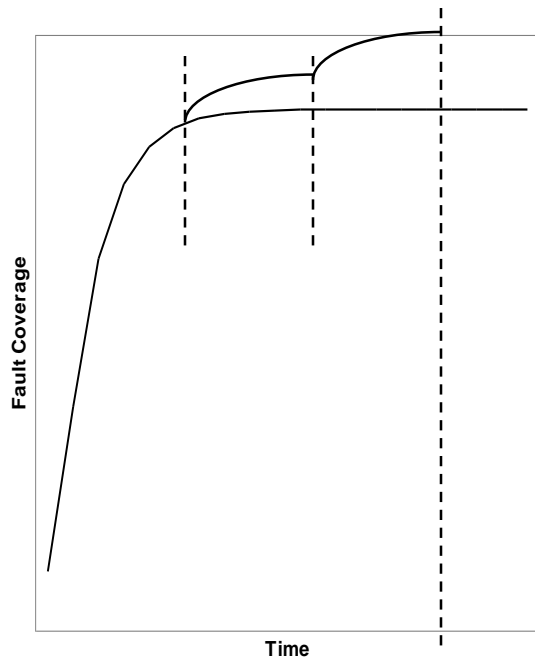


<i>Circuit</i>	L_{MAX}	L_{OPT}	S_{MAX}	S_{OPT}	B_k	G_{TOTAL}
C432	780	91	80	21	4	175
C499	2036	78	132	60	6	438
C880	5589	121	77	48	8	505
C1355	1522	121	126	52	6	433
C1908	5803	105	143	123	5	720
C2670	6581	444	155	77	30	2754
C3540	8734	297	211	110	7	1067
C5315	2318	711	171	12	23	987
C6288	210	20	45	20	4	100
C7552	18704	583	267	61	51	3694

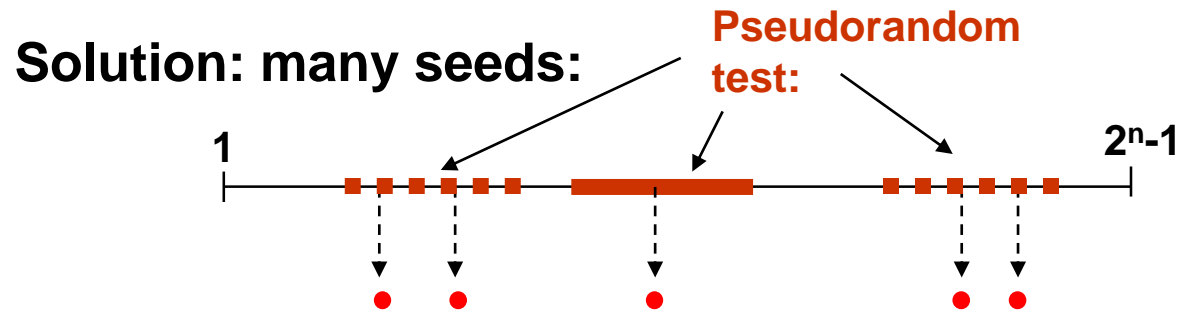
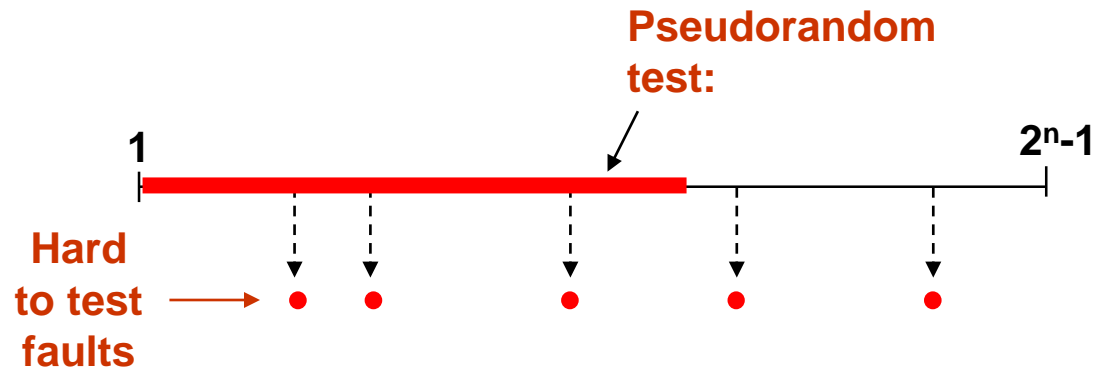
Hybrid BIST with Reseeding

The motivation of using random patterns is:

- low generation cost
- high initial efficiency

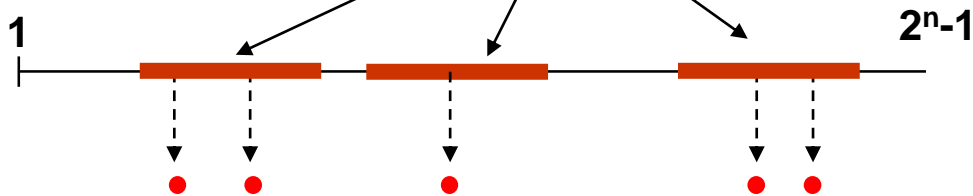


Problem: **low fault coverage** → long PR test

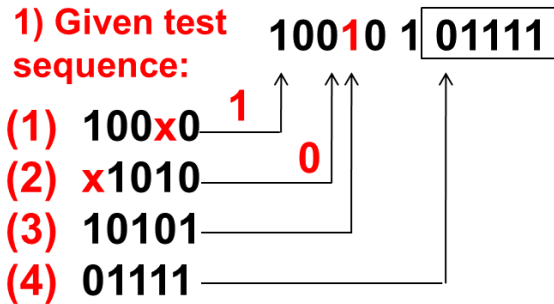


Hybrid BIST with Reseeding

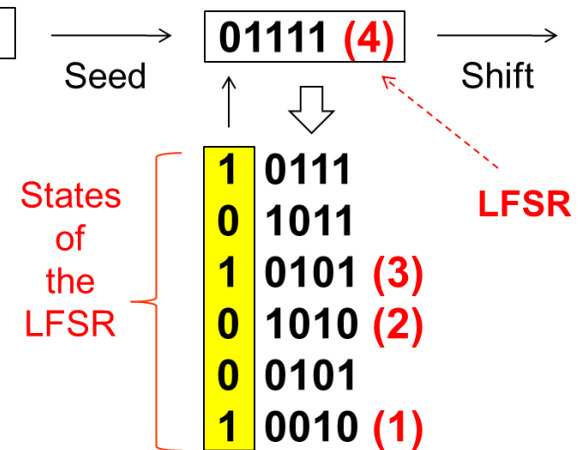
Using many seeds: **Pseudorandom test (with different polynomials):**



Creation of the shortest bit-stream:



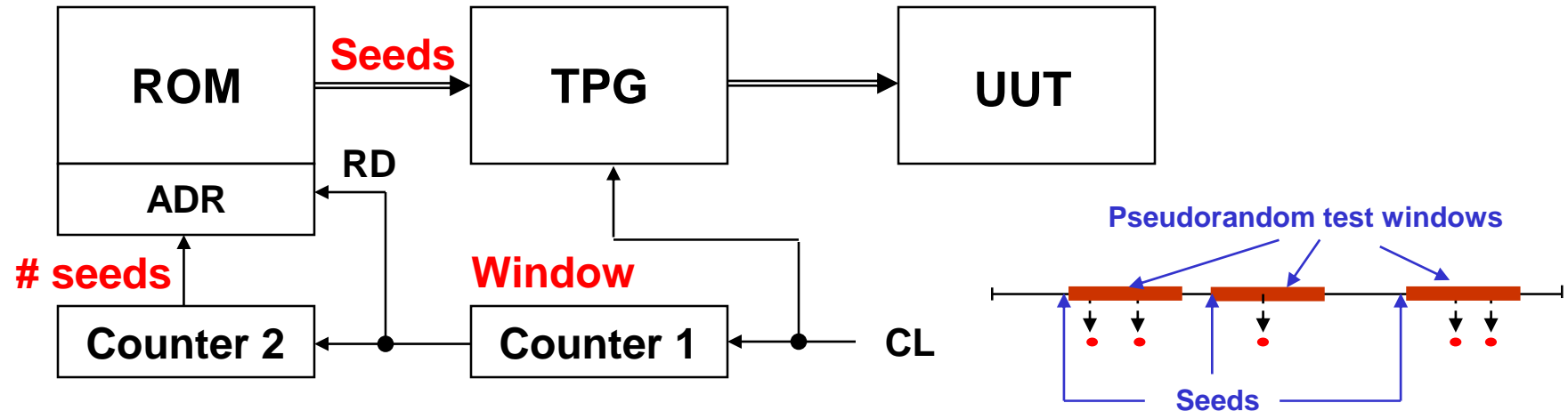
Expected shortest LFSR sequence:



Problems:

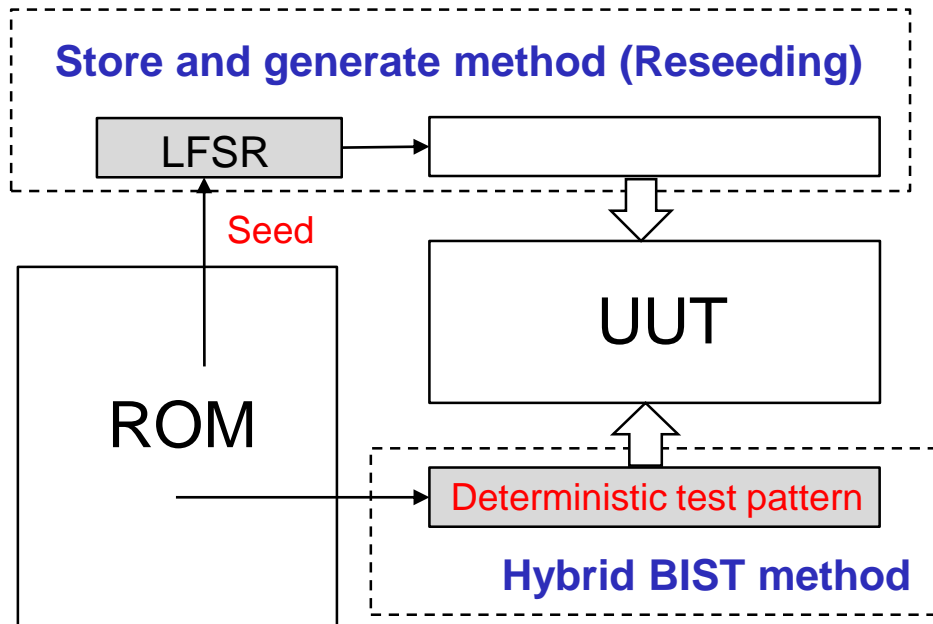
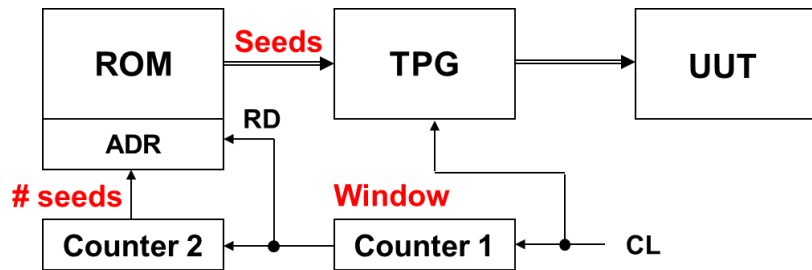
Which polynomials and seeds should be used for the blocks?

Store-and-Generate Test Architecture

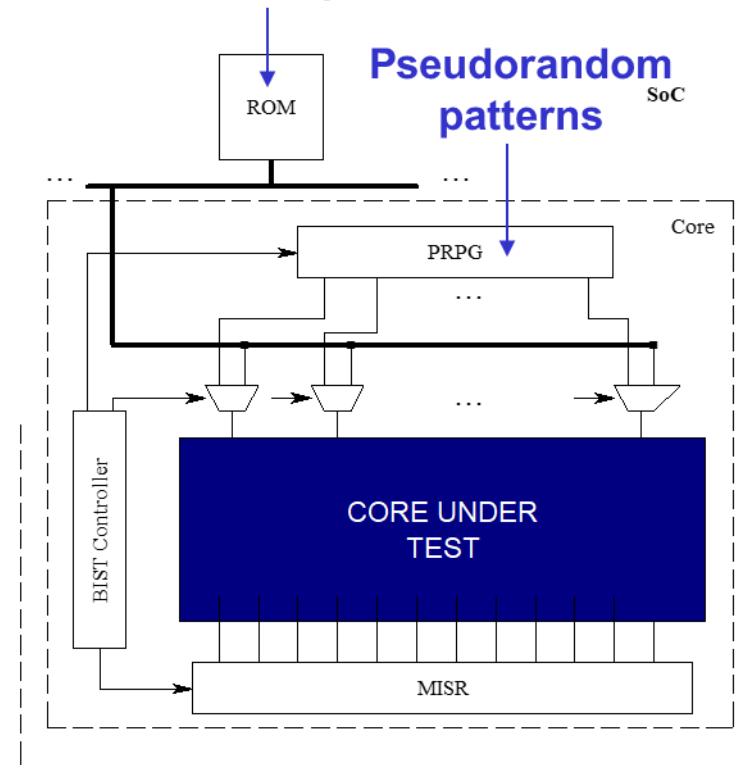


- **ROM** contains deterministic data for BIST control to target **hard-to-test-faults**
- Each pattern P_k in ROM serves as an initial state of the LFSR for test pattern generation (TPG) - **seeds**
- Counter 1 counts the number of pseudorandom patterns generated starting from P_k - **width of the windows**
- After finishing the cycle for Counter 2 is incremented for reading the next pattern P_{k+1} – for **starting the new window**

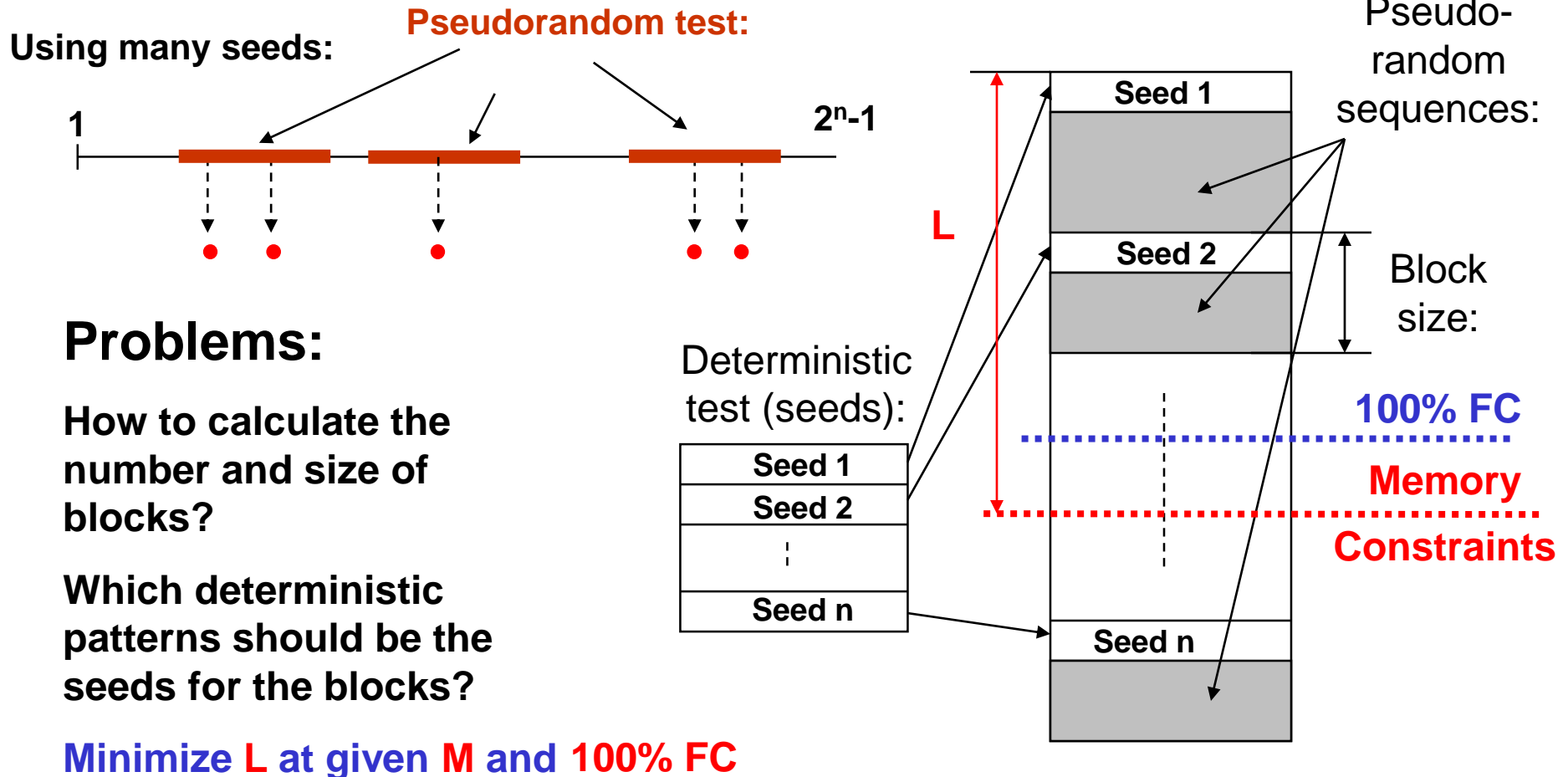
Store-and-Generate vs. Hybrid BIST



Deterministic patterns

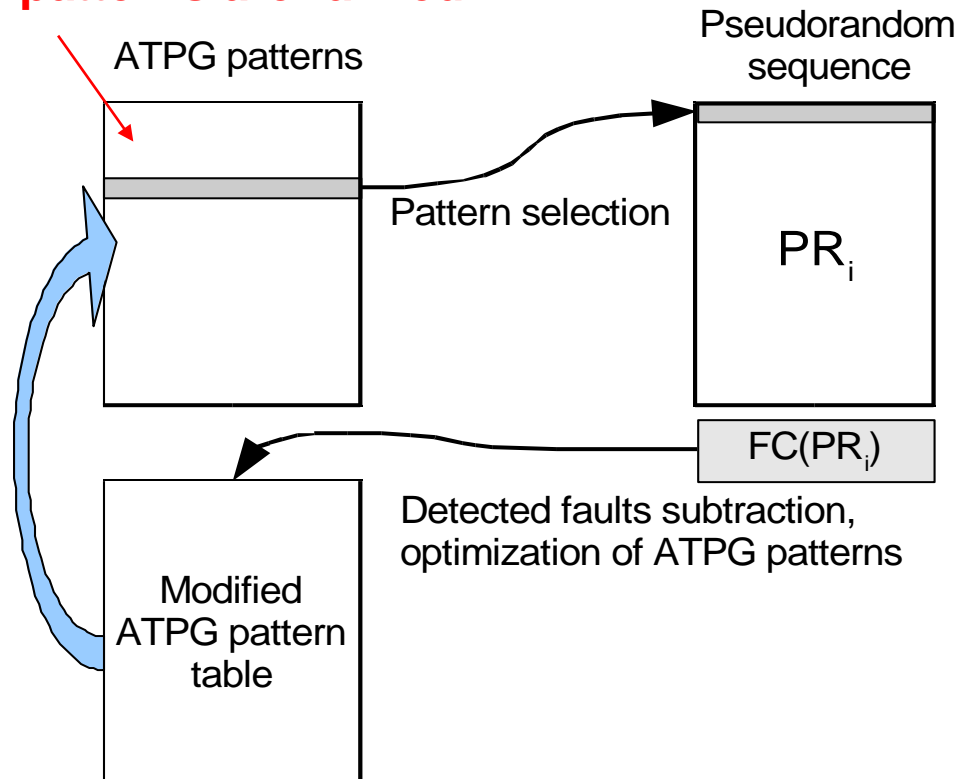


HBIST Optimization Problem



Hybrid BIST Optimization Algorithm 1

D-patterns are ranked



Algorithm is based on **D-patterns ranking**

Deterministic test patterns with 100% quality are generated by ATPG

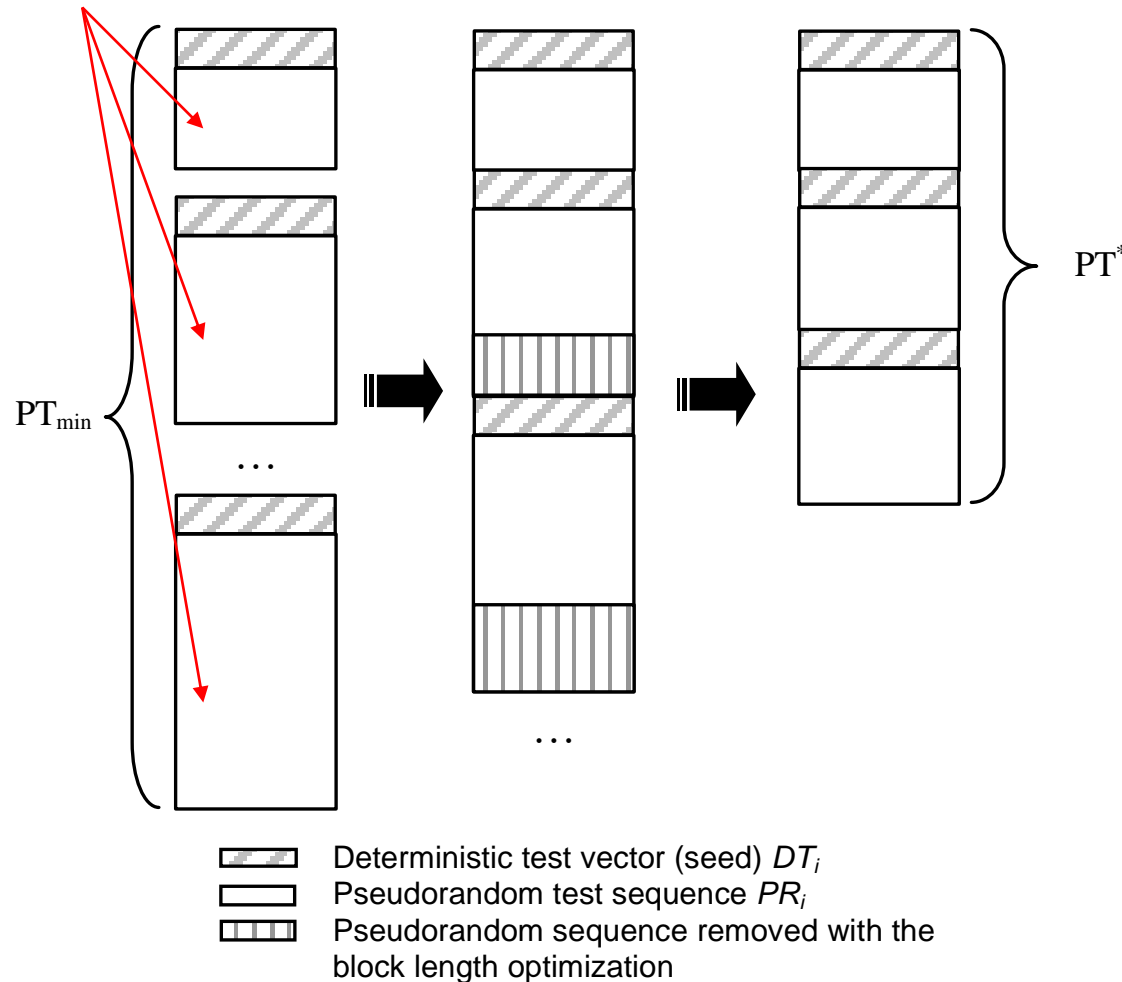
The best pattern is selected as a seed

A pseudorandom block is produced and the fault table of ATPG patterns is updated

The procedure ends when 100% fault coverage is achieved

Hybrid BIST Optimization Algorithm 2

P-blocks are ranked



Algorithm is based on **P-blocks ranking**

Deterministic test patterns with 100% quality are generated by ATPG

All P-blocks are generated for all D-patterns and ranked

The best P-block is selected included into sequence and updated

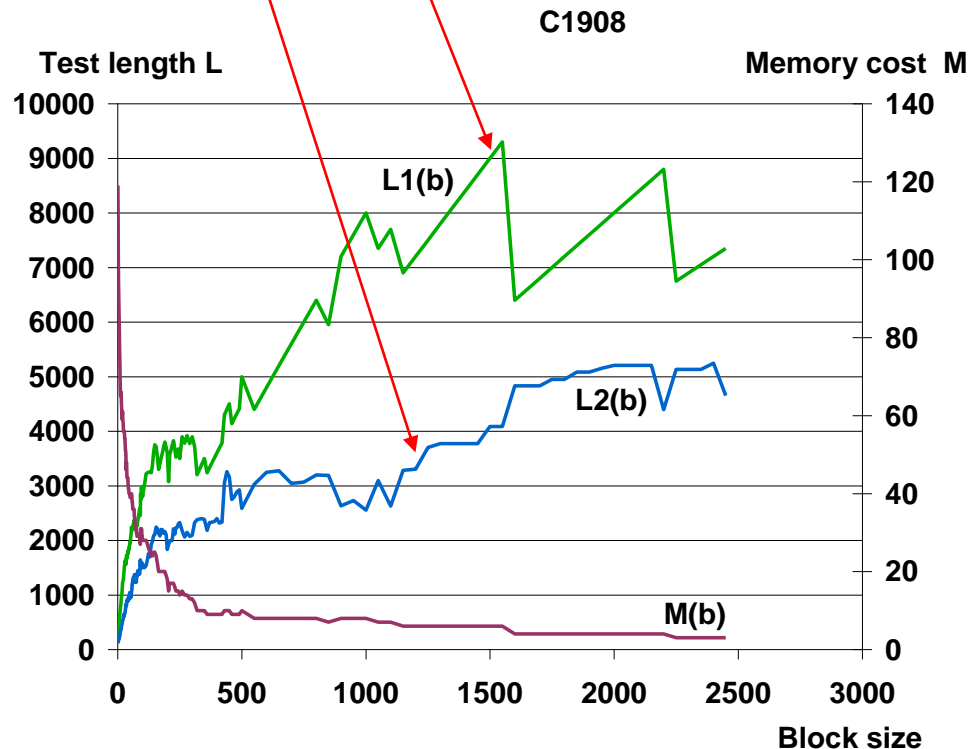
The procedure ends when **100% fault coverage is achieved**

Cost Curves for Hybrid BIST with Reseeding

Two possibilities for reseeding:

Constant block length (less HW overhead)

Dynamic block length (more HW overhead)



Functional Self-Test

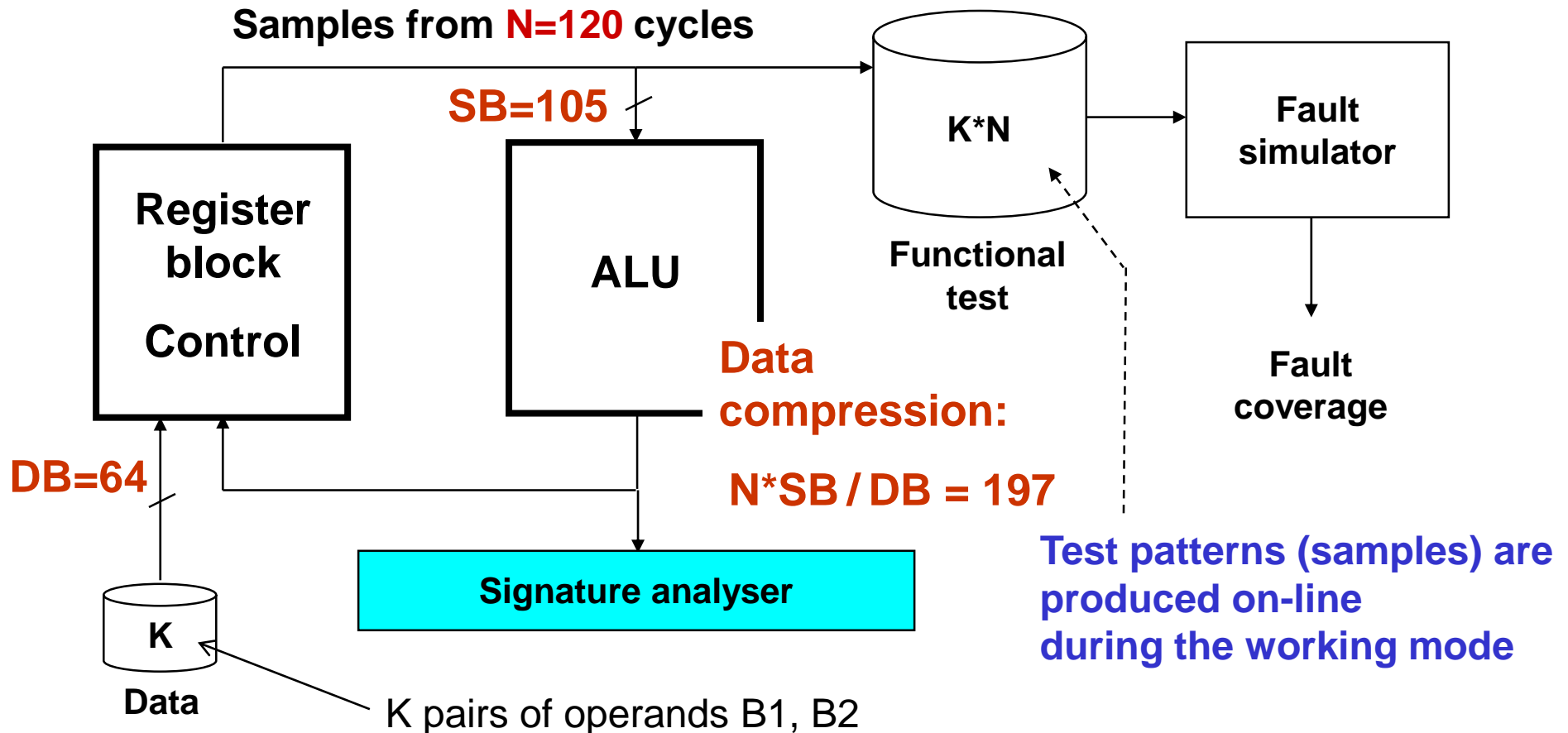
- **Traditional BIST** solutions use **special hardware** for pattern generation on chip, this may introduce area overhead and performance degradation
- New methods have been proposed which **exploit specific functional units** like arithmetic blocks or processor cores for on-chip test generation
- It has been shown that **adders** can be used as test generators for pseudorandom and deterministic patterns
- Today, there is **no general method** how to use arbitrary functional units for built-in test generation

Hybrid Functional BIST

- To **improve the quality** of FBIST we introduce the method of Hybrid FBIST
- The **idea of Hybrid FBIST** consists in using the mixture of
 - functional patterns produced by the microprogram (no additional HW is needed), and
 - additional stored deterministic test patterns to improve the total fault coverage (HW overhead: MUX-es, Memory)
- Tradeoffs **should be found** between
 - the testing time and
 - the HW/SW overhead cost

Example: Functional BIST for Divider

Functional BIST quality analysis for

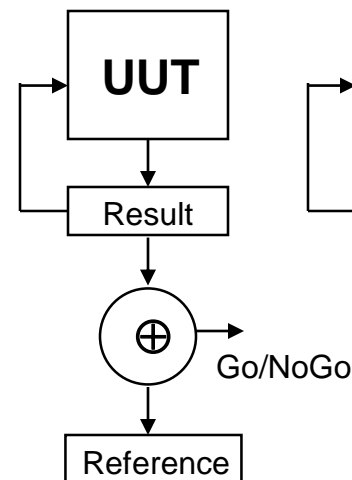


Example: Functional BIST Quality for Divider

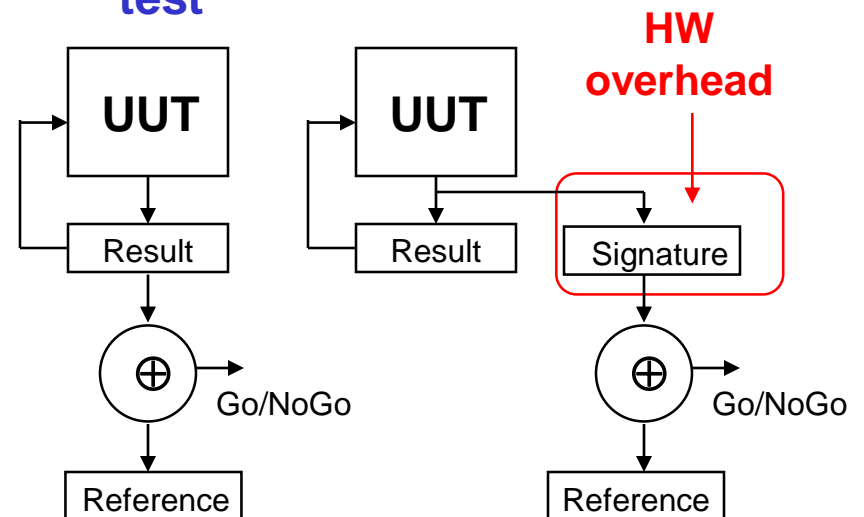
Fault coverage of FBIST compared to Functional test

Data	Functional testing			Functional BIST		
	B1	B2	Total	B1	B2	Total
4/2	13.21	15.09	14.15	35.14	40.57	29.72
7/2	21.23	16.98	19.10	38.44	47.64	29.25
6/3	19.34	31.6	25.47	41.04	39.62	42.45
8/2	25.47	10.38	17.92	32.07	40.57	25.00
9/4	8.96	5.66	7.31	36.56	47.64	25.47
9/3	32.55	26.89	29.72	43.63	46.07	40.57
12/6	13.44	8.02	18.87	36.08	39.62	32.55
14/2	18.16	25.00	11.32	37.50	49.06	25.94
15/3	29.48	31.13	27.83	47.88	50.00	45.75
2/4	7.8	7.55	8.02	29.01	20.75	33.02
Aver.	18.96	17.83	17.97	37.74	42.15	32.97
Gain	1.0	1.0	1.0	2.0	2.4	1.8

Traditional Functional test



FBIST

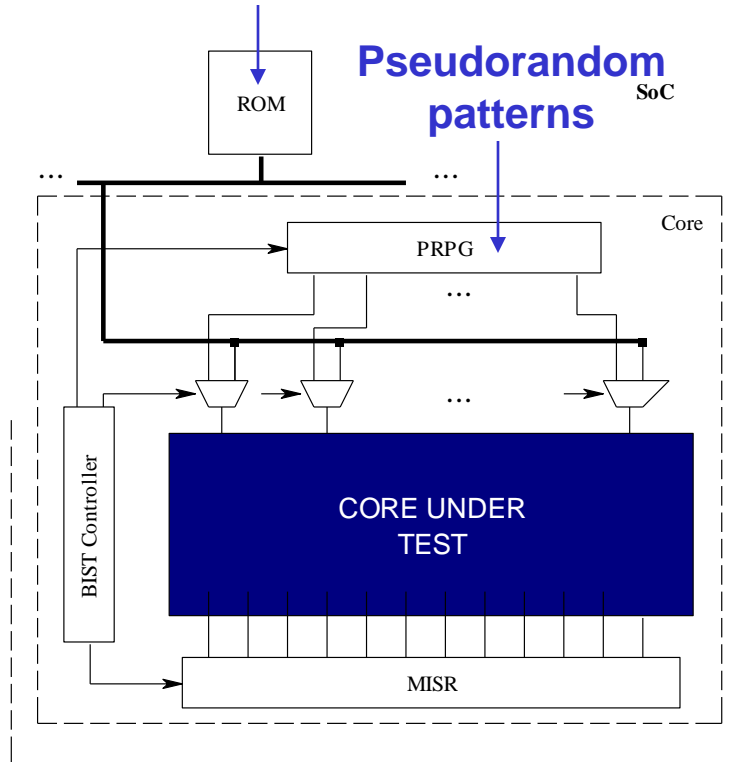


FBIST: collection and analysis of samples during the working mode

Fault coverage is better, however, still very low (ranging from 42% to 70%)

Hybrid Built-In Self-Test

Deterministic patterns



Hybrid test set contains
pseudorandom and
deterministic vectors

Pseudorandom test is improved
by a stored test set which is
specially generated to target the
random resistant faults

Optimization problem:

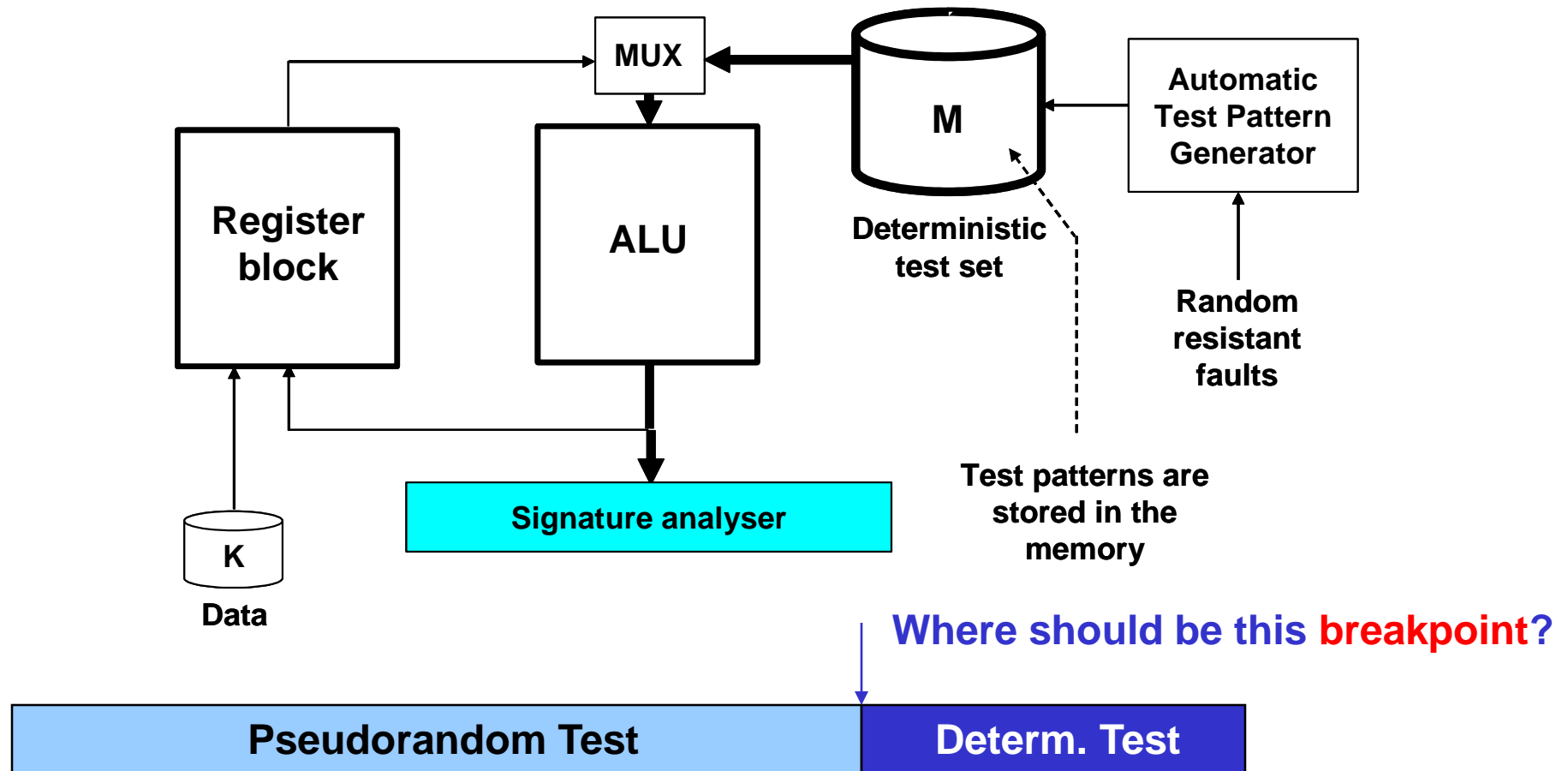
Where should be this **breakpoint**?

Pseudorandom Test

Determ. Test

Hybrid Functional BIST for Divider

Hybrid Functional BIST implementation



Cost Functions for Hybrid Functional BIST

Total cost:

$$C_{Total} = C_{FB_Total} + C_{D_Total}$$

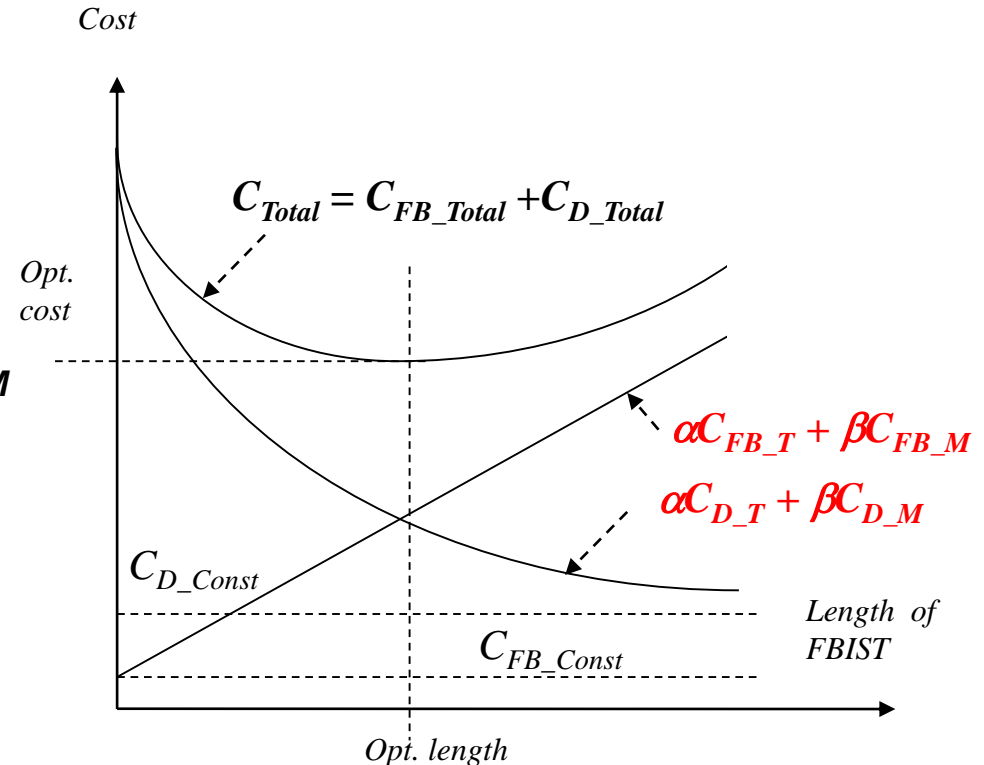
The cost of **functional test** part:

$$C_{FB_Total} = C_{FB_Const} + \alpha C_{FB_T} + \beta C_{FB_M}$$

The cost of **deterministic test** part:

$$C_{D_Total} = C_{D_Const} + \alpha C_{D_T} + \beta C_{D_M}$$

- | | | |
|-----------------|----------------|---------------------------------------|
| C_{FB_Const} | C_{D_Const} | - HW/SW overhead |
| C_{FB_T} | C_{D_T} | - testing time cost |
| α, β | | - weights of time and memory expenses |



Problem: minimize C_{Total}

Hybrid Functional BIST Quality

Hyb FBIST with multiple seeds (data operands)

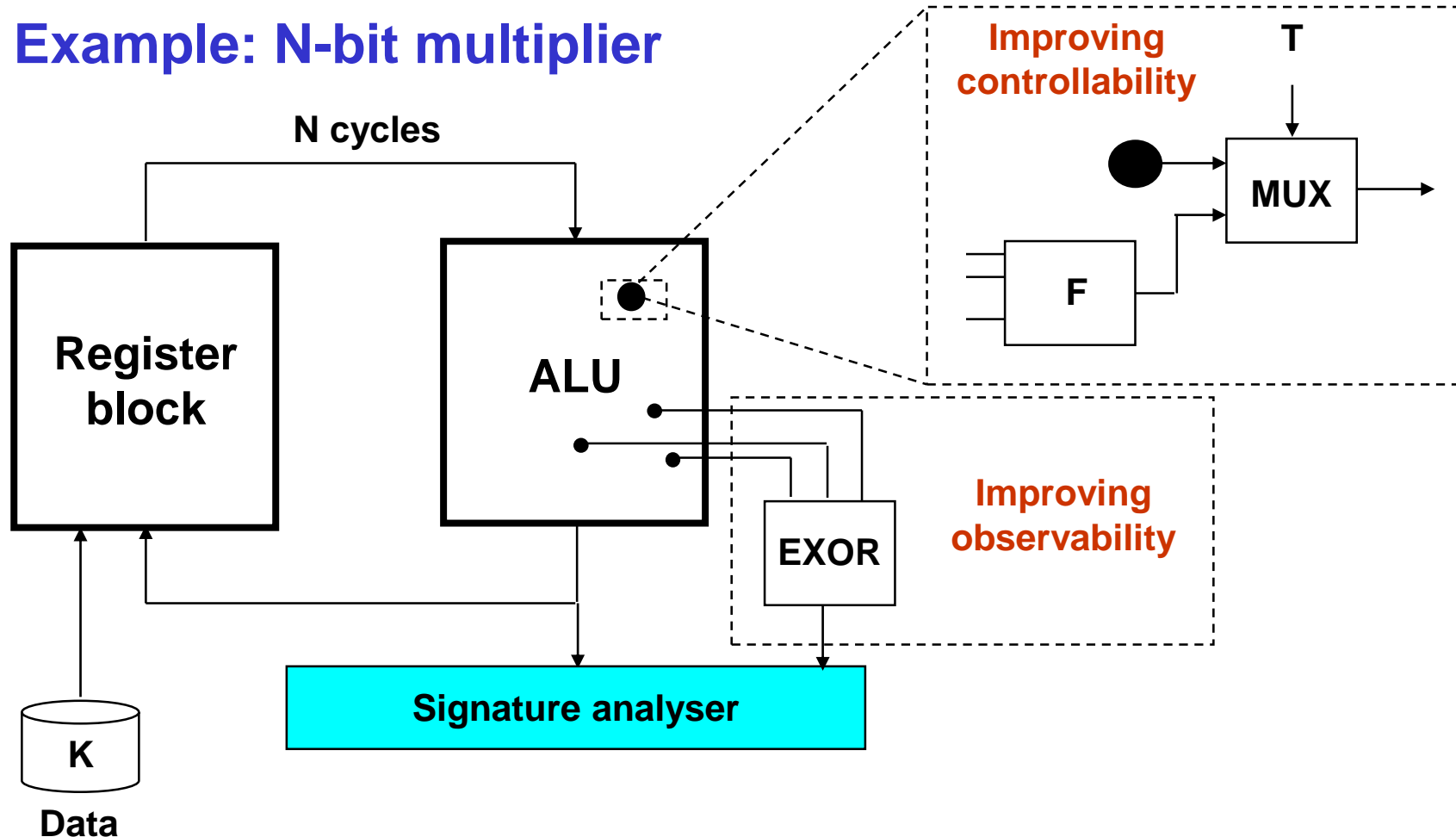
Functional test part					Determ. test part		Total cost
k	N_j	N	FC %	Total cost	D	Total cost	
0	0	0	100	0	58	6148	6148
1	108	108	66,8	140	24	2544	2684
2	105	213	76,7	277	18	1908	2185
3	113	326	83,3	518	17	1802	2320
4	108	434	85,5	690	16	1696	2386
5	110	544	88,4	864	15	1590	2454

k – number of operands used in the FBIST

The fault coverage increases if **k** increases

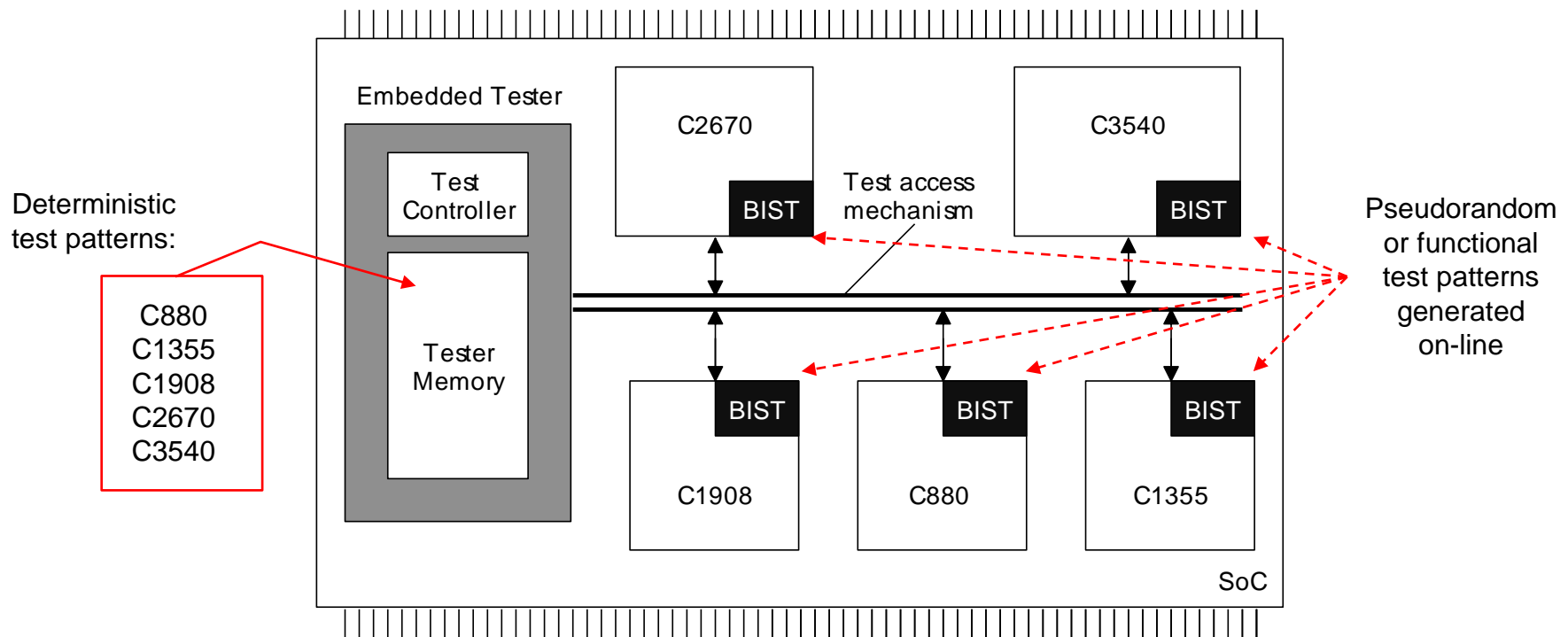
Functional Self-Test with DFT

Example: N-bit multiplier

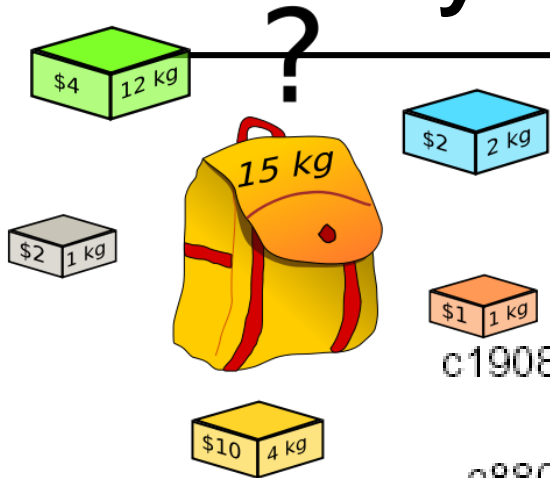


Hybrid BIST for Multiple Cores

Embedded tester for testing multiple cores



Hybrid BIST for Multiple Cores



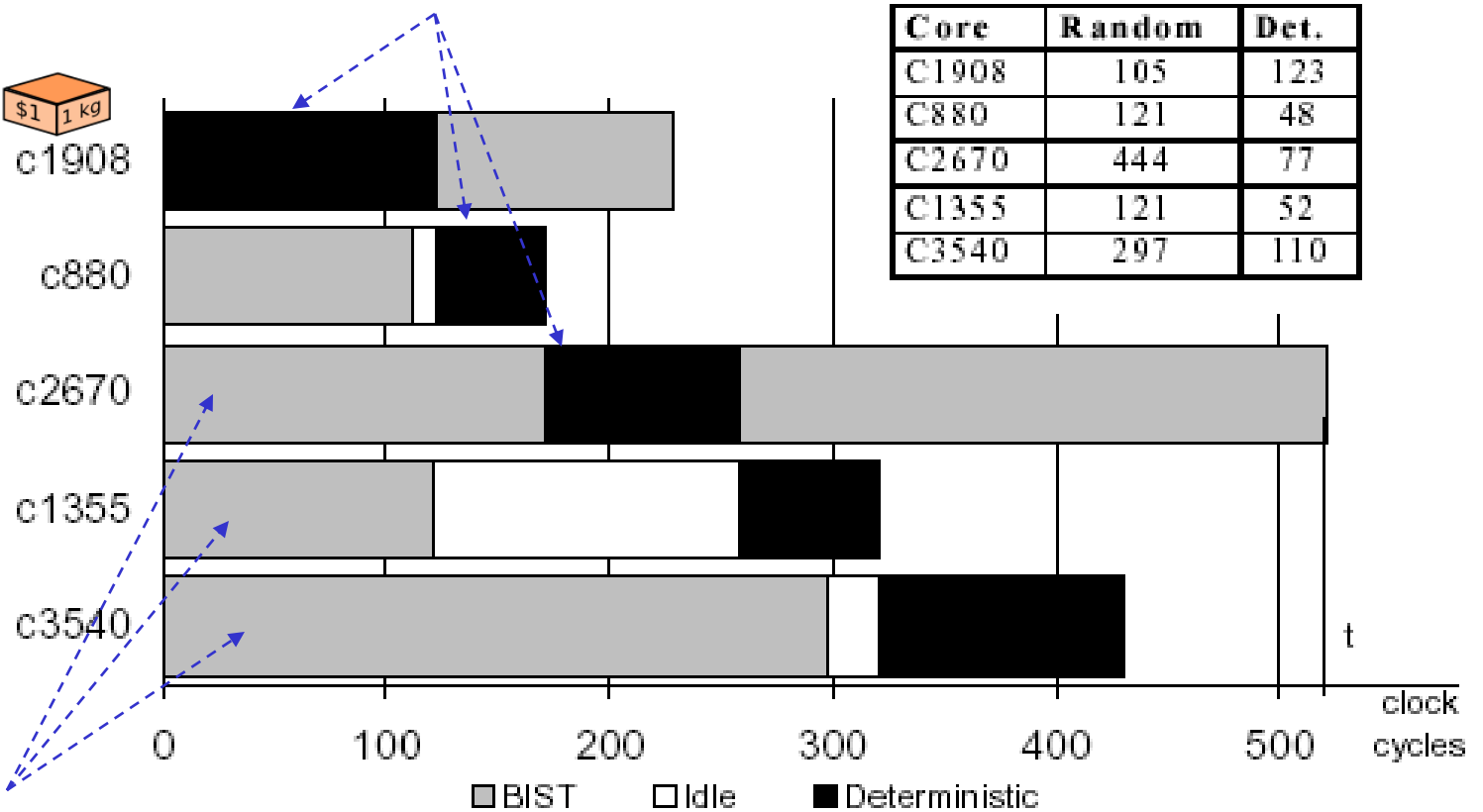
How to pack knapsack?

How to compress the test sequence?

Deterministic test (DT)

The optimal test set for each core

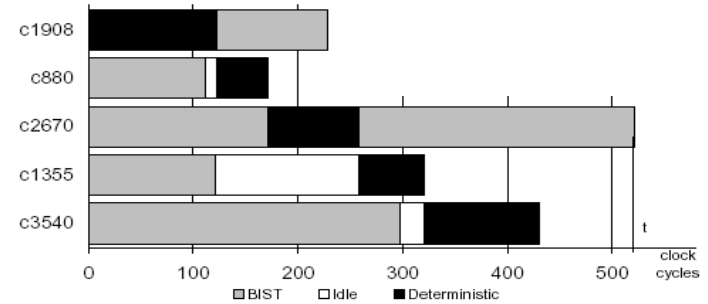
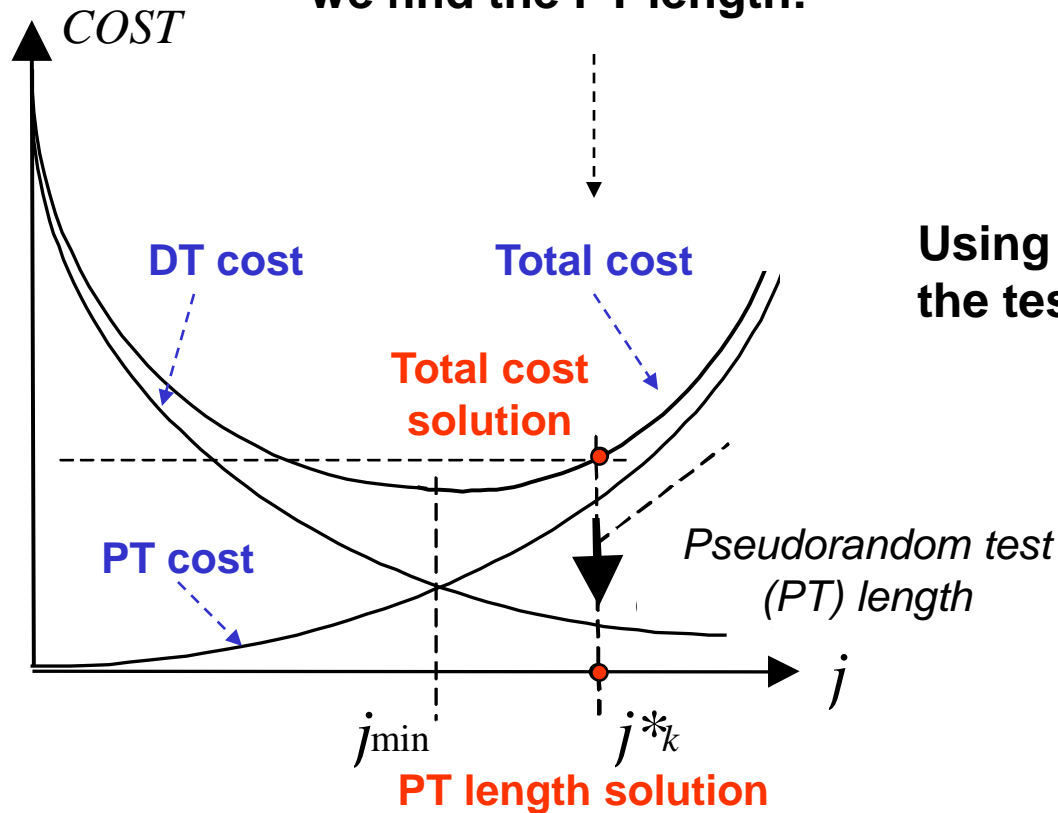
Core	Random	Det.
C1908	105	123
C880	121	48
C2670	444	77
C1355	121	52
C3540	297	110



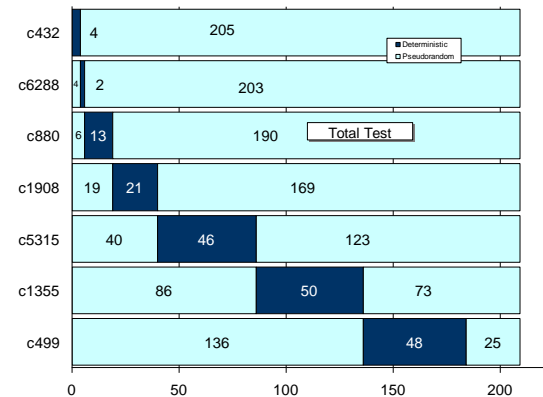
Pseudorandom test (PT)

Total Test Cost Estimation

Using total cost solution we find the PT length:

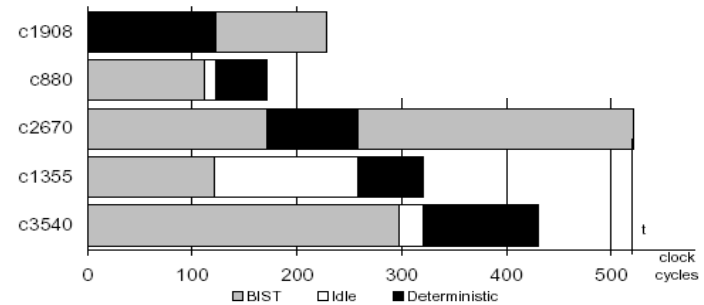
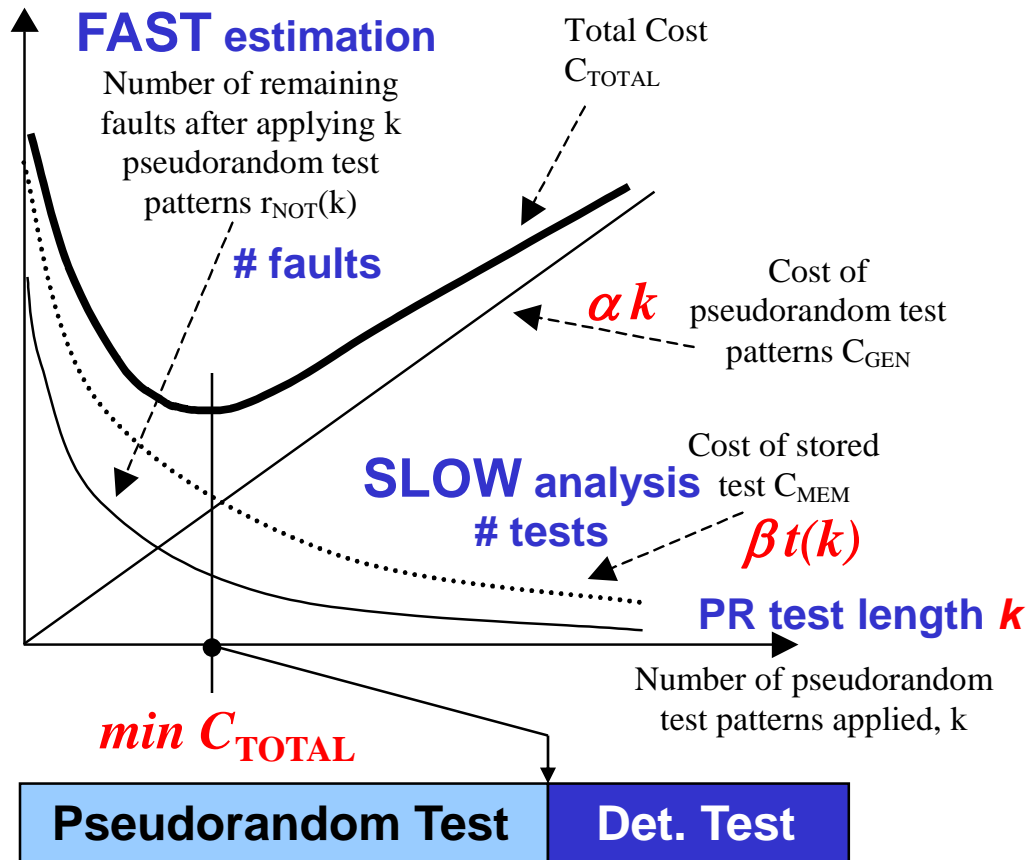


Using PT length, we calculate the test processes for all cores:



Multi-Core Hybrid BIST Optimization

Cost of BIST: $C_{TOTAL} = \alpha k + \beta t(k)$

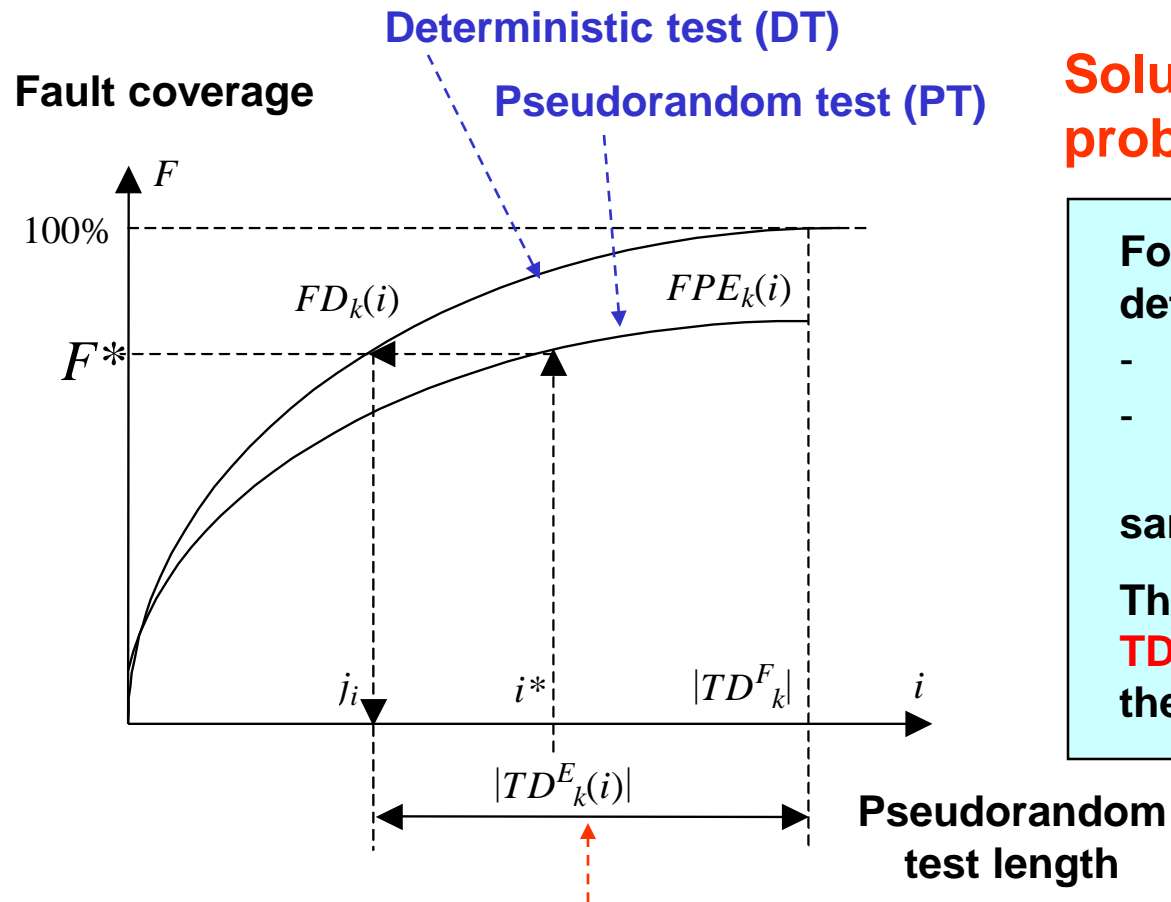


Two problems:

- 1) Calculation of DT $\beta t(k)$ cost is difficult
- 2) We have to optimize n (!) processes

How to avoid the calculation of the very expensive full DT $\beta t(k)$ cost curve?

Deterministic Test Length Estimation



Solution of the first problem:

For each PT length i^* we determine

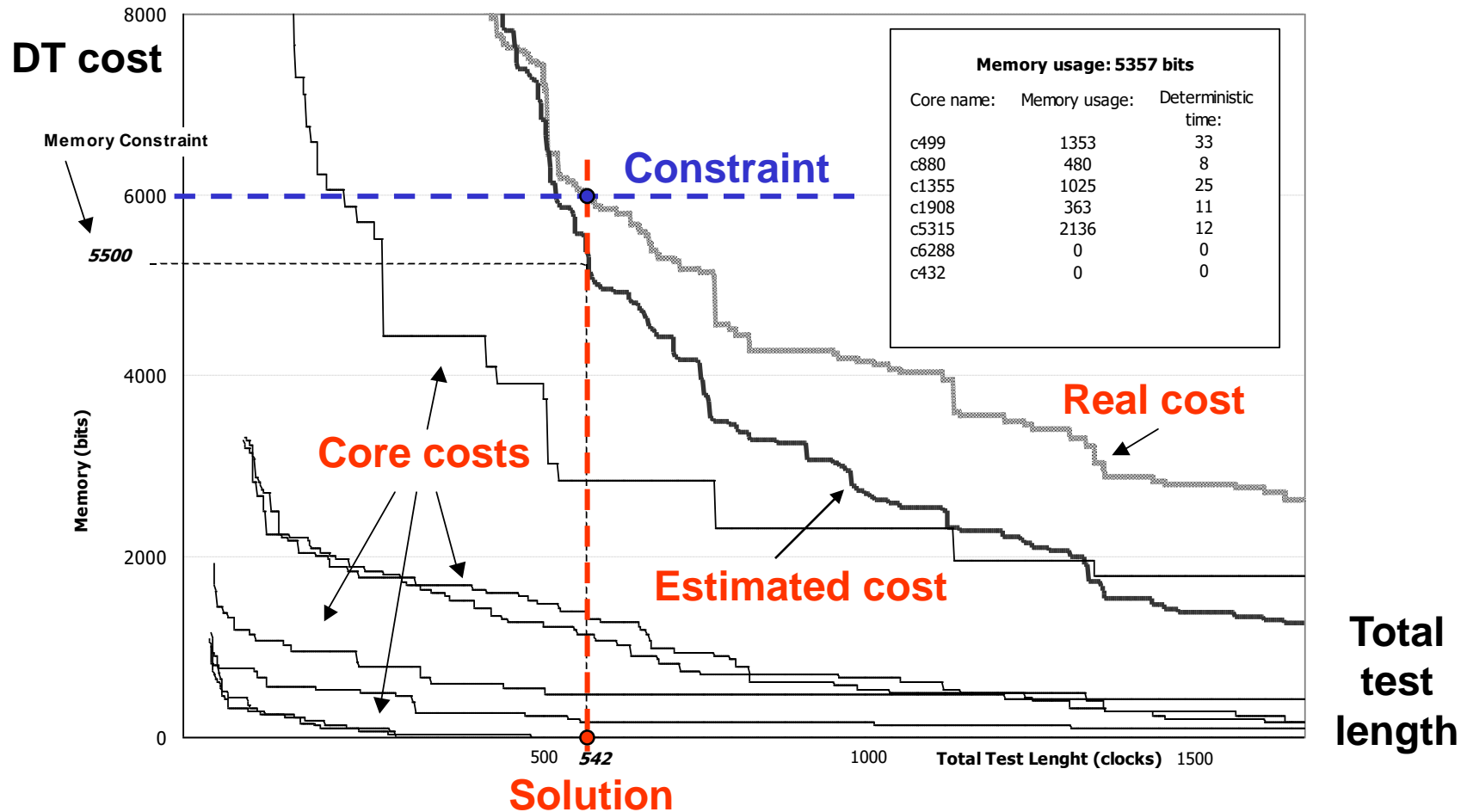
- PT fault coverage F^* , and
- the imaginable part of DT $FD_k(i)$ to be used for the same fault coverage

Then the remaining part of DT $TD_k^E(i)$ will be the **estimation** of the DT length

Deterministic test length estimation for a single core

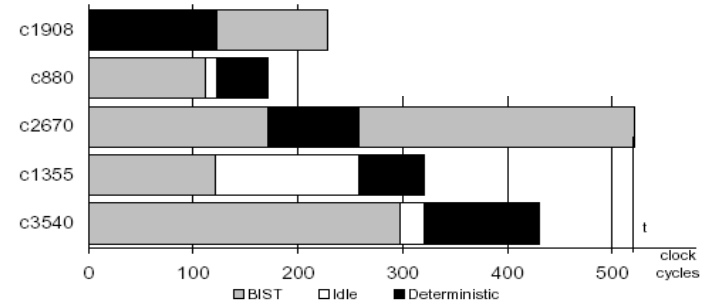
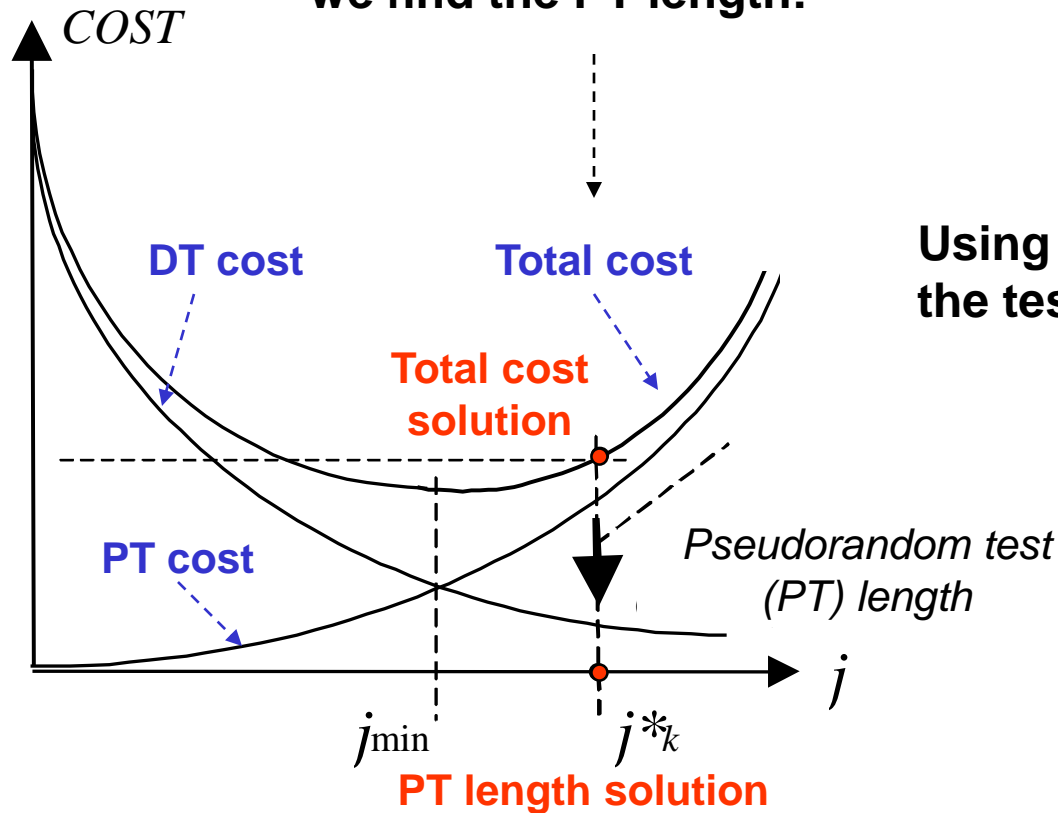
Deterministic Test Cost Estimation

Total cost calculation of core costs:

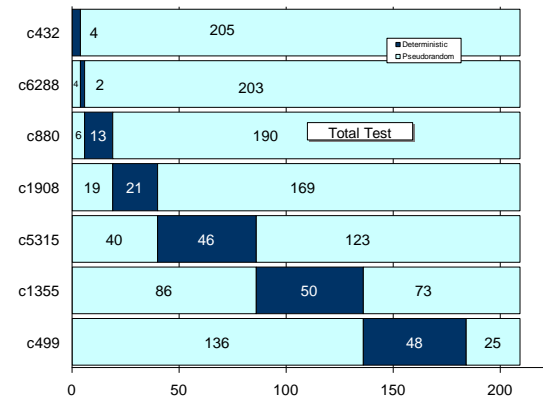


Total Test Cost Estimation

Using total cost solution we find the PT length:

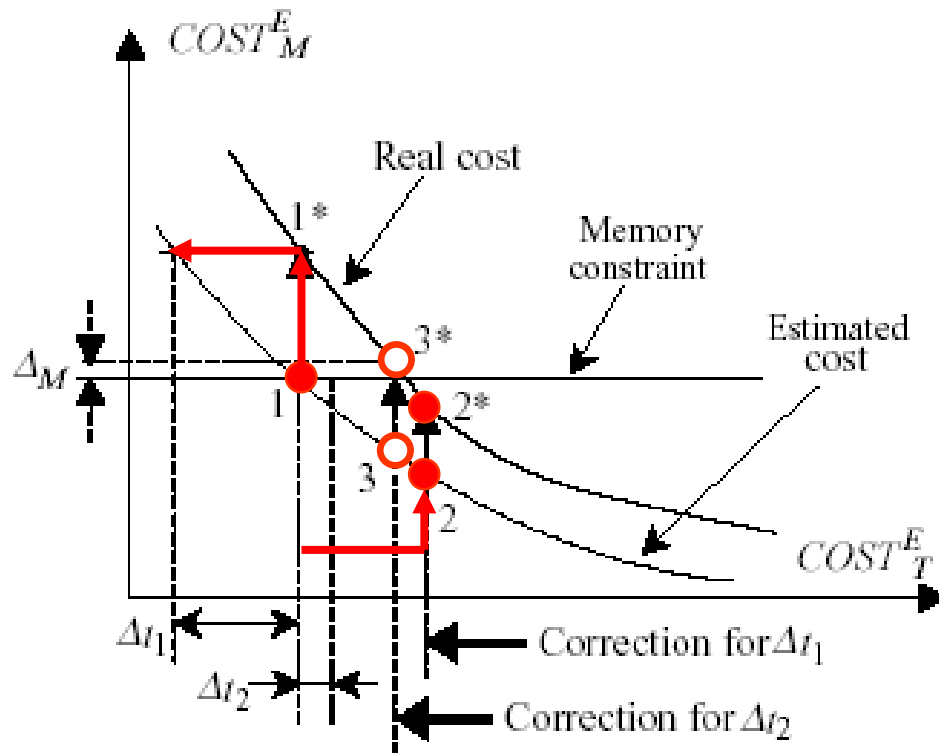


Using PT length, we calculate the test processes for all cores:



Multi-Core Hybrid BIST Optimization

Iterative optimization process:



1 - **First estimation**

1* - **Real cost calculation**

2 - **Correction of the estimation**

2* - **Real cost calculation**

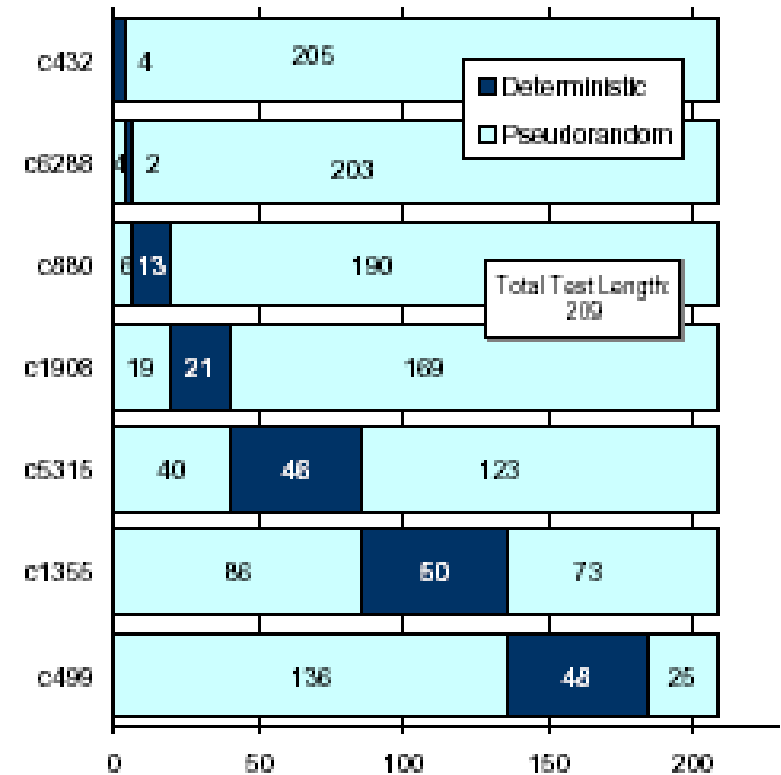
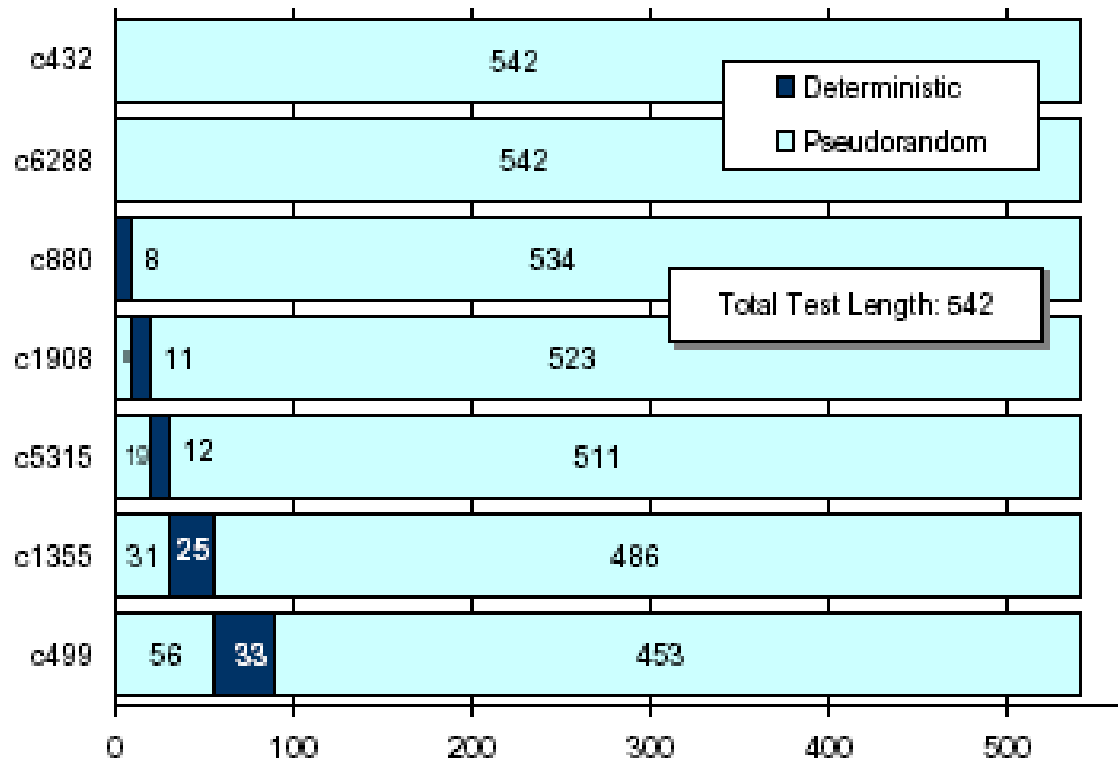
3 - **Correction of the estimation**

3* - **Final real cost**

G.Jervan, P.Eles, Z.Peng, R.Ubar, M.Jenihhin. Test Time Minimization for Hybrid BIST of Core-Based Systems. *Asian Test Symposium 2003*, Xi'an, China, November 17-19, 2003,

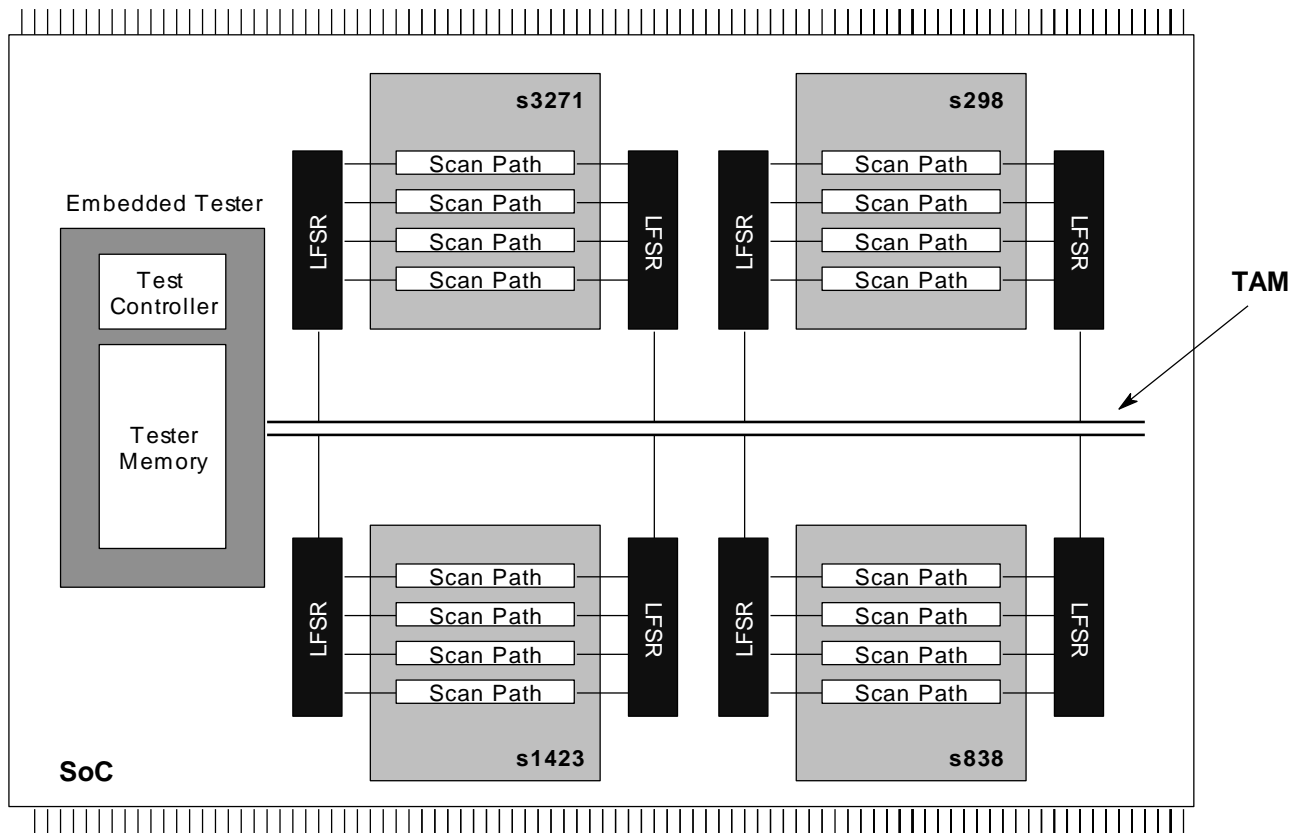
Optimized Multi-Core Hybrid BIST

Pseudorandom test is carried out in parallel,
deterministic test - sequentially



Test-per-Scan Hybrid BIST

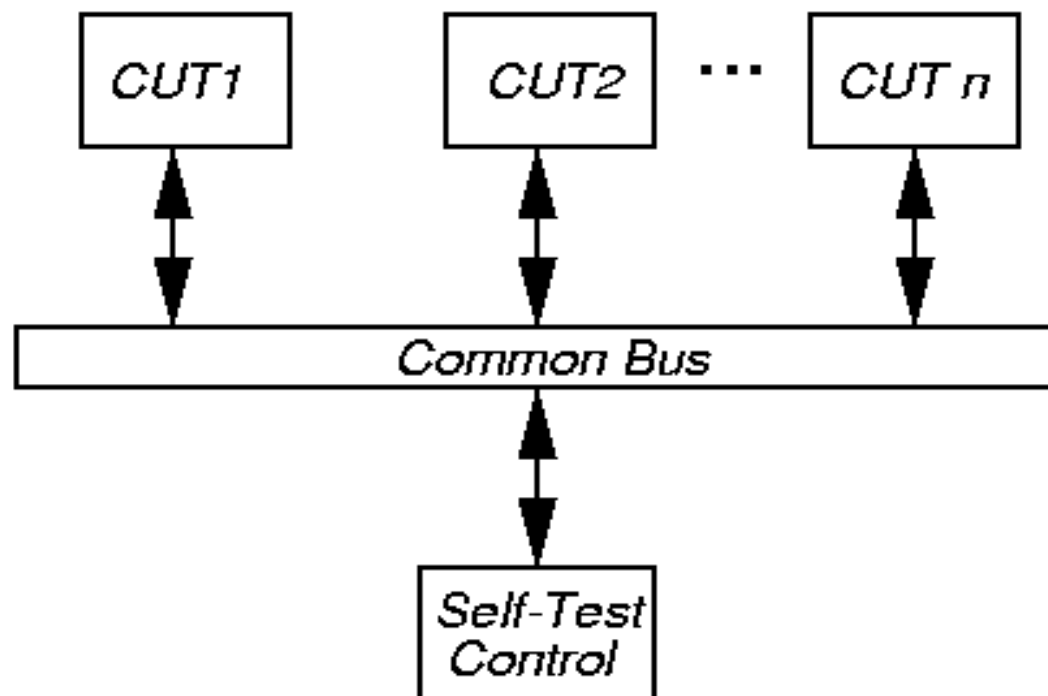
Every core's BIST logic is capable to produce a set of independent pseudorandom test
The pseudorandom test sets for all the cores can be carried out simultaneously



Deterministic tests can only be carried out for one core at a time

Only one test access bus at the system level is needed.

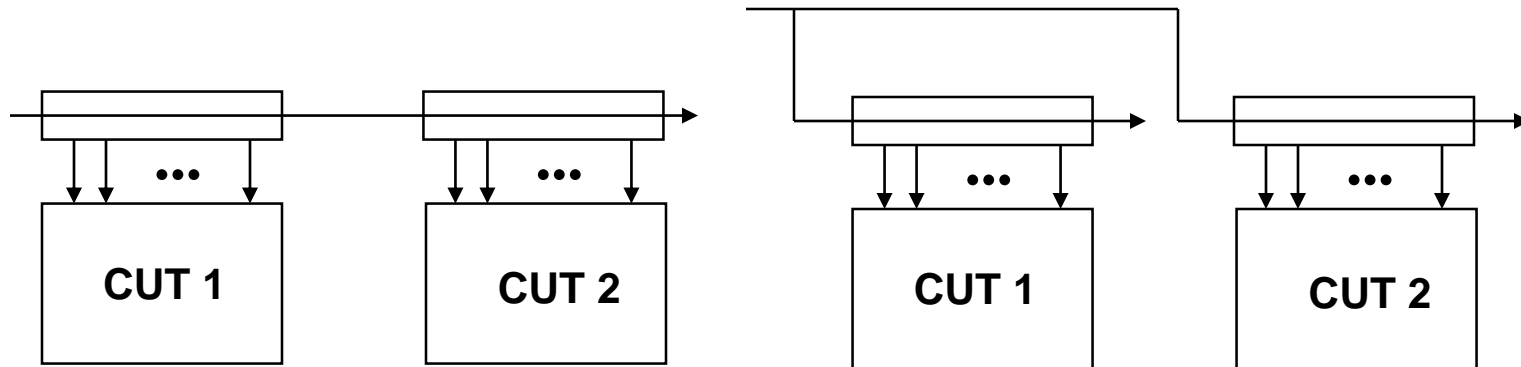
Bus-Based BIST Architecture



- ***Self-test control*** broadcasts patterns to each CUT over bus – parallel pattern generation
- **Awaits bus transactions showing CUT's responses to the patterns: serialized compaction**

Broadcasting Test Patterns in BIST

Concept of test pattern sharing via novel scan structure – to reduce the test application time:



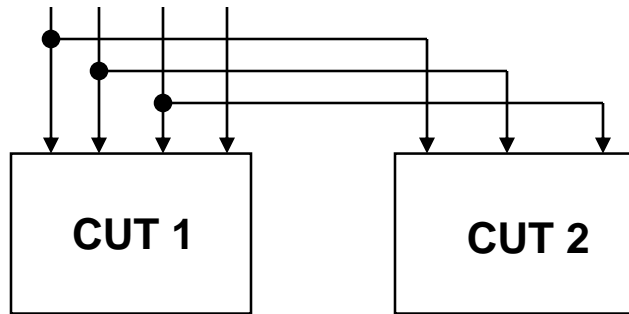
Traditional single scan design

Broadcast test architecture

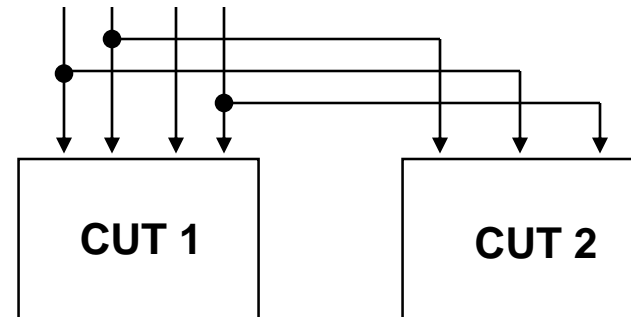
While one module is tested by its test patterns, the same test patterns can be applied simultaneously to other modules in the manner of pseudorandom testing

Broadcasting Test Patterns in BIST

Examples of connection possibilities in Broadcasting BIST:



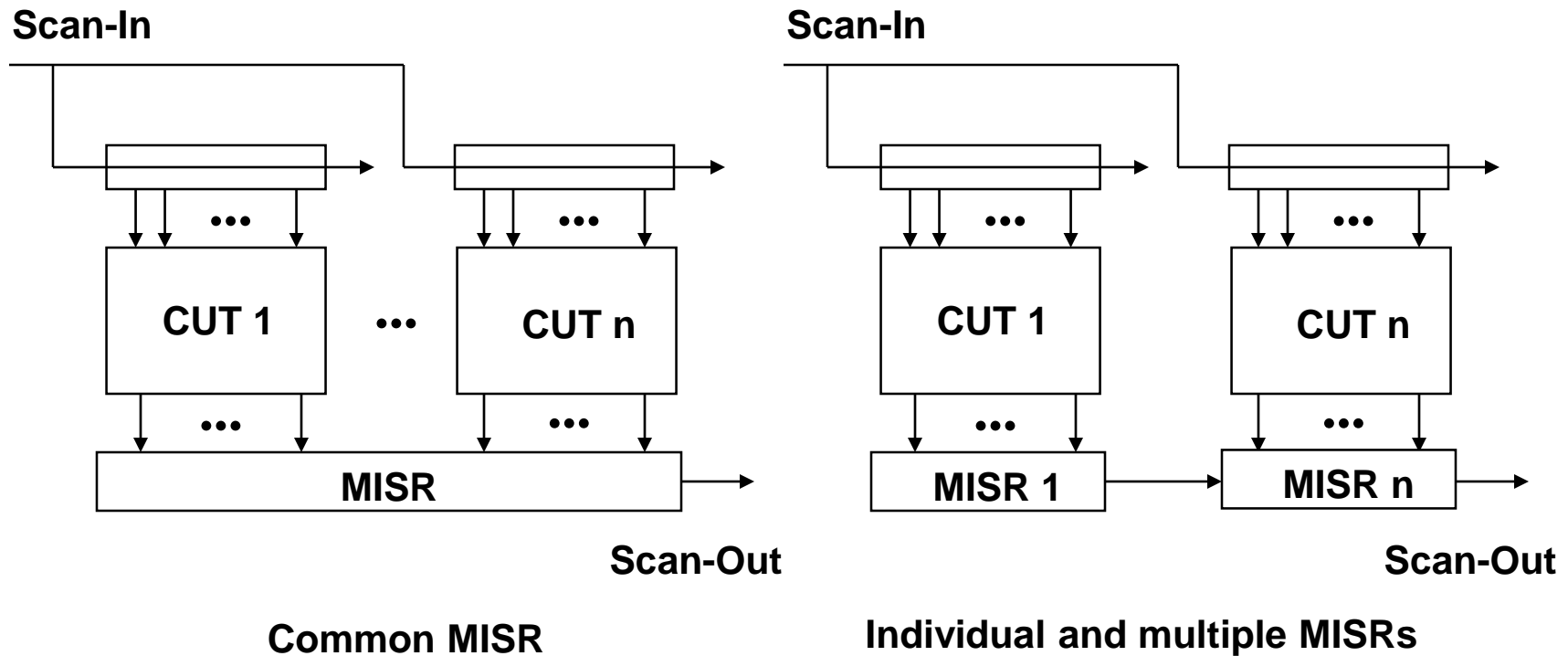
j-to-j connections



Random connections

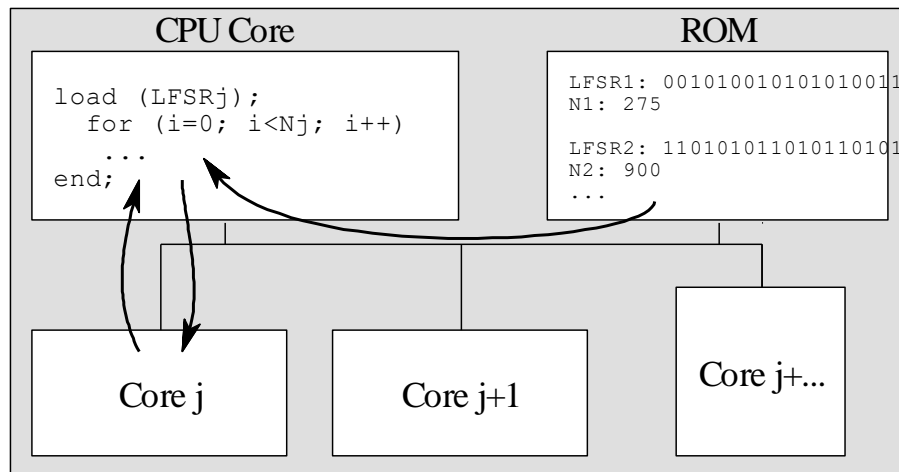
Broadcasting Test Patterns in BIST

Scan configurations in Broadcasting BIST:



Software BIST

Software based test generation:



To reduce the hardware overhead cost in the BIST applications the hardware LFSR can be replaced by software

Software BIST is especially attractive to test SoCs, because of the availability of computing resources directly in the system (a typical SoC usually contains at least one processor core)

The TPG software is the same for all cores and is stored as a single copy

All characteristics of the LFSR are specific to each core and stored in the ROM

They will be loaded upon request.

For each additional core, only the BIST characteristics for this core have to be stored

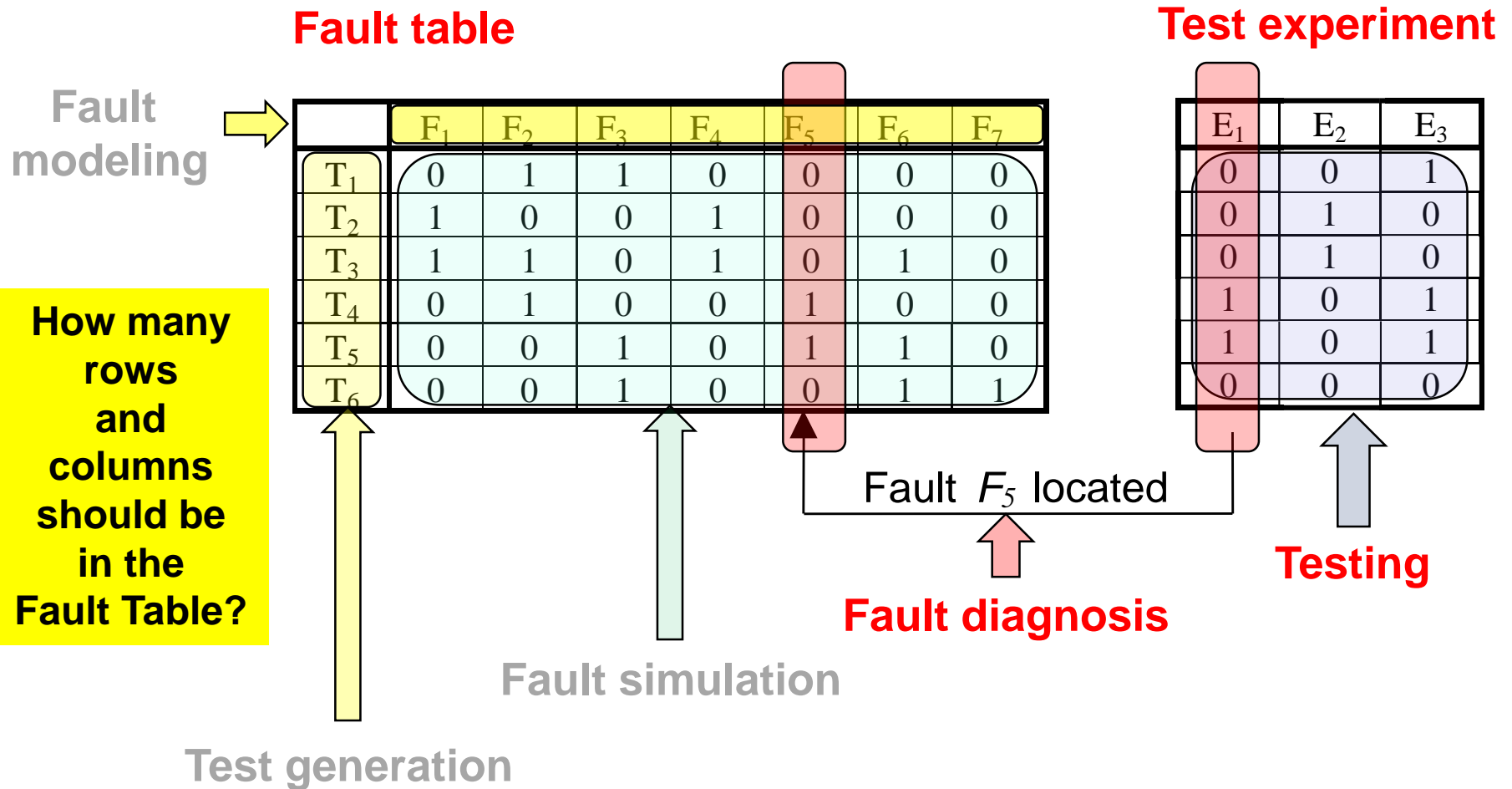
Embedded Built-in Self-Diagnosis (BISD)

- **Introduction to Fault Diagnosis**
 - **Combinational diagnosis (effect-cause approach)**
 - **Sequential (adaptive) diagnosis (cause-effect approach)**
- **General conception of embedded BISD**
- **Diagnostic resolution**
 - **Intersection based on test subsequences**
 - **Intersection based on using signature analyzers**
- **Fault model free diagnosis**
- **Fault evidence based diagnosis**

Why Fault Masking is Important Issue?

Diagnosis method	Fault table					Test result	
Devil's advocate approach		Tested faults					Passed
				Tested faults			Failed
			Tested faults				Failed
Single fault assumption					Fault candidates		Diagnosis
Multiple faults allowed		?	Fault candidates				
Angel's advocate		Proved OK			Fault candidates		

Fault Diagnosis



Sequential Fault Diagnosis

Sequential fault diagnosis by Edge-Pin Testing (cause-effect)

	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇
T ₁	0	1	1	0	0	0	0
T ₂	1	0	0	1	0	0	0
T ₃	1	1	0	1	0	1	0
T ₄	0	1	0	0	1	0	0
T ₅	0	0	1	0	1	1	0
T ₆	0	0	1	0	0	1	1

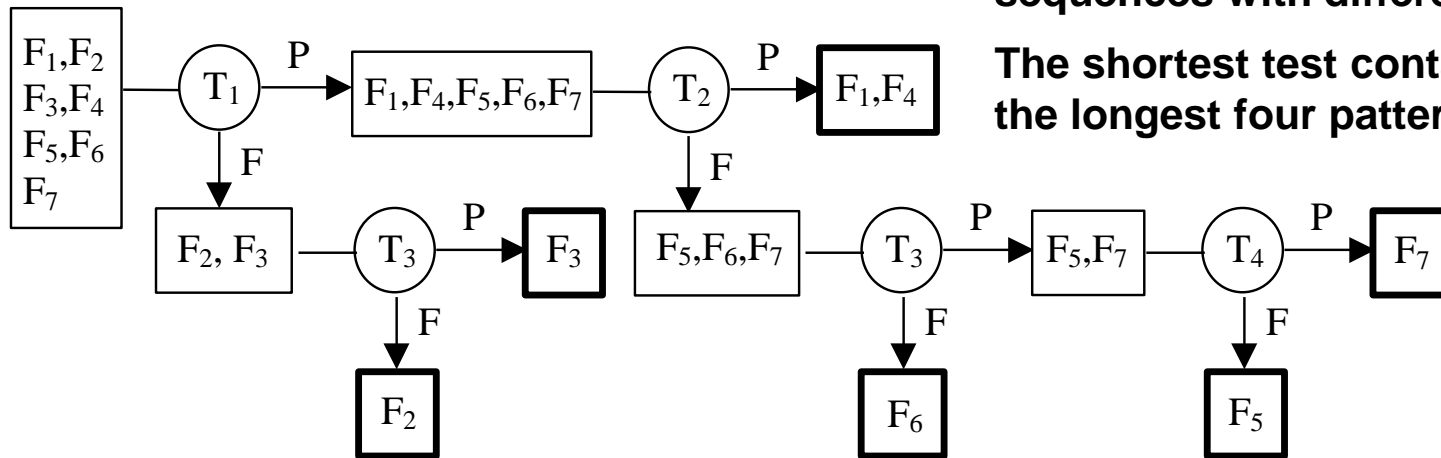
Diagnostic tree

Two faults F_1, F_4 remain indistinguishable

Not all test patterns used in the fault table are needed

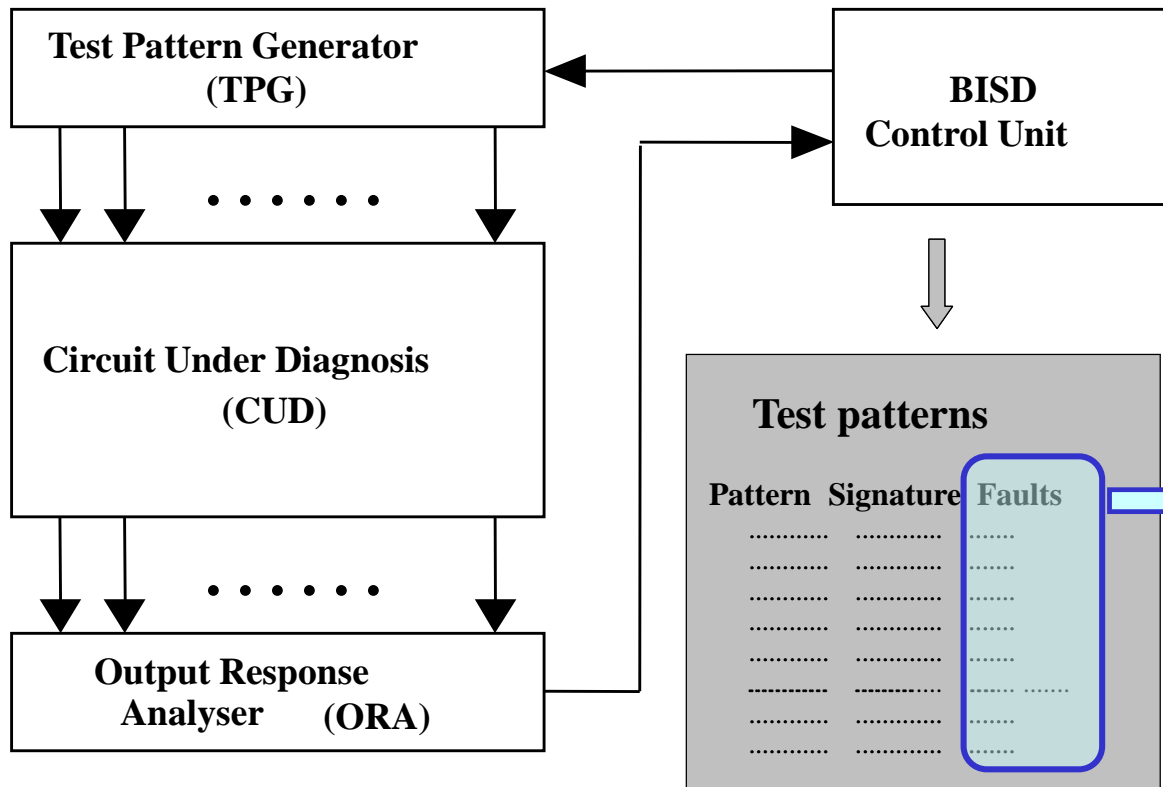
Different faults need for identifying test sequences with different lengths

The shortest test contains two patterns, the longest four patterns

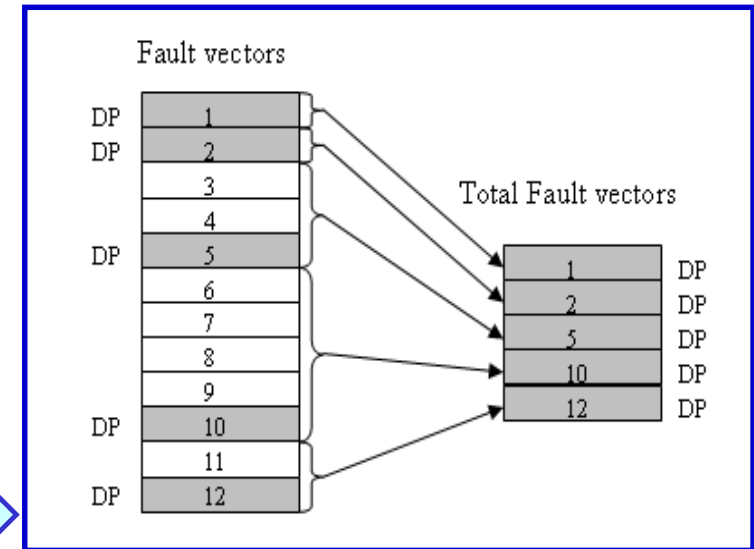


Embedded BIST Based Fault Diagnosis

BISD scheme:

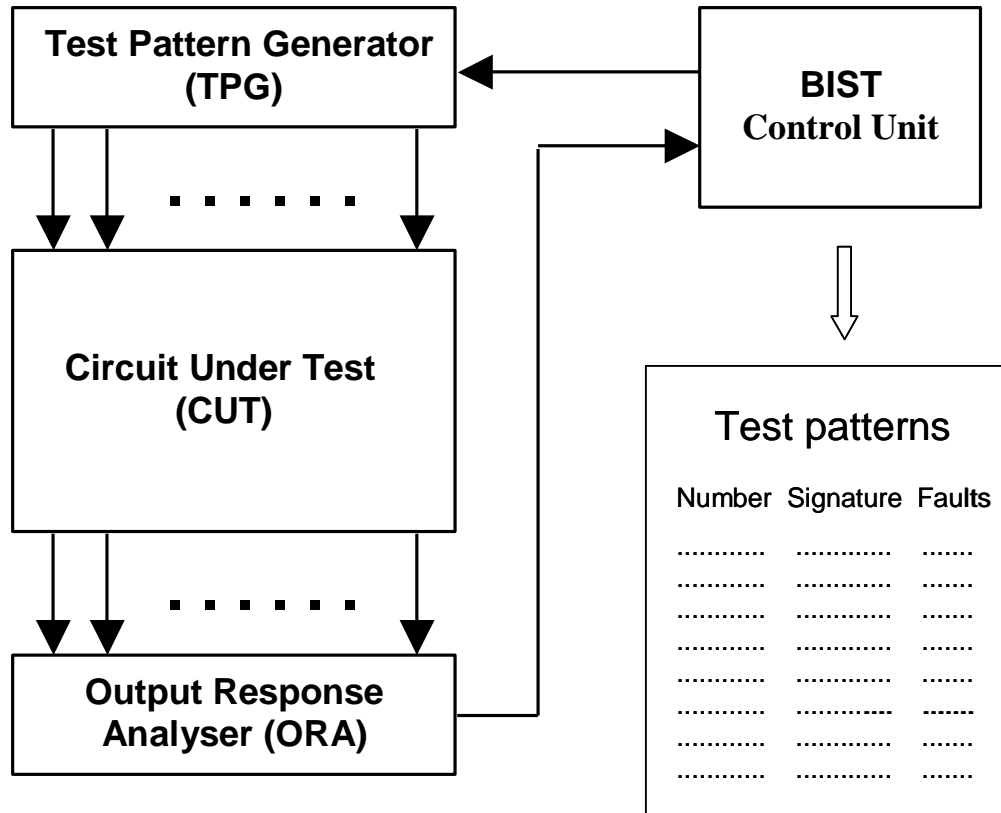


Pseudorandom test sequence:

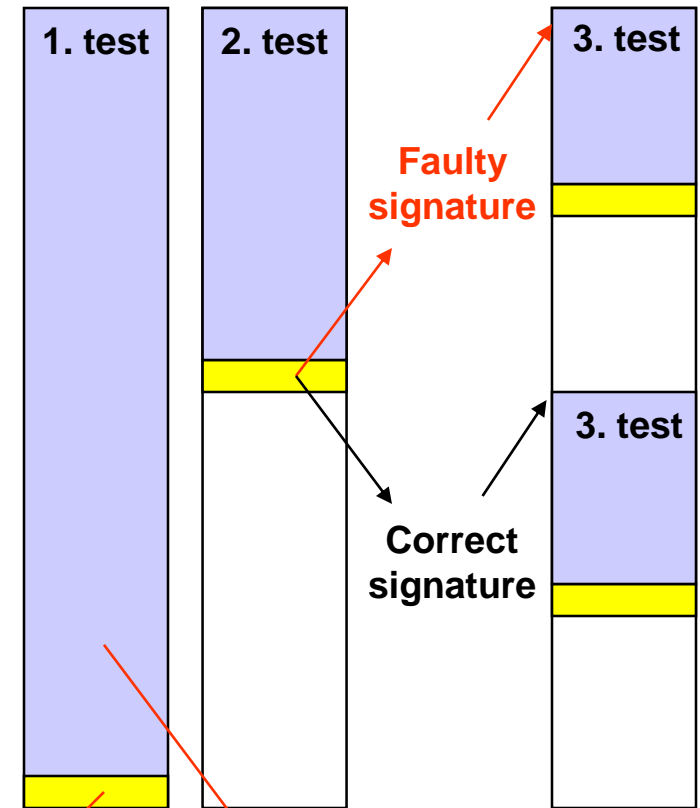


Diagnostic Points (DPs) – patterns that detect new faults
Further minimization of DPs – as a tradeoff with diagnostic resolution

Built-In Fault Diagnosis



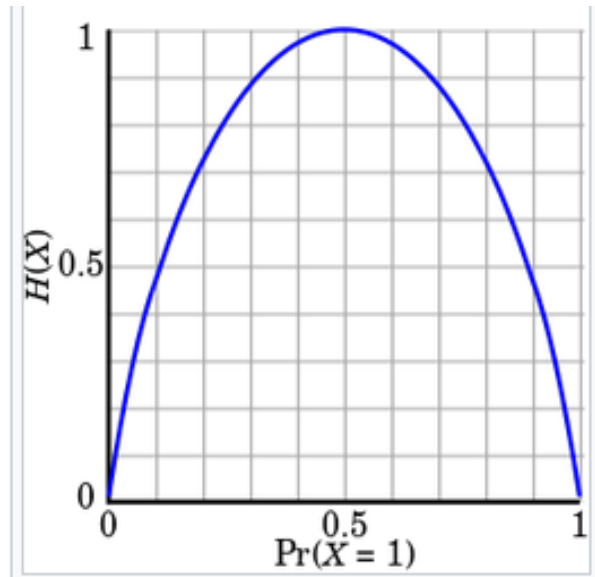
Diagnosis procedure:



Faulty signature

Pseudorandom test sequence

Introduction to Information Theory



Entropy H_X of a discrete random variable X is a measure of the amount of **uncertainty** associated with the value of X

$$H = - \sum_i p_i \log_2 (p_i)$$

where p_i is the probability of occurrence of the i -th possible value of the source symbol; (the entropy is given in the units of "bits" (per symbol) because it uses log of base 2)

$$H_X = - p \log_2 p - (1-p) \log_2 (1-p)$$

$$I = - p \log_2 p - (1-p) \log_2 (1-p)$$

p – probability of detecting a fault

Built-In Fault Diagnosis

Measuring of information we get from the test:

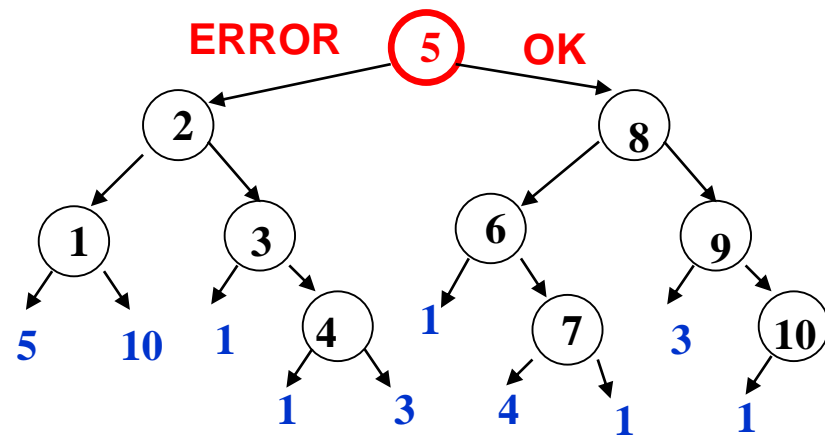
$$I = -p \log_2 p - (1-p) \log_2 (1-p)$$

p – probability of detecting a fault

Pseudorandom test fault simulation (detected faults)

No	All faults	New faults	Coverage
1	5	5	16.67%
2	15	10	50.00%
3	16	1	53.33%
4	17	1	56.67%
5	20	3	66.67%
6	21	1	70.00%
7	25	4	83.33%
8	26	1	86.67%
9	29	3	96.67%
10	30	1	100.00%

Binary search with bisectioning of **test patterns**



Average number of test sessions: **3,3**

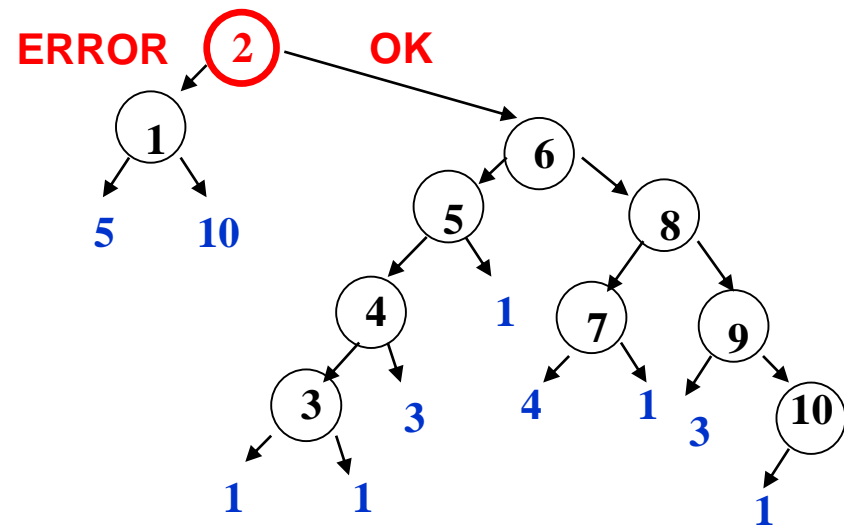
Average number of clocks: **8,67**

Built-In Fault Diagnosis

Pseudorandom test fault simulation (detected faults)

No	All faults	New faults	Coverage
1	5	5	16.67%
2	15	10	50.00%
3	16	1	53.33%
4	17	1	56.67%
5	20	3	66.67%
6	21	1	70.00%
7	25	4	83.33%
8	26	1	86.67%
9	29	3	96.67%
10	30	1	100.00%

Binary search with bisectioning of **faults**

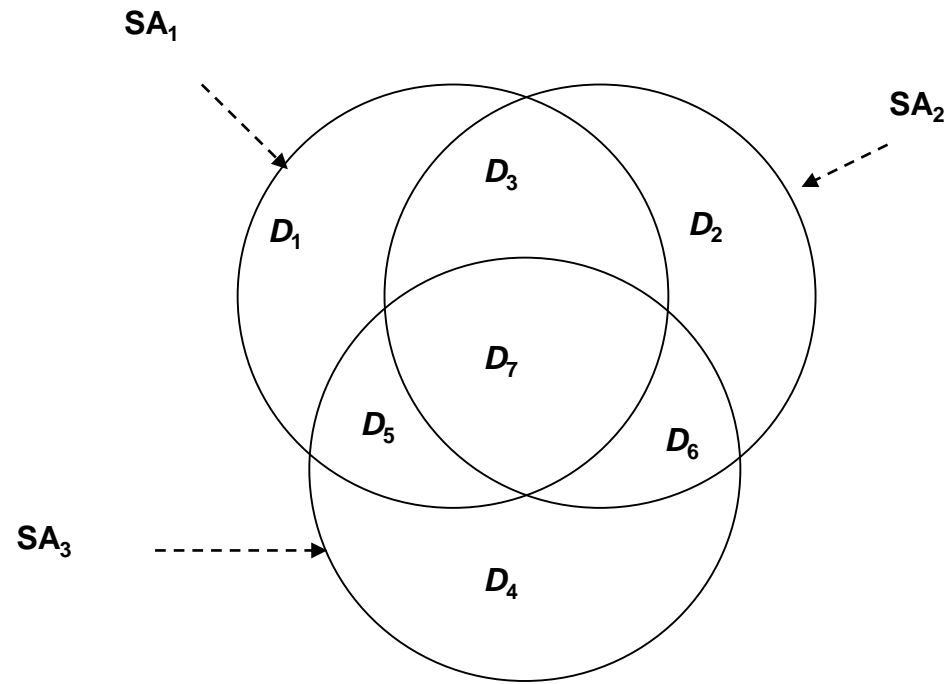
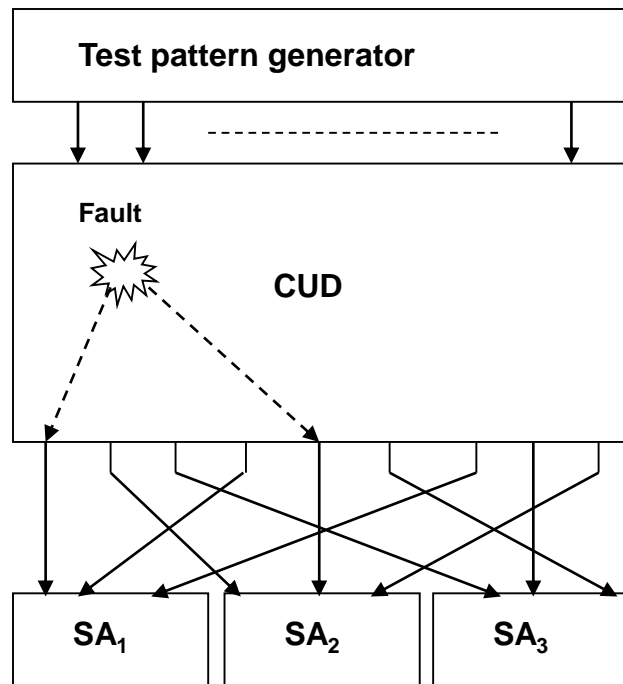


Average number of test sessions: **3,06**

Average number of clocks: **6,43**

Built-In Fault Diagnosis

Diagnosis with multiple signatures
(based on reasoning of spacial information):

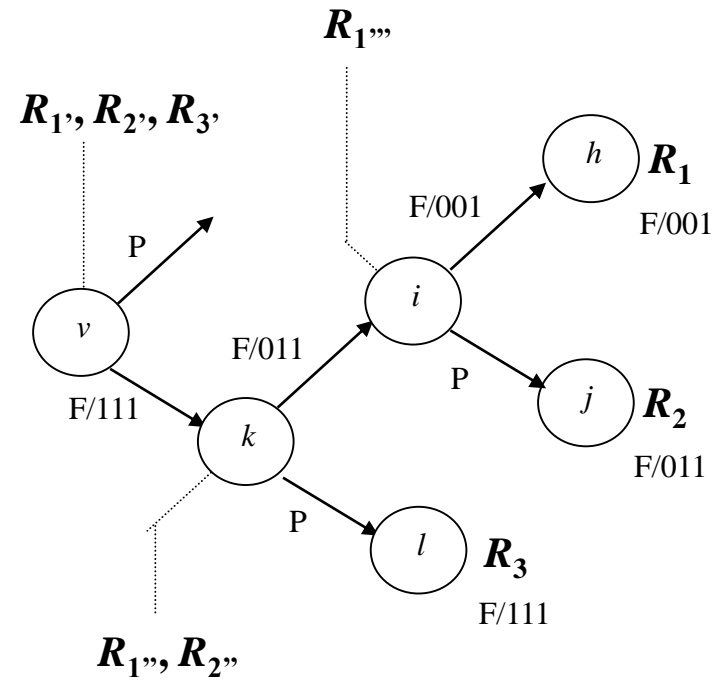


Built-In Fault Diagnosis

Diagnosis with multiple signatures:

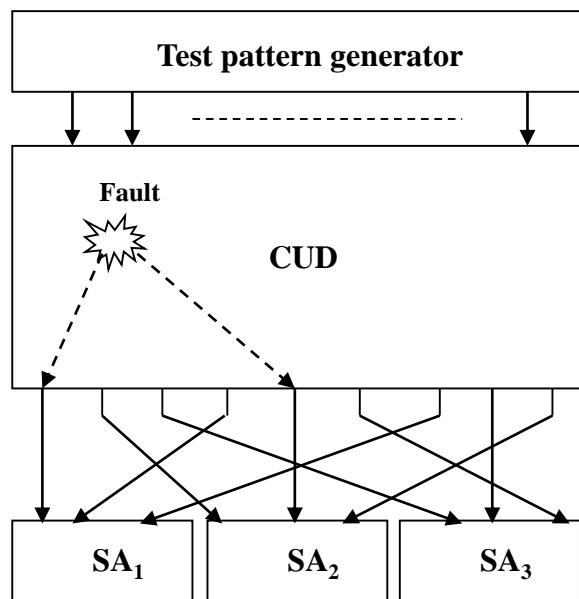
No	Codeword			Diagnosis
h	0	0	1	R_1
i	0	0	1	R_1''
j	0	1	1	R_2
k	0	1	1	R_1''', R_2''
l	1	1	1	R_3
v	1	1	1	R_1', R_2', R_3'

Diagnostic tree



Built-In Fault Diagnosis

BIST with multiple signature analyzers



Optimization of the interface between CUD and SA-s

Optimization in time dimension

Intersection using tests

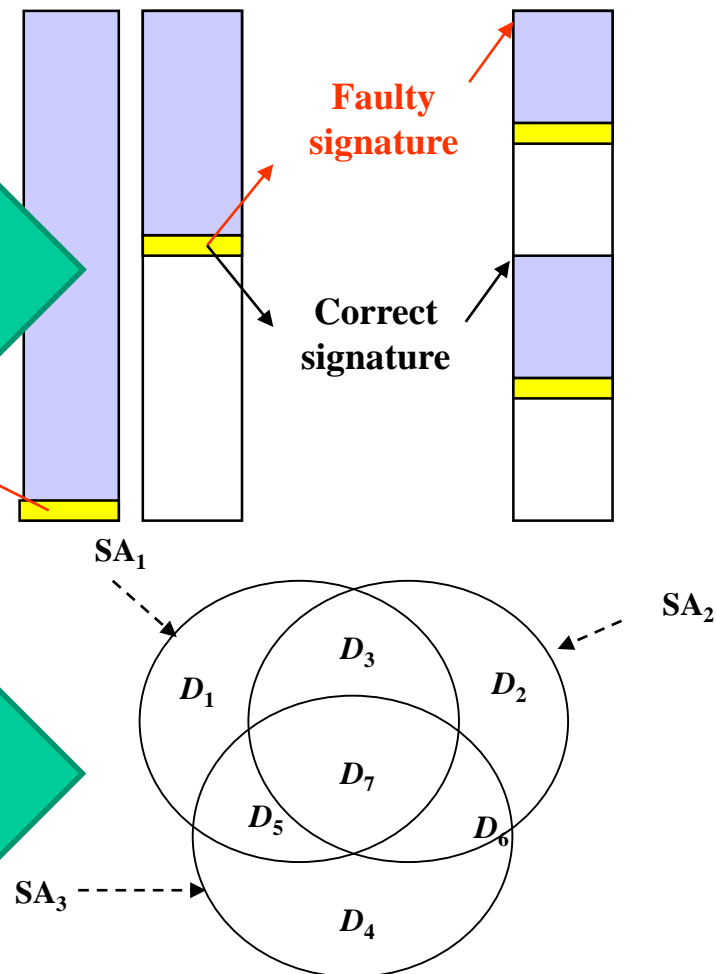
Faulty signature

Faulty signature

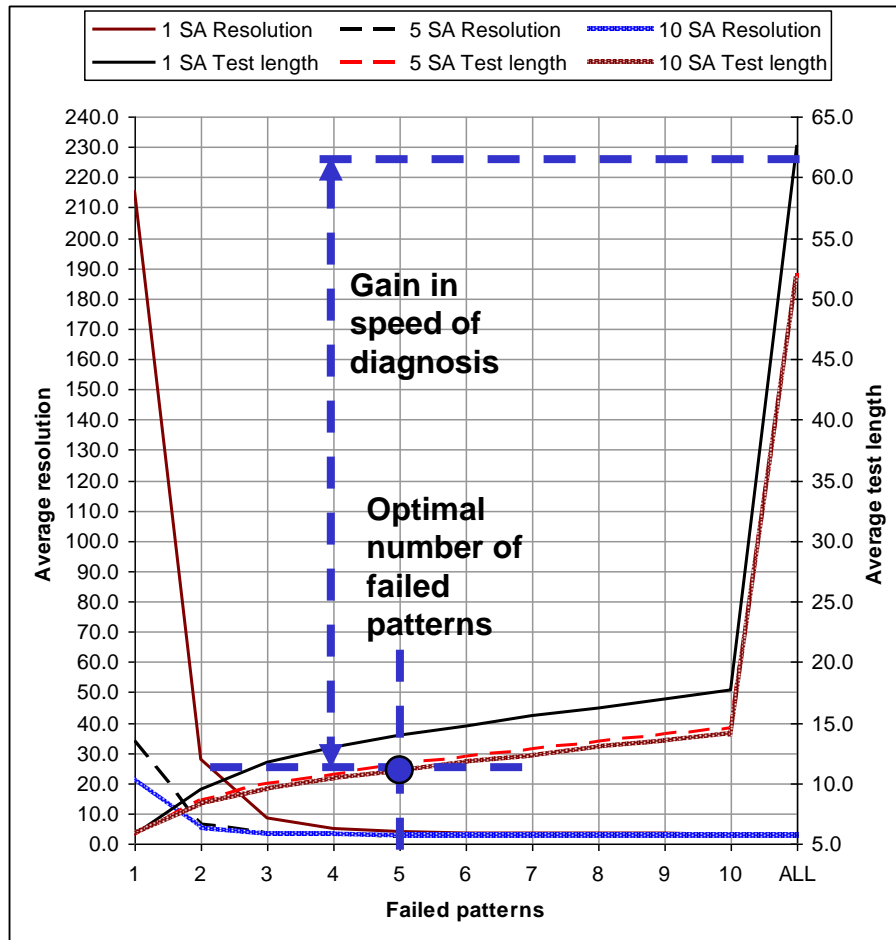
Correct signature

Optimization in space dimension

Intersection using SA-s



Built-In Fault Diagnosis



Diagnosis with multiple signatures:

Measured:

- average resolution
- average test length

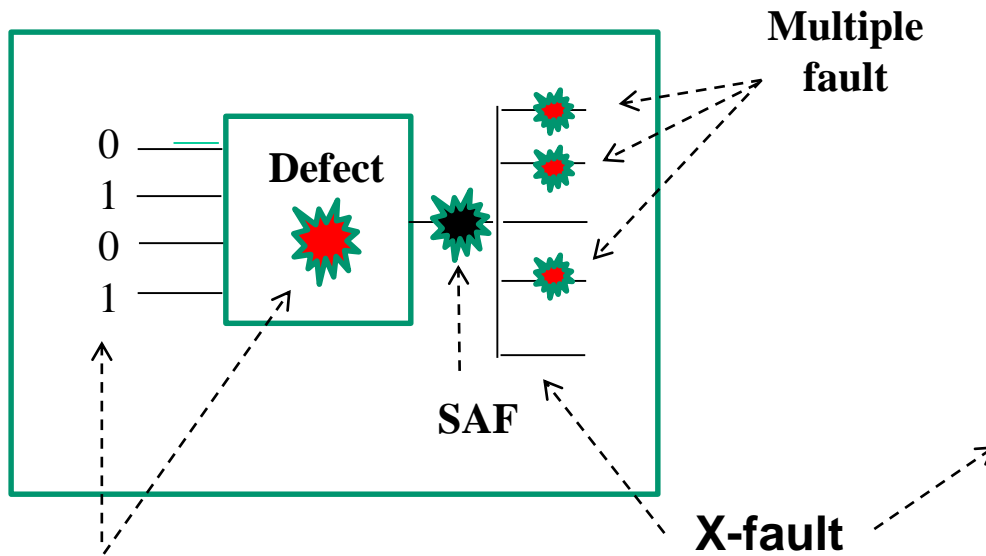
Compared: 1SA, 5SA, 10SA

Gain in test length: 6 times

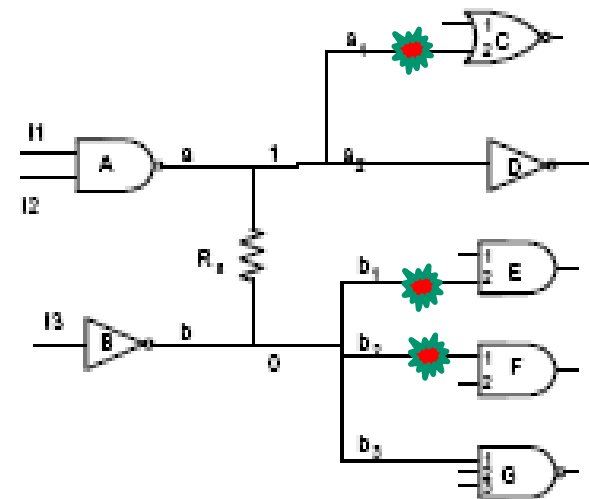
R.Ubar, S.Kostin, J.Raik. Embedded Fault Diagnosis in Digital Systems with BIST. J. of Microprocessors and Microsystems, Volume 32, August 2008, pp. 279-287.

Extended Fault Models

Extensions of the parallel critical path tracing for two large general fault classes for modeling physical defects:



Resistive bridge fault



Conditional fault

Pattern fault

Constrained SAF

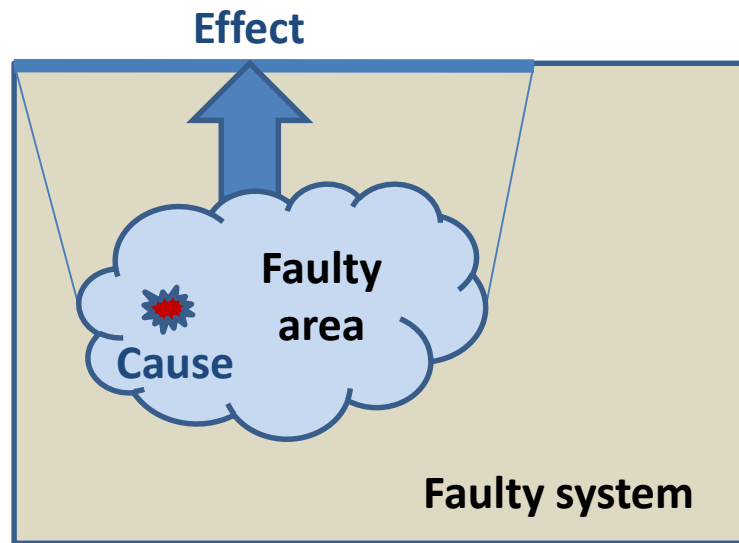
Single faulty signal

X-fault
Byzantine fault
Bridges
Stuck-opens

Multiple faulty signal

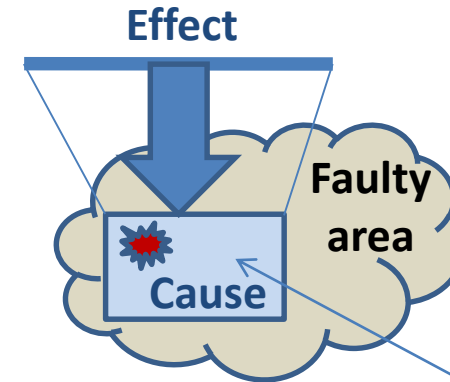
Fault-Model Free Fault Diagnosis

Combined cause-effect and effect-cause diagnosis



1) Cause-Effect Fault Diagnosis

Suspected faulty area is located based on the fault table (dictionary)



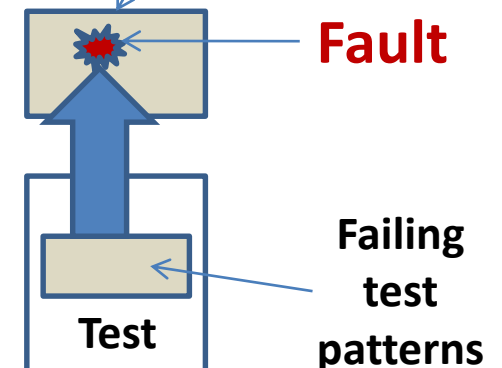
2) Effect-Cause Fault Diagnosis

Faulty block is located in the suspected faulty area

Faulty block

3) Fault Reasoning

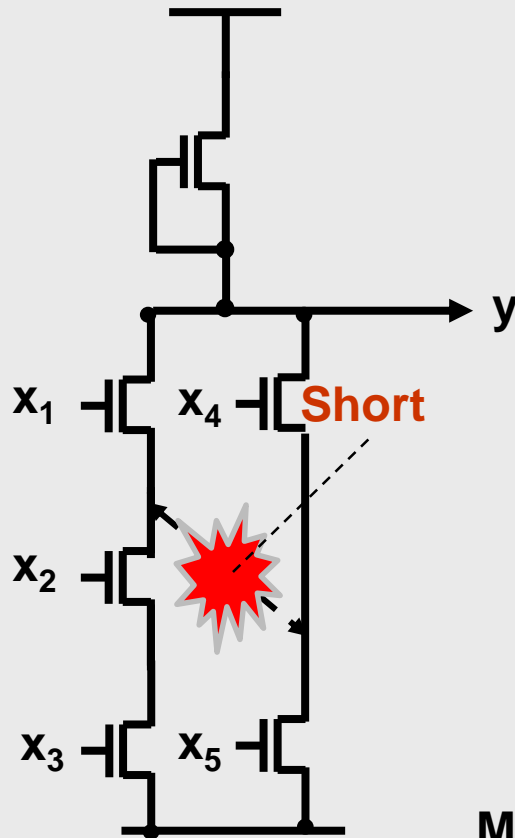
Failing test patterns are mapped into the suspected defect or into a set of suspected defects in the faulty block



Fault

Failing test patterns

Practical Use of Boolean Differences



A transistor fault causes a change in a logic function not representable by SAF model

Correct function: $y = x_1 x_2 x_3 \vee x_4 x_5$

Faulty function: $y^d = (x_1 \vee x_4)(x_2 x_3 \vee x_5)$

Defect variable: $d = \begin{cases} 0 - \text{defect } d \text{ is missing} \\ 1 - \text{defect } d \text{ is present} \end{cases}$

Generic function with defect:

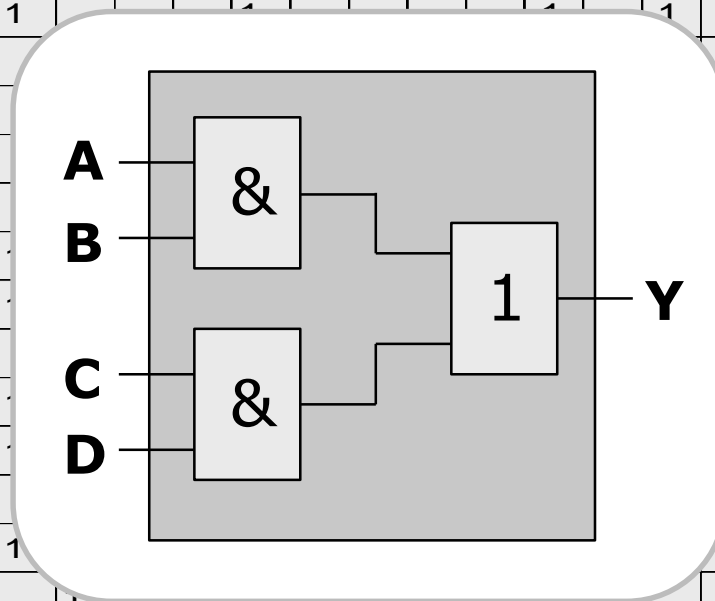
$$y^* = (y \wedge \bar{d}) \vee (y^d \wedge d)$$

Mapping the physical defect onto the logic level by solving the equation:

$$\frac{\partial y^*}{\partial d} = 1$$

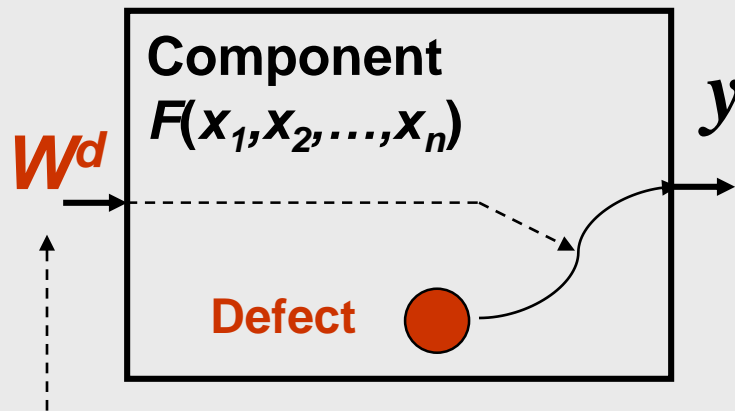
Fault Table: Mapping Defects to Faults

i	Fault d_i	Erroneous function f^{di}	p_i	Input patterns t_j															
				0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	B/C	$\text{not}((B * C) * (A + D))$	0.010307065				1								1	1	1		
2	B/D	$\text{not}((B * D) * (A + C))$	0.000858922				1								1	1		1	
3	B/N9	$B * (\text{not}(A))$	0.043375564	1	1	1						1	1	1	1				
4	B/Q	$B * (\text{not}(C * D))$	0.007515568	1	1	1							1	1	1		1	1	1
5	B/VDD	$\text{not}(A + (C * D))$	0.001717844										1	1	1				
6	B/VSS	$\text{not}(C * D)$	0.035645265													1	1	1	
7	A/C	$\text{not}((A * C) * (B + D))$	0.098990767				1					1				1	1		
8	A/D	$\text{not}((A * D) * (B + C))$	0.013098561				1					1				1		1	
9	A/N9	$A * (\text{not}(B))$	0.038651492	1	1	1													
10	A/Q	$A * (\text{not}(C * D))$	0.025982392	1	1	1													
11	A/VDD	$\text{not}(B + (C * D))$	0.000214731																
12	C/N9	$\text{not}(A + B + D) + (C * (\text{not}((A * B) + D)))$	0.020399399			1													
13	C/Q	$C * (\text{not}(A * B))$	0.033927421	1	1														
14	C/VSS	$\text{not}(A * B)$	0.005153532																
15	D/N9	$\text{not}(A + B + C) + (D * (\text{not}((A * B) + C)))$	0.007730298				1												
16	D/Q	$D * (\text{not}(A * B))$	0.149452437	1		1													
17	N9/Q	$\text{not}((A * B) + (B * C * D) + (A * C * D))$	0.143654713																
18	N9/VDD	$\text{not}((C * D) + (A * B * D) + (A * B * C))$	0.253382006																
19	Q/VDD	SA1 at Q	0.014386944																1
20	Q/VSS	SA0 at Q	0.095555078	1	1	1													



Generalization: Functional Fault Model

Conditional Stuck-at-Fault model Constrained SAF



Logical constraints

$$W^d = \frac{\partial y^*}{\partial d} = 1$$

Fault model:

(dy, W^d)

SAF

Constraint

$(dy, \{W_k^{d^k}\})$

SAF

All

constraints
for all
defects

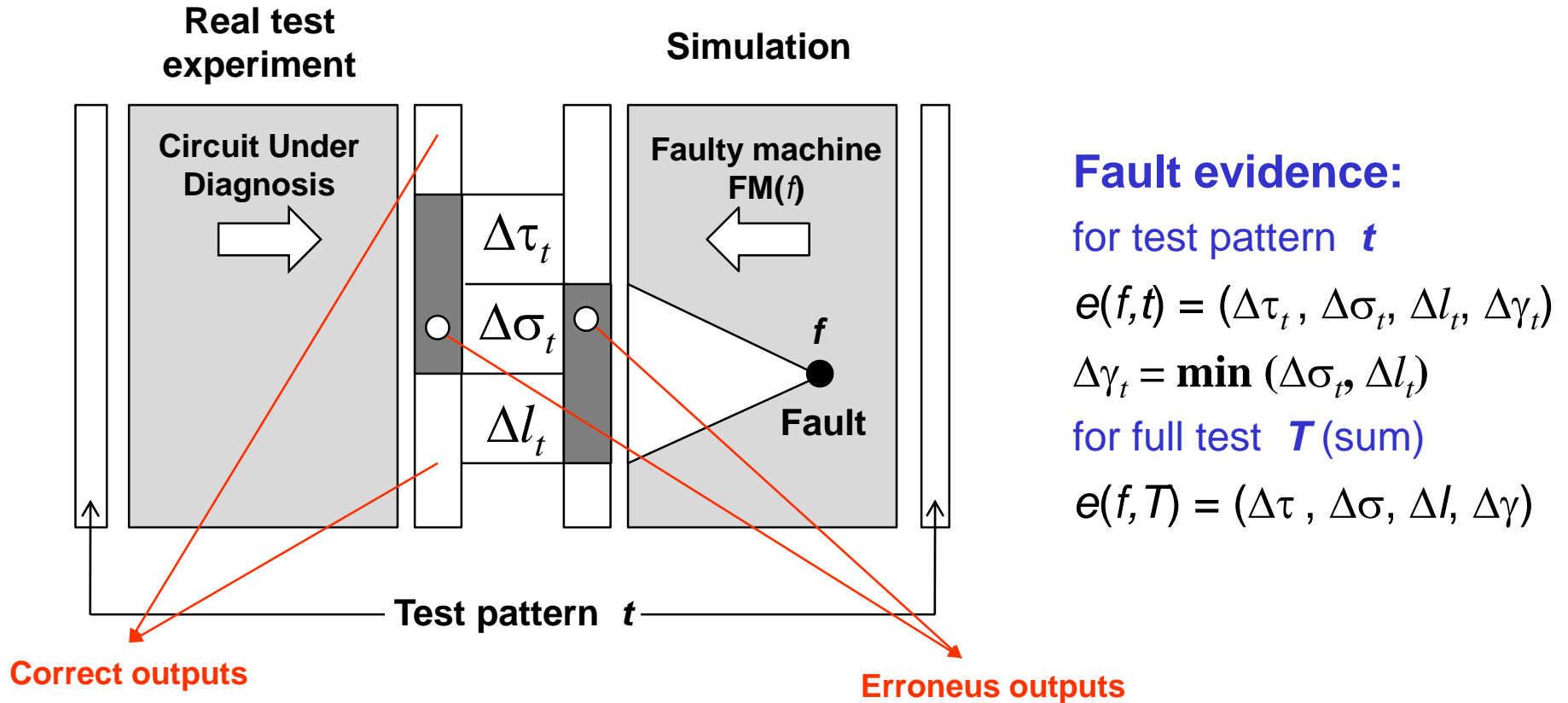
Constraints calculation:

$$y^* = F^*(x_1, x_2, \dots, x_n, d) = \bar{d}F \vee dF^d$$

Fault-free \bar{d} Faulty d

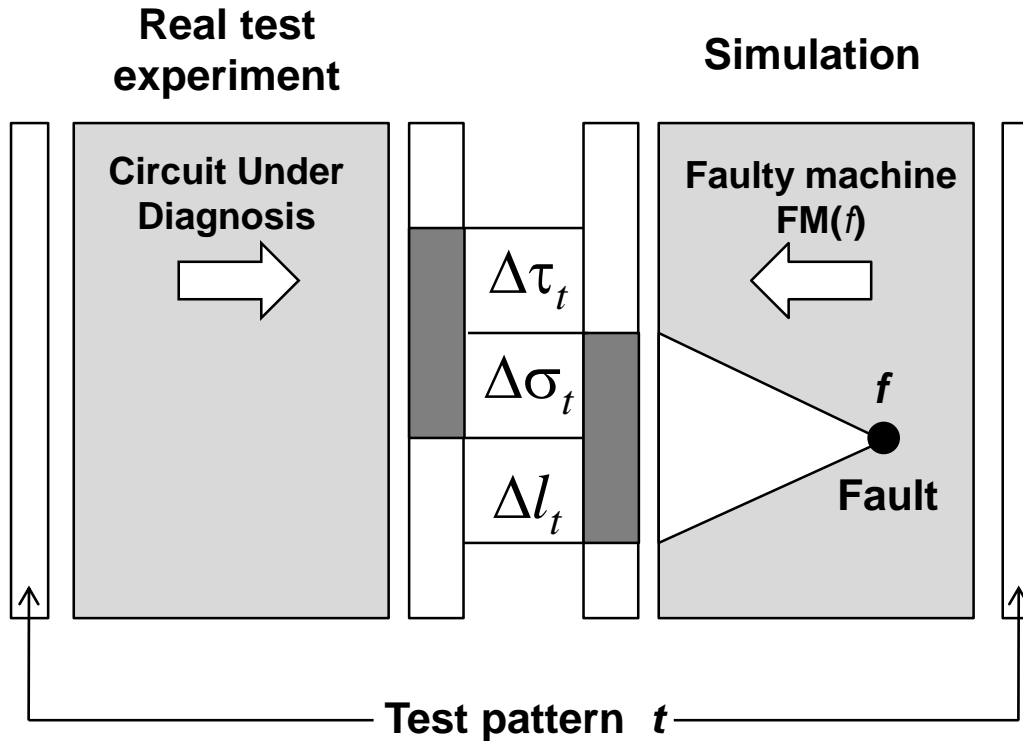
$d = 1$, if the defect is present

Diagnosis of Fault Model Free Defects



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Diagnosis of Fault Model Free Defects

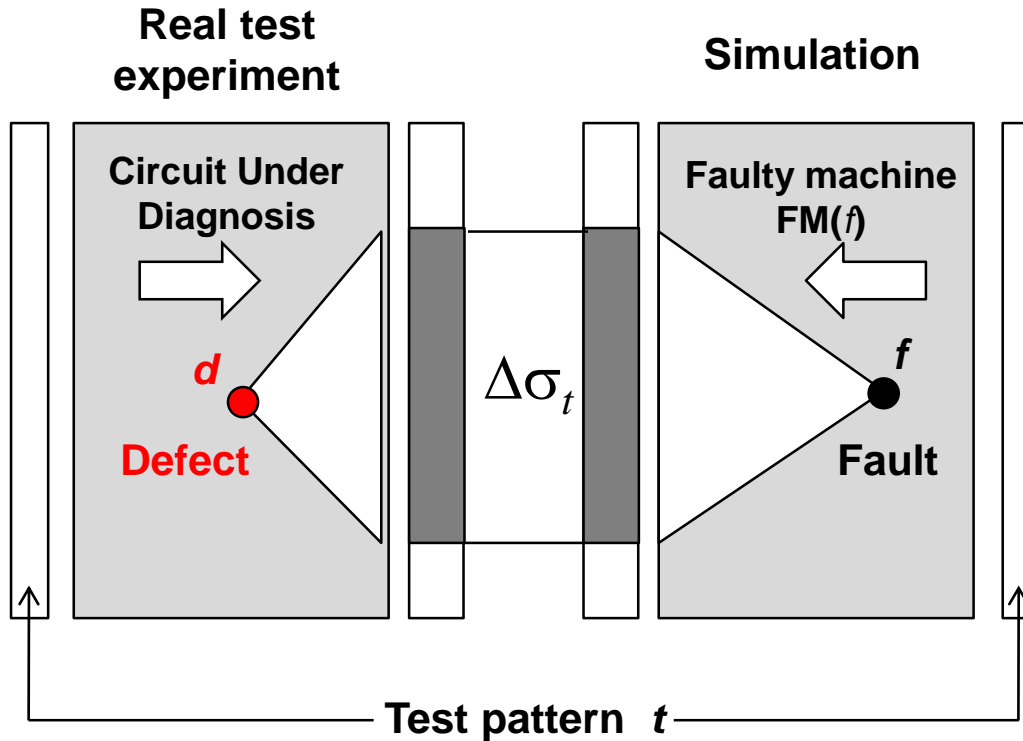


Different classical fault cases

Classic model	l_t	τ_t	γ_t
Single SAF	0	0	0
Multiple SAF	0	>0	0
Single conditional SAF	>0	0	0
Multiple cond. SAF	>0	>0	0
Delay fault	>0	0	>0
General case	>0	>0	>0

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Diagnosis of Fault Model Free Defects

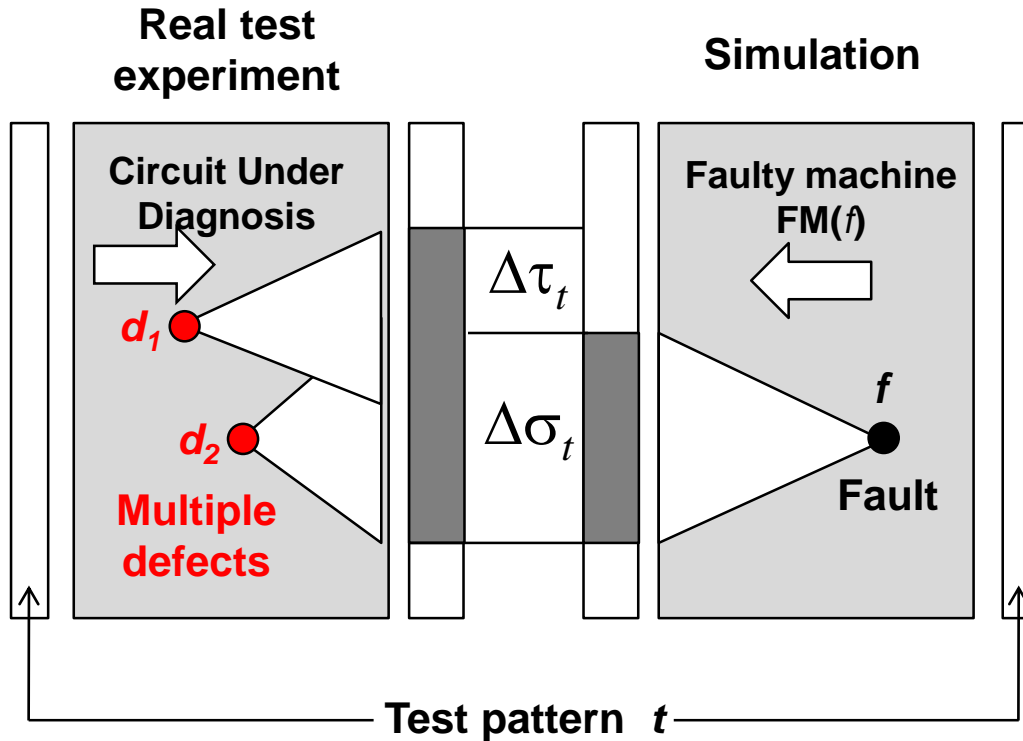


Different classical fault cases

Classic model	I_t	τ_t	γ_t
Single SAF	0	0	0
Multiple SAF	0	>0	0
Single conditional SAF	>0	0	0
Multiple cond. SAF	>0	>0	0
Delay fault	>0	0	>0
General case	>0	>0	>0

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Diagnosis of Fault Model Free Defects

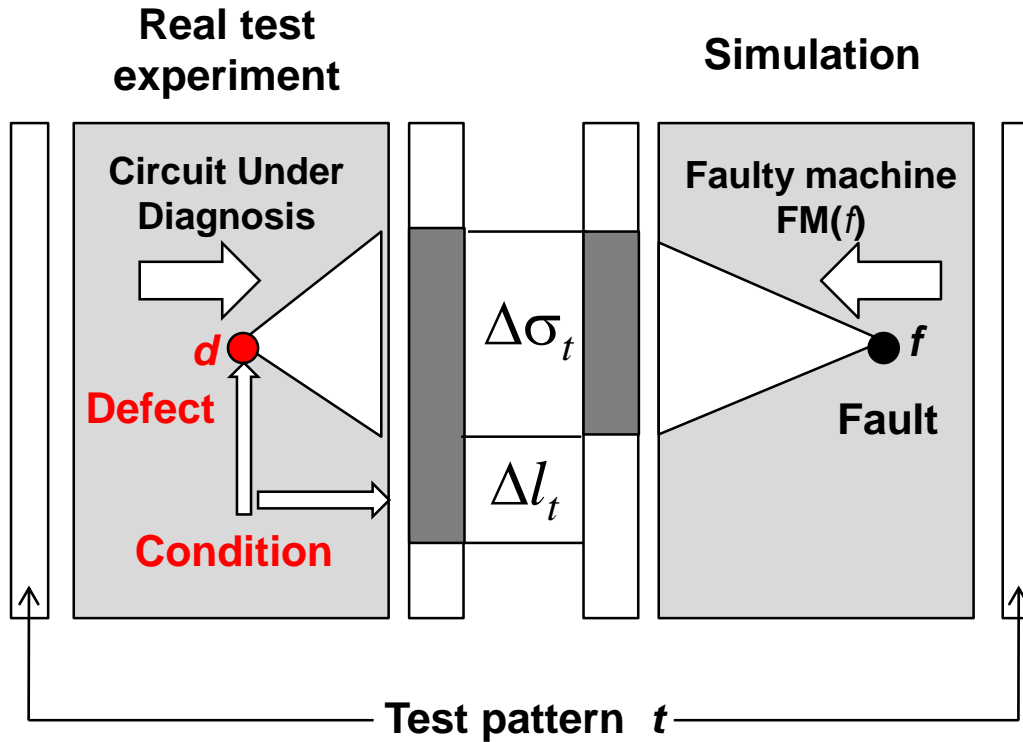


Different classical fault cases

Classic model	I_t	τ_t	γ_t
Single SAF	0	0	0
Multiple SAF (defects)	0	>0	0
Single conditional SAF	>0	0	0
Multiple cond. SAF	>0	>0	0
Delay fault	>0	0	>0
General case	>0	>0	>0

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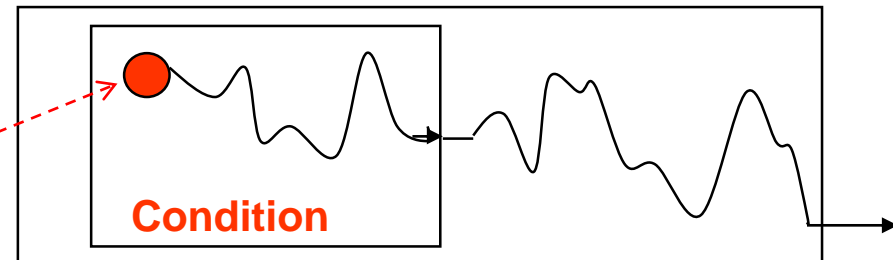
Diagnosis of Fault Model Free Defects



Different classical fault cases

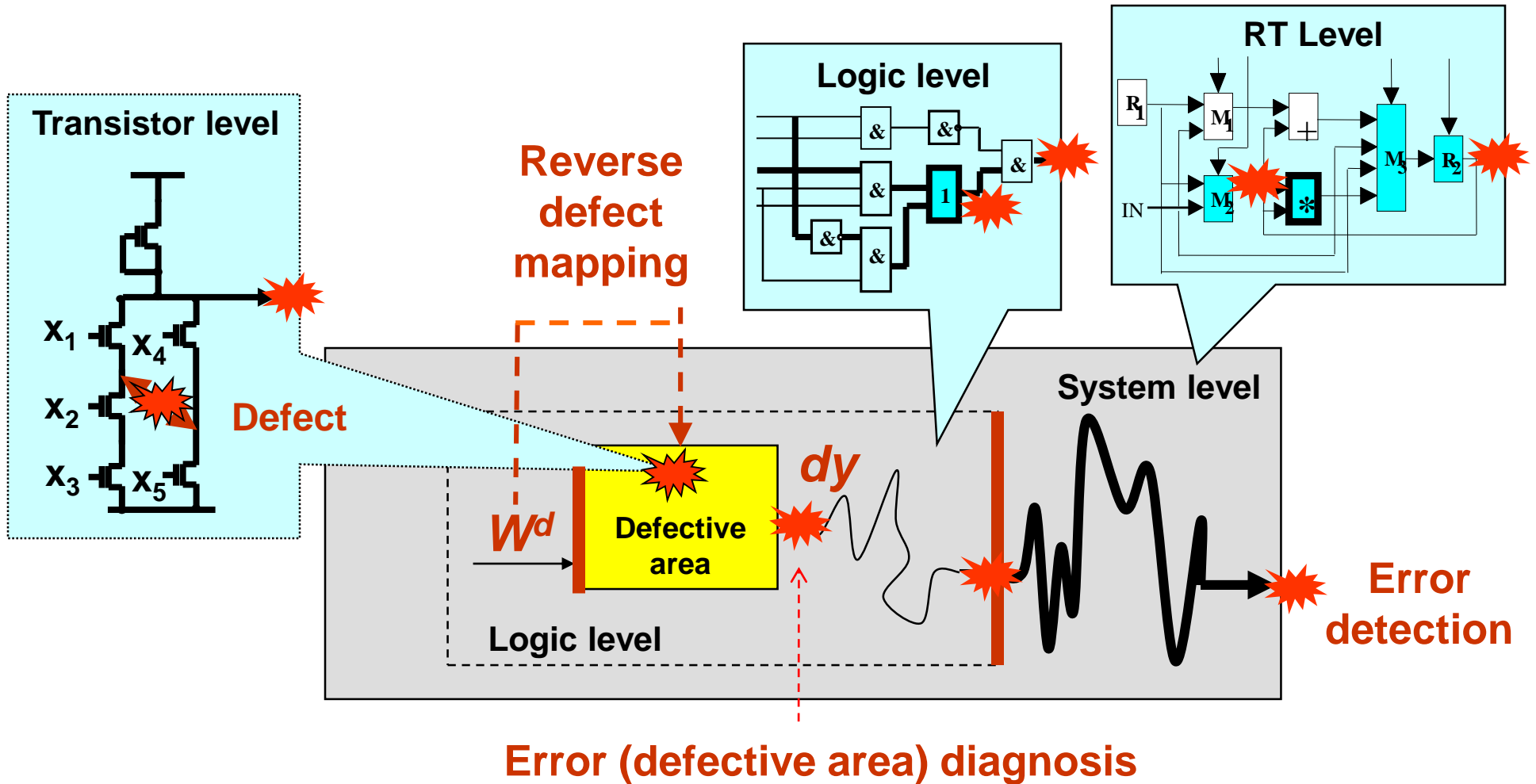
Classic model	l_t	τ_t	γ_t
Single SAF	0	0	0
Multiple SAF	0	>0	0
Single conditional SAF	>0	0	0
Multiple cond. SAF	>0	>0	0
Delay fault	>0	0	>0
General case	>0	>0	>0

Defect

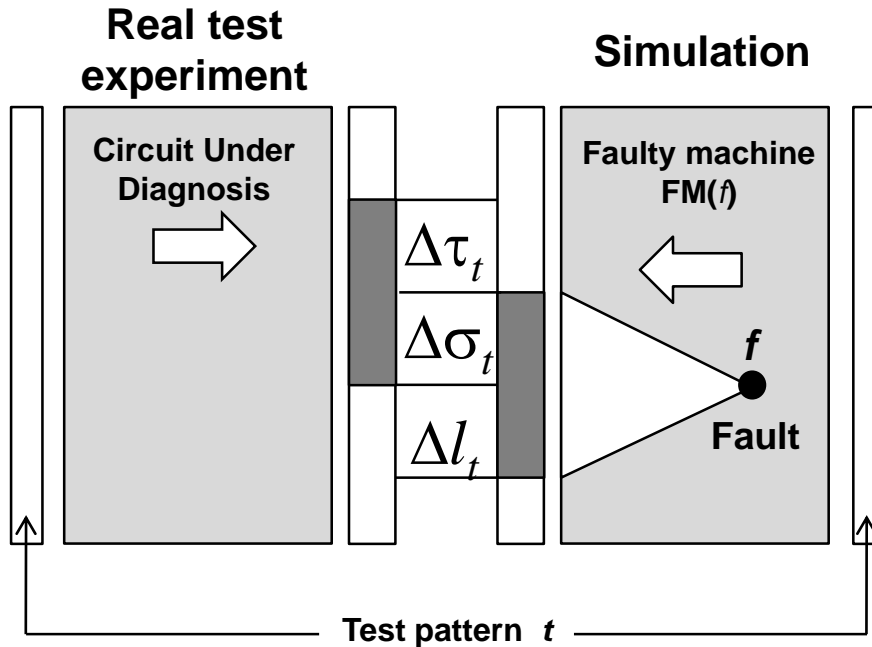


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Fault Diagnosis Without Fault Models



Diagnosis of Fault Model Free Defects



Ranking

(on the top the most suspicious faults):

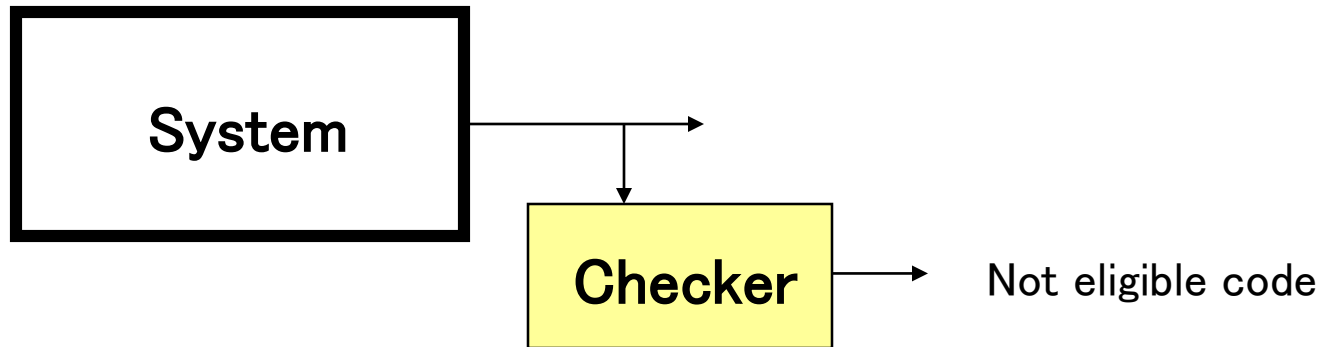
- (1) By increasing γ_T
(single SAF on top)
- (2) If γ_T are equal then
by decreasing σ_T
- (3) If γ_T and σ_T are
equal then by
increasing l_T

Example:

SAF	γ_T	σ_T	l_T
f_1	0	42	0
f_2	30	42	15
f_3	30	42	25
f_4	30	42	30
f_5	30	36	38
f_6	38	23	22
f_7	38	23	23

$$\Delta\gamma_t = \min(\Delta\sigma_t, \Delta l_t)$$

Fault Tolerance: Error Detecting Codes



Examples:

Decimal digits:

Eligible: 0,1,2,..., 9

Not eligible: 10,11,..., 15

Parity check:

Parity bit

	↓		
00	0	0	1
01	1	3	2
10	1	5	4
11	0	6	7

↑ Eligible

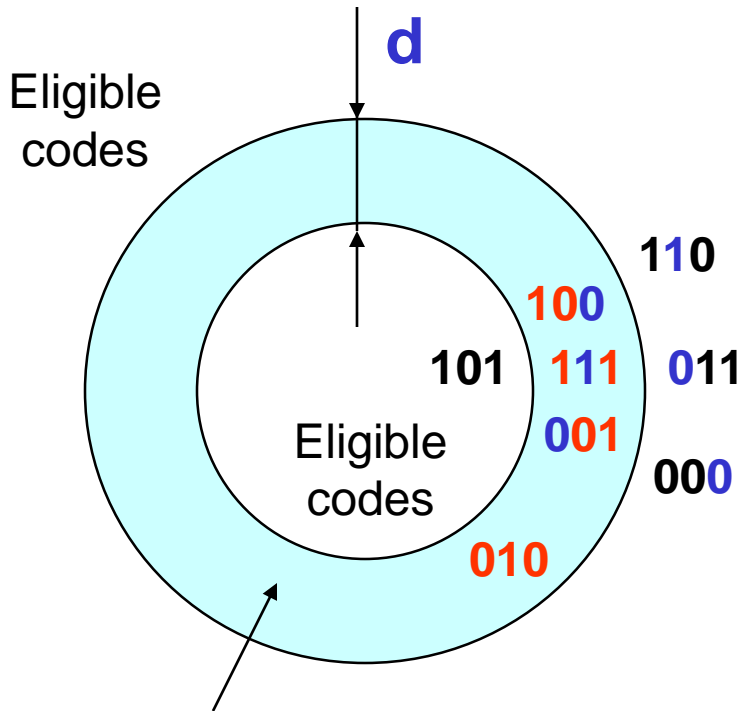
↑ Not

eligible

Error Detecting/Correcting Codes

Hamming distance between codes:

Minimal number of bits
how two codes differ
from each other



Not eligible codes

Parity check:

$d = 2$

Parity bit

00	0	0	1
01	1	3	2
10	1	5	4
11	0	6	7

Eligible

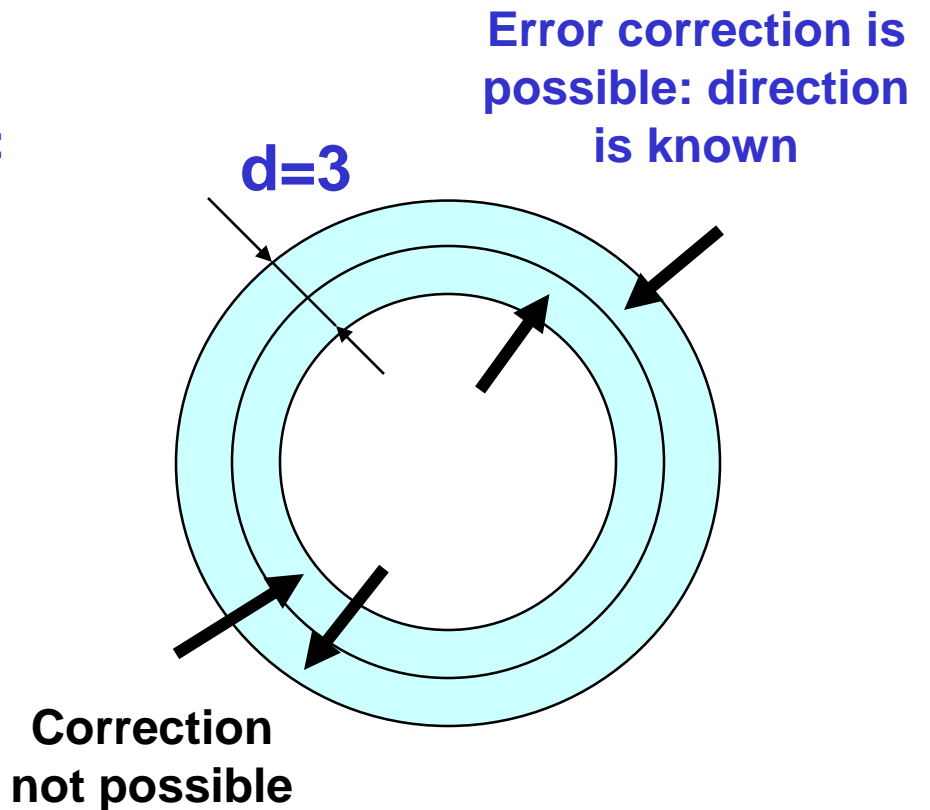
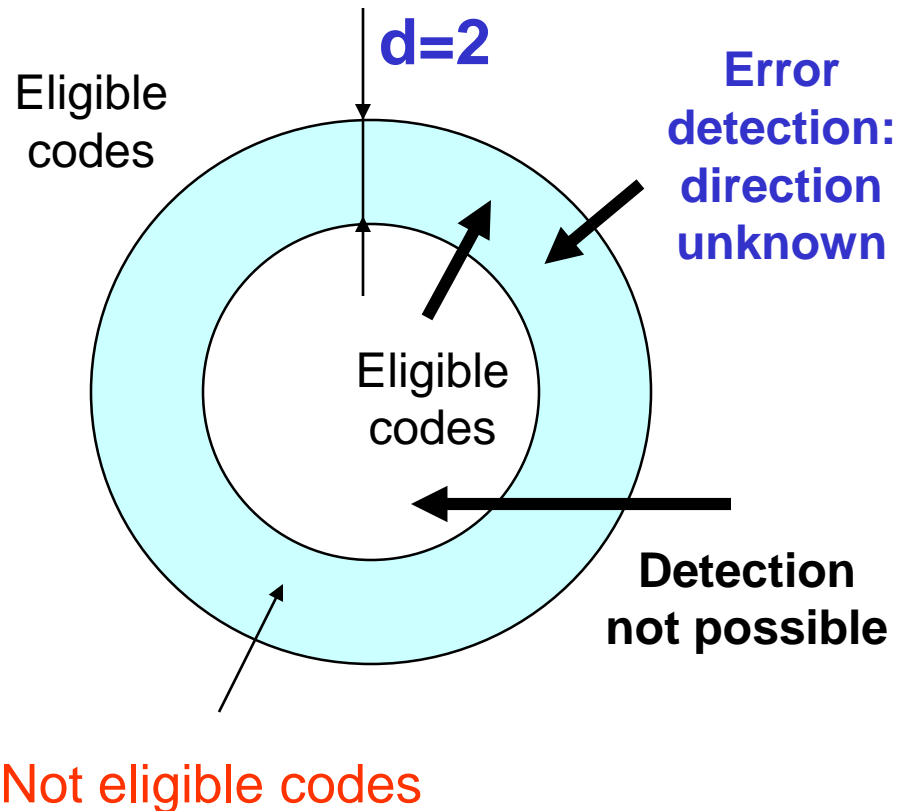
Not

eligible

Error Detecting/Correcting Codes

Error detecting codes:

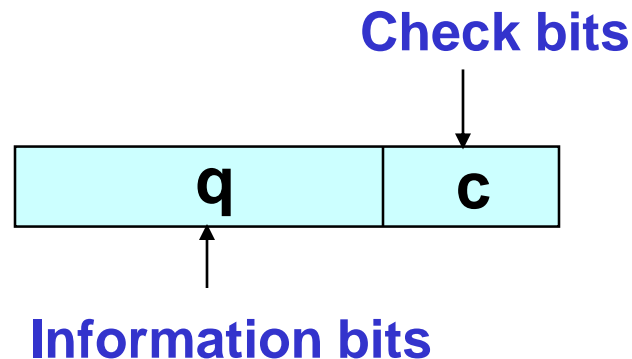
Error correcting codes:



Fault Tolerance: Error Correcting Codes

$d = 2e + 1$ - $2e$ - error detection
 e - error correction

One error correction code: $2^c \geq q + c + 1$



For addressing of the erroneous bit

Error free

Fault Tolerance: One Error Correcting Code

Location of erroneous bit:



Check bits

$$b_{2^i}, i = 1, \dots, c$$

$$P_1 = b_1 \oplus b_3 \oplus b_5 \oplus b_7 = 0$$

$$P_2 = b_2 \oplus b_3 \oplus b_6 \oplus b_7 = 0$$

$$P_3 = b_4 \oplus b_5 \oplus b_6 \oplus b_7 = 0$$

Check bits have to be independently assigned

Analogy with fault diagnosis
by using fault table:

1	0	1	0	1	0	1
7	6	5	4	3	2	1

Initial code

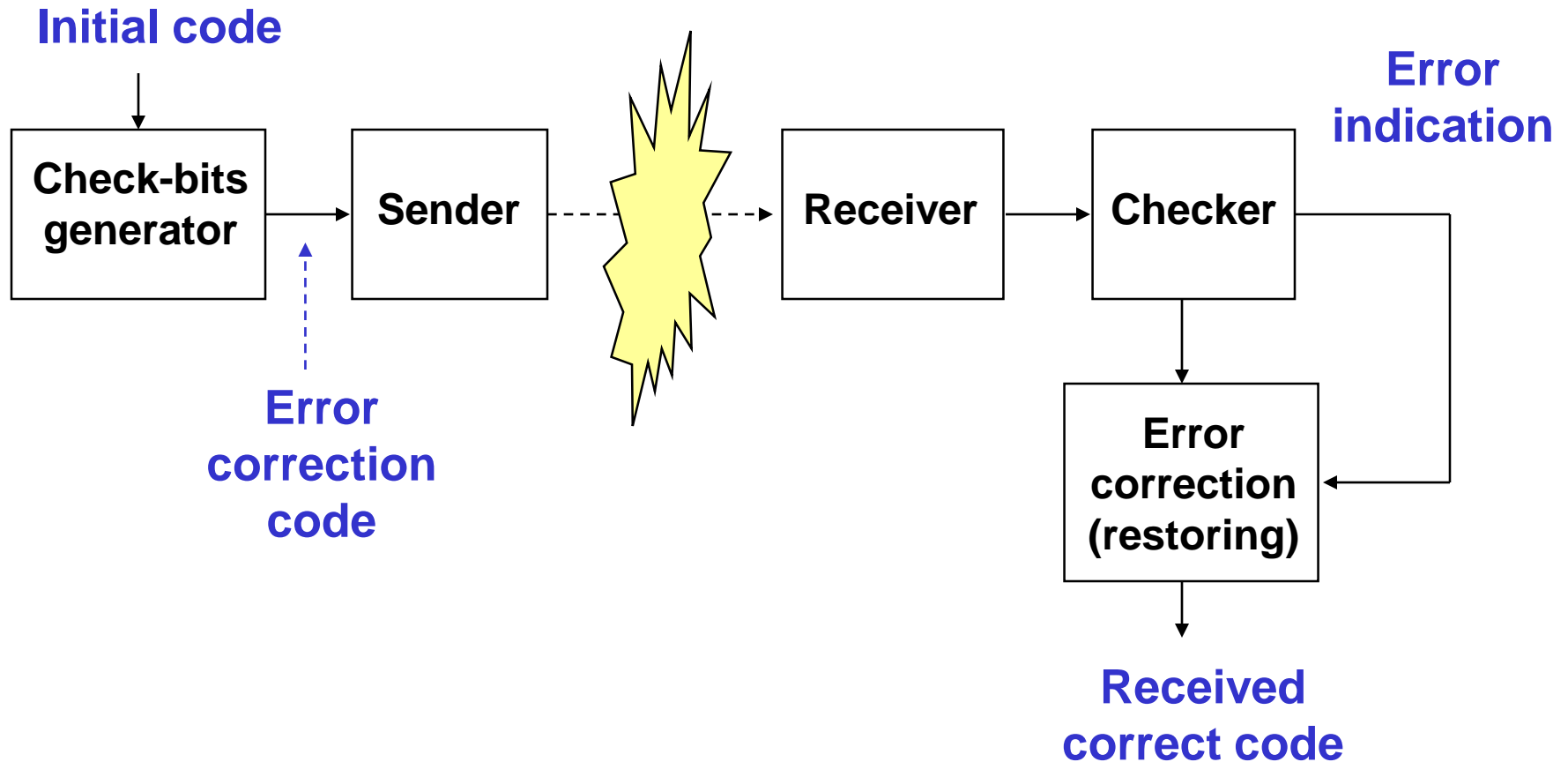
1	0	1	1	1	0	1
---	---	---	---	---	---	---

Received code

	7	6	5	4	3	2	1	0	Test
P_1	1		1		1		1		0
P_2	1	1			1	1			0
P_3	1	1	1	1					1

Diagnosis

Fault Tolerant Communication System



Error Detection in Arithmetic Operations

Residue codes

N – information code

C = (N) mod m - check code

m – residue of the code

p = $\lceil \log_2 m \rceil$ – number of check bits

Example

Information bits: **I_2, I_1, I_0**

m = 3, p = 2

Check bits: **C_1, C_0**

Information bits				Check bits		
I_2	I_1	I_0	I	C	C_1	C_0
0	0	0	0	0	0	0
0	0	1	1	1	0	1
0	1	0	2	2	1	0
0	1	1	3	0	0	0
1	0	0	4	1	0	1
1	0	1	5	2	1	0
1	1	0	6	0	0	0
1	1	1	7	1	0	1

Error Detection in Arithmetic Operations

Addition:

Information bits	Check bits	
0 0 1 0	1 0	2.2
0 1 0 0	0 1	4.1
<hr/>		
0 1 1 0	1 1	6.3
$(6) \bmod 3 = 0$	$(3) \bmod 3 = 0$	

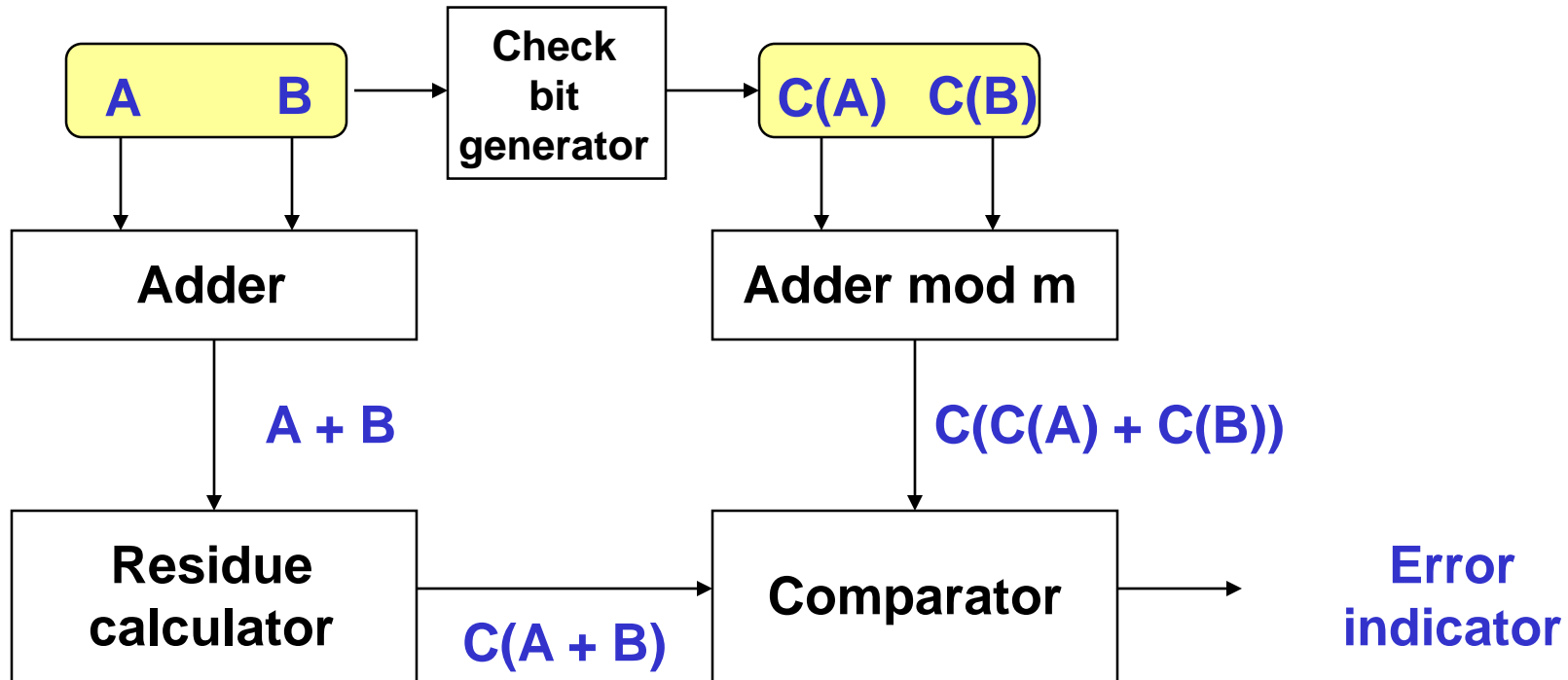
Multiplication:

Information bits	Check bits	
0 0 1 0	1 0	2.2
0 1 0 0	0 1	4.1
<hr/>		
1 0 0 0	1 0	8.2
$(8) \bmod 3 = 2$	$(2) \bmod 3 = 2$	

Information bits	Check bits	
0 0 1 0	1 0	2.2
0 1 0 0	0 1	4.1
<hr/>		
0 1 0 0	1 1	4.3
$(4) \bmod 3 = 1$	$(3) \bmod 3 = 0$	
Error!		

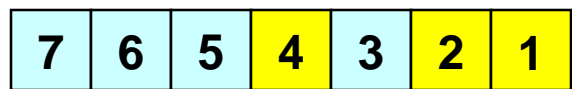
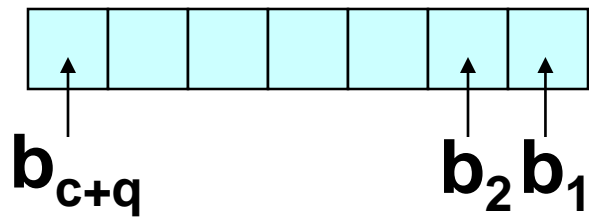
Information bits	Check bits	
0 0 1 0	1 0	2.2
0 1 0 0	0 1	4.1
<hr/>		
1 0 0 1	1 0	9.2
$(9) \bmod 3 = 0$	$(2) \bmod 3 = 2$	
Error!		

Error Detection in Arithmetic Operations



Fault Tolerance: One Error Correcting Code

One error correction code: $2^c \geq q + c + 1$



Check bits

Calculation of check sums:

$$\sum_{k \in P_i} b_k = 0, i = 1, \dots, c$$

Parity bits for $c = 3$:

$$P_1 = b_1 \oplus b_3 \oplus b_5 \oplus b_7 = 0$$

$$P_2 = b_2 \oplus b_3 \oplus b_6 \oplus b_7 = 0$$

$$P_3 = b_4 \oplus b_5 \oplus b_6 \oplus b_7 = 0$$

Theory of LFSR

Characteristic Polynomials:

$$G(x) = c_0 + c_1x + c_2x^2 + \dots + c_mx^m + \dots = \sum_{m=0}^{\infty} c_mx^m$$

**Multiplication of
polynomials**

$$\begin{array}{r} x^2 + x + 1 \\ \underline{x^2 + 1} \\ x^2 + x + 1 \\ \underline{x^4 + x^3 + x^2} \\ x^4 + x^3 \quad \quad + x + 1 \end{array}$$

Fault Tolerant Communication System

