Additive Decomposition

corresponds to a given partition on the set of states of prototype FSM

Decomposition problem

- Often convenient to realize a sequential circuit as an interconnection of sub-circuits that realize the same terminal behavior

- A large hardware behavioral description is decomposed into several smaller ones

- First, decide on how the overall circuit is to be broken up and what function each of the sub-circuits must serve

- Then, treat each of the sub-circuits as a separate and independent design problem

FSM decomposition problem

- Decomposition has been a classic problem of discrete system theory for many years. FSM decomposition is a topic that waxes and wanes in importance. The fundamental works were done in the 1960s, became less interesting during the era beginning of era of VLSI, and is becoming more important again with pervasive use of programmable logic in digital design.
- One goal is to make the synthesis problem more tractable by providing smaller sub-problems that can be solved efficiently.
- Another goal is to create descriptions that can be synthesized into a structure that meets the design constraints.
- In the past, synthesis focused on quality measures based on area and performance. The continuing decrease in feature size and increase in chip density in recent years have given rise to consider decomposition theory for low power as new dimension of the design process.

FSM decomposition

- Given a FSM description of a desired terminal behavior, the decomposition problem is to find two or more machines which, when interconnected in a prescribed way, will display that terminal behavior.

- Hardware behavior description is decomposed into a network of interconnected FSMs targeting optimization by various criteria (performance, measurements, power consumption).

- We should emphasize the fact that the FSM decomposition and the reduction of variable dependence (for example: for state assignment) are virtually identical concepts.

Prototype FSM model

Formally, FSM is a quintuple $\langle S, X, Y, \delta, \lambda \rangle$, where

- $S = \{ s_1, \ldots, s_n \}$ is a set of states;
- $X = \{ x_1, \ldots, x_l \}$ is a set of primary input variables;
- $Y = \{ y_1, \ldots, y_m \}$ is a set of primary output variables;
- $\delta : D(\delta) \rightarrow S$ is a multiple valued next state function with

  domain $D(\delta) = \{0, 1\}^l \times S$ and

  codomain S;

  $\{0, 1\}$ represents a set of values (symbols) each input variable x_i may assume;
- $\lambda : D(\lambda) \rightarrow R(\lambda)$ is an output function with

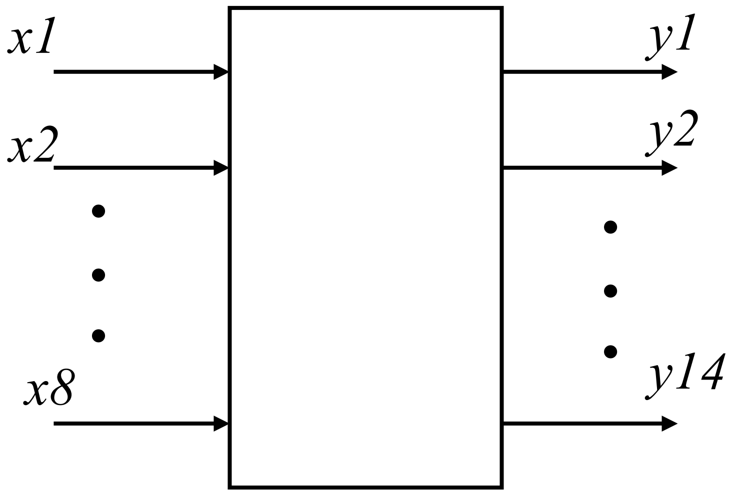
  domain $D(\lambda) = \{0, 1\}^l \times S$ and

  codomain $R(\lambda) = \{0, 1\}^m$;

  $\{0, 1\}$ is a set of values each output variable y_i may assume.

Example Prototype FSM

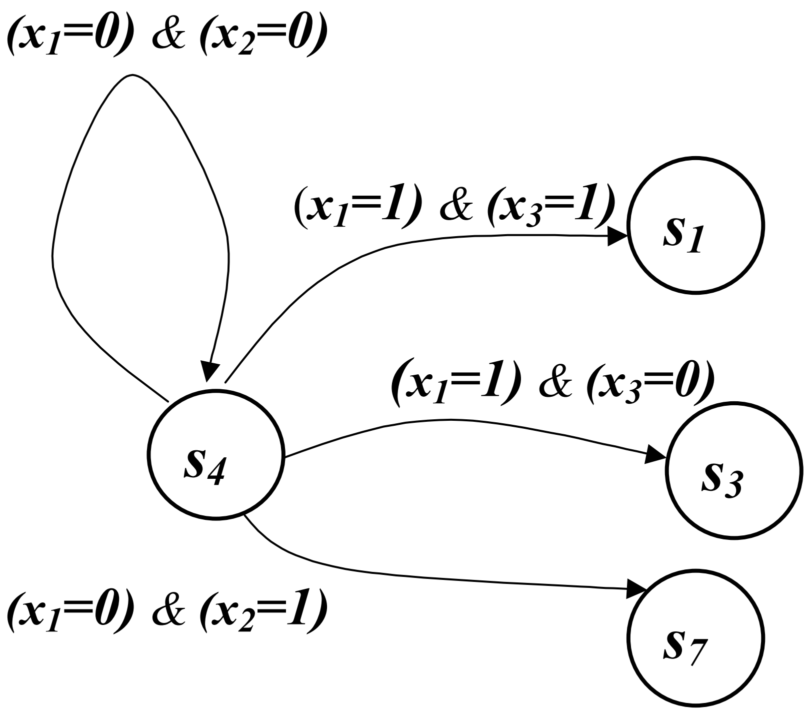
- $S=\{ s_1 , s_2 , \ldots , s_8 \}$
- $X=\{x_1 , x_2 , \ldots , x_8 \}$
- $Y= \{y_1 , y_2 , \ldots , y_{14} \}$



Present state s_p	Next state s_n	Input condition $X_h(s_p, s_n)$	Output signals $Y_h(s_p, s_n)$	h
s_1	s_1	x_1	y_7	1
	s_3	$\hat{} x_1$	-	2
s_2	s_3	1	$y_{10}\ y_{11}$	3
s_3	s_6	x_7	$y_{10}\ y_{11}$	4
	s_8	$\hat{} x_7 \,\&\, x_8$	-	5
	s_2	$\hat{} x_7 \,\&\, \hat{} x_8$	$y_2\ y_5\ y_{10}$	6
s_4	s_1	$x_1 \,\&\, x_3$	$y_3\ y_4$	7
	s_3	$x_1 \,\&\, \hat{} x_3$	$y_1\ y_3\ y_4$	8
	s_7	$\hat{} x_1 \,\&\, x_2$	$y_3\ y_4$	9
	s_4	$\hat{} x_1 \,\&\, \hat{} x_2$	y_1	10
s_5	s_4	$x_4 \,\&\, x_6$	$y_6\ y_{13}$	11
	s_5	$x_4 \,\&\, \hat{} x_6$	$y_6\ y_{13}$	12
	s_8	$\hat{} x_4$	$y_6\ y_8$	13
s_6	s_2	x_5	$y_{10}\ y_{11}$	14
	s_3	$\hat{} x_5 \,\&\, x_7$	y_{12}	15
	s_8	$\hat{} x_5 \,\&\, \hat{} x_7$	$y_{10}\ y_{11}$	16
s_7	s_5	1	y_1	17
s_8	s_8	x_6	-	18
	s_5	$\hat{} x_6$	$y_9\ y_{14}$	19

FSM description

s_4	s_1	x_1 & x_3	y_3 y_4	7
	s_3	x_1 & ^x_3	y_1 y_3 y_4	8
	s_7	^x_1 & x_2	y_3 y_4	9
	s_4	^x_1 & ^x_2	y_1	10



FSM description

The search for the next state means the evaluation of the Boolean functions. It is necessary to evaluate which of these functions has value “true” for a given input combination σ from $\{0, 1\}^l$.

We use the representation of Boolean functions with complexes of cubes. A *cube* (product term) in $\{0, 1\}^{nv}$ of dimension p (p-cube) is a collection of any 2^p points (minterms) that have exactly $nv-p$ bits all the same (bound components).

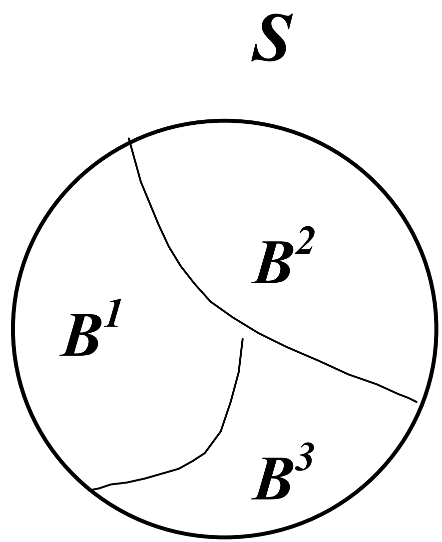
FSM description

s_4	s_1	$x_1 \& x_3$	$y_3\ y_4$	7
	s_3	$x_1 \& \hat{}x_3$	$y_1\ y_3\ y_4$	8
	s_7	$\hat{}x_1 \& x_2$	$y_3\ y_4$	9
	s_4	$\hat{}x_1 \& \hat{}x_2$	y_1	10

		$x_1\, x_2\, x_3\, x_4\, x_5\, x_6$	$y_1\, y_2\, y_3\, y_4 \ldots y_{13}\, y_{14}$	
s_4	s_1	1 - 1 - - -	00110000000000	7
	s_3	1 - 0 - - -	10110000000000	8
	s_7	0 1 - - - -	00110000000000	9
	s_4	0 0 - - - -	10000000000000	10

Partition

- A *partition* of a *set* ***S*** is collection of nonempty and pairwise disjoint subsets of ***S*** which exhaust the set ***S***.
- We can give a diagrammatic representation of partitions. If the set ***S*** is represented by an enclosed area of paper, we can draw lines to divide the area into nonoverlapping regions. Each region of the resulting diagram will correspond to a block of partition.
- Distinct partition of ***S*** induce distinct *equivalence relation* on a nonempty set ***S***.



Example:

$S = \{ s_1, s_2, \ldots, s_8 \}$

$\pi = \{ \{ s_1, s_4, s_7 \}, \{ s_2, s_3, s_6 \}, \{ s_5, s_8 \} \}$

Approach for FSM decomposition

Let put network *N* with *n* component automata in accordance to pair ***(A, π)***. The number of component automata is equal to the number of blocks in the partition *π*.

So, in our example there will be three component automata B^1, B^2 and B^2, since there are three blocks (B^1, B^2 *and* B^3) in the partition *π*.

$$\pi = \{ \{ s_1, s_4, s_7 \}, \{ s_2, s_3, s_6 \}, \{ s_5, s_8 \} \}$$

Decomposition procedure

- Generation of the initial partition
- Definition of states in component FSMs
- Definition of the set of external input variables of component FSM
- Generation of the set of the set of internal (additional) input variables
- Definition of the set of output variables of component FSMs
- Generation of transition and output functions of component FSMs
- Realization of FSM network.

The set of states of (component) sub-FSM

Define set of states in component automata as:

$$S^m = B^m \cup \{a_m\}$$

where:

B^m - is the block of the partition π,

a_m - is the additional state that exists in each component automata.

Let define states of component automata in our example:

$S = \{ s_1 , s_2 , \ldots , s_8 \}$

$\pi = \{ \{ s_1, s_4, s_7 \}, \{ s_2 , s_3, s_6 \}, \{ s_5 , s_8 \} \}$

$S^1 = \{ s_1, s_4, s_7, a_1 \}$

$S^2 = \{ s_2, s_3, s_6, a_2 \}$

$S^3 = \{ s_5, s_8, a_3 \}$

Set of external input variables

X (*B^m*) is the set of external input variables at all transitions from the states of block *B^m* in the transition table of the prototype FSM *A*:

$$X(B^m) = \cup_{s \in B^m} X(s)$$

In our example:

$X(B^1) = \{x_1, x_2, x_3\}$ ⟷ $\{s_1, s_4, s_7\}$

$X(B^2) = \{x_5, x_7, x_8\}$ ⟷ $\{s_2, s_3, s_6\}$

$X(B^3) = \{x_4, x_6\}$ ⟷ $\{s_5, s_8\}$

$$\pi = \{ \{s_1, s_4, s_7\}, \{s_2, s_3, s_6\}, \{s_5, s_8\} \}$$

Present state s_p	Input condition $X_h(s_p, s_n)$	h
s_1	x_1	1
	$\wedge x_1$	2
s_2	1	3
s_3	x_7	4
	$\wedge x_7 \& x_8$	5
	$\wedge x_7 \& \wedge x_8$	6
s_4	$x_1 \& x_3$	7
	$x_1 \& \wedge x_3$	8
	$\wedge x_1 \& x_2$	9
	$\wedge x_1 \& \wedge x_2$	10
s_5	$x_4 \& x_6$	11
	$x_4 \& \wedge x_6$	12
	$\wedge x_4$	13
s_6	x_5	14
	$\wedge x_5 \& x_7$	15
	$\wedge x_5 \& \wedge x_7$	16
s_7	1	17
s_8	x_6	18
	$\wedge x_6$	19

Set of external output variables

$Y(B^m)$ is the set of external output variables at all transitions from the states of block B^m in the transition table of the FSM A:

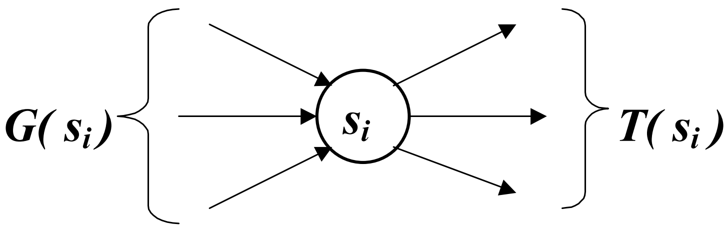
$$Y(B^m) = \cup_{s \in B^m} \; Y(s)$$

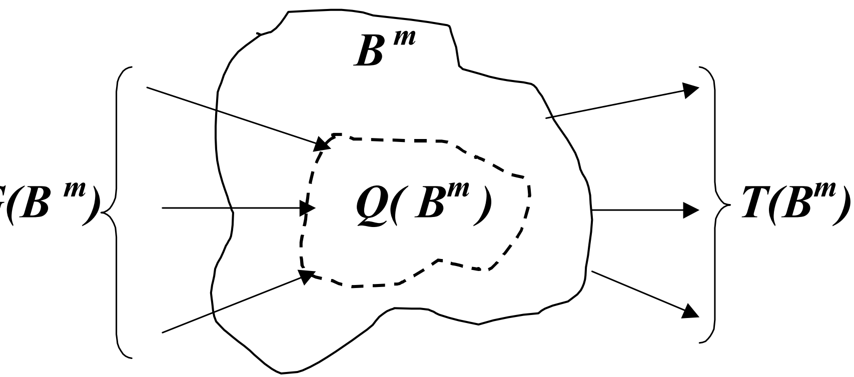
In our example:

$Y(B^1) = \{ y_3, y_4, y_7 \}$
$Y(B^2) = \{ y_2, y_5, y_{10}, y_{11}, y_{12} \}$
$Y(B^3) = \{ y_6, y_8, y_9, y_{13}, y_{14} \}$

Present state s_p	Output signals $Y_h(s_p, s_n)$	h
s_1	y_7	1
	-	2
s_2	y_{10} y_{11}	3
s_3	y_{10} y_{11}	4
	-	5
	y_2 y_5 y_{10}	6
s_4	y_3 y_4	7
	y_1 y_3 y_4	8
	y_3 y_4	9
	y_1	10
s_5	y_6 y_{13}	11
	y_6 y_{13}	12
	y_6 y_8	13
s_6	y_{10} y_{11}	14
	y_{12}	15
	y_{10} y_{11}	16
s_7	y_1	17
s_8	-	18
	y_9 y_{14}	19

Generation of additional inputs and outputs





- G^m is the set of states not included in B^m, from which there are transitions to the states of B^m:

In our example:

$G^1=\{ s_5 \}$, $G^2=\{ s_1, s_4 \}$, $G^3=\{ s_3, s_6, s_7 \}$

- T^m is the set of states not included in B^m, to which there are transitions from the states of B^m:

In our example:

$T^1 = \{ s_3, s_5 \}$, $T^2 = \{ s_8 \}$, $T^3 = \{ s_4 \}$

- Q^m is the subset of states of B_m, to which there are transitions from states not included in B^m :

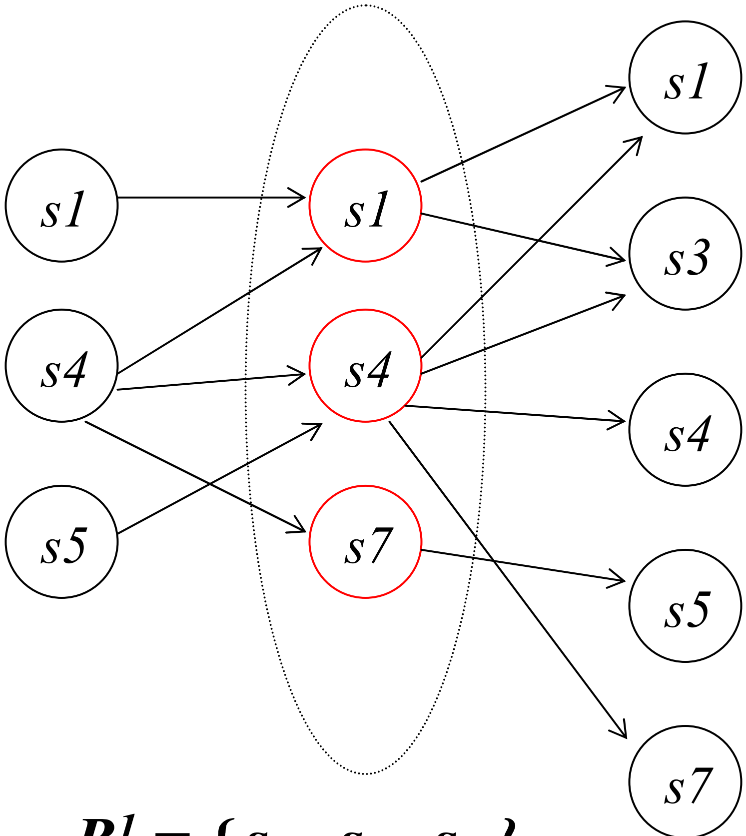
In our exmple:

$Q^1 = \{ s_4 \}$ $Q^2 = \{ s_3 \}$, $Q^3 = \{ s_5, s_8 \}$

Example FSM parameters

s_i	$X(s_i)$	$Y(s_i)$	$G(s_i)$	$T(s_i)$
s_1	x_1	y_7	s_4	s_3
s_2	-	y_{10} y_{11}	s_3 s_6	s_3
s_3	x_7 x_8	y_2 y_5 y_{10} y_{11}	s_1 s_2 s_4 s_6	s_2 s_6 s_8
s_4	x_1 x_2 x_3	y_1 y_3 y_4	s_5	s_1 s_3 s_7
s_5	x_4 x_6	y_6 y_8 y_{13}	s_7 s_8	s_4 s_8
s_6	x_5 x_7	y_{10} y_{11} y_{12}	s_3	s_2 s_3 s_8
s_7	-	y_1	s_4	s_5
s_8	x_6	y_9 y_{14}	s_3 s_5 s_6	s_5

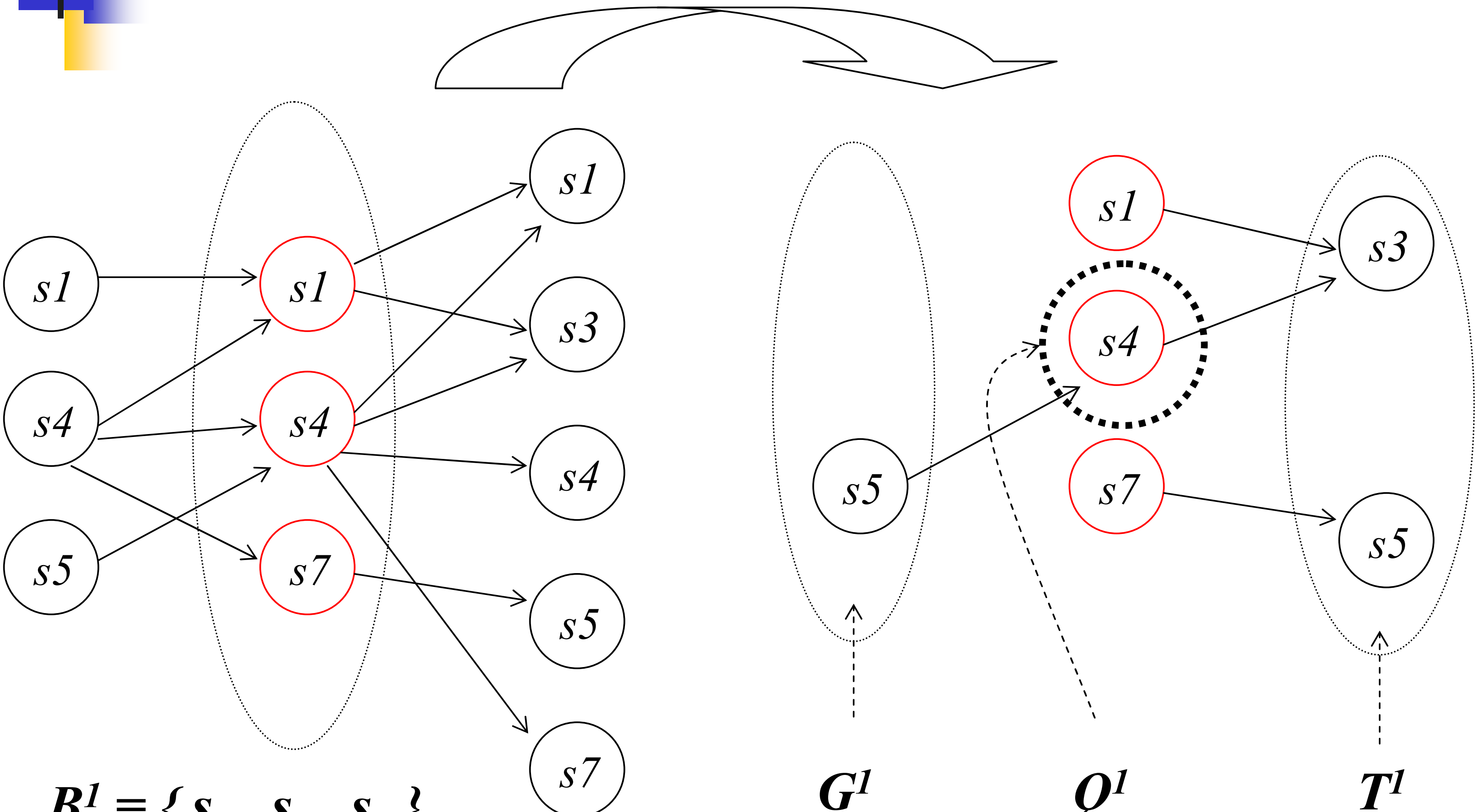
Additional variables for the first sub-FSM



$$B^1 = \{ s_3, \ s_4, \ s_7 \}$$

Present state s_p	Next state s_n	h
s_1	s_1	1
s_1	s_3	2
s_2	s_3	3
s_3	s_6	4
s_3	s_8	5
s_3	s_2	6
s_4	s_1	7
s_4	s_3	8
s_4	s_7	9
s_4	s_4	10
s_5	s_4	11
s_5	s_5	12
s_5	s_8	13
s_6	s_2	14
s_6	s_3	15
s_6	s_8	16
s_7	s_5	17
s_8	s_8	18
s_8	s_5	19

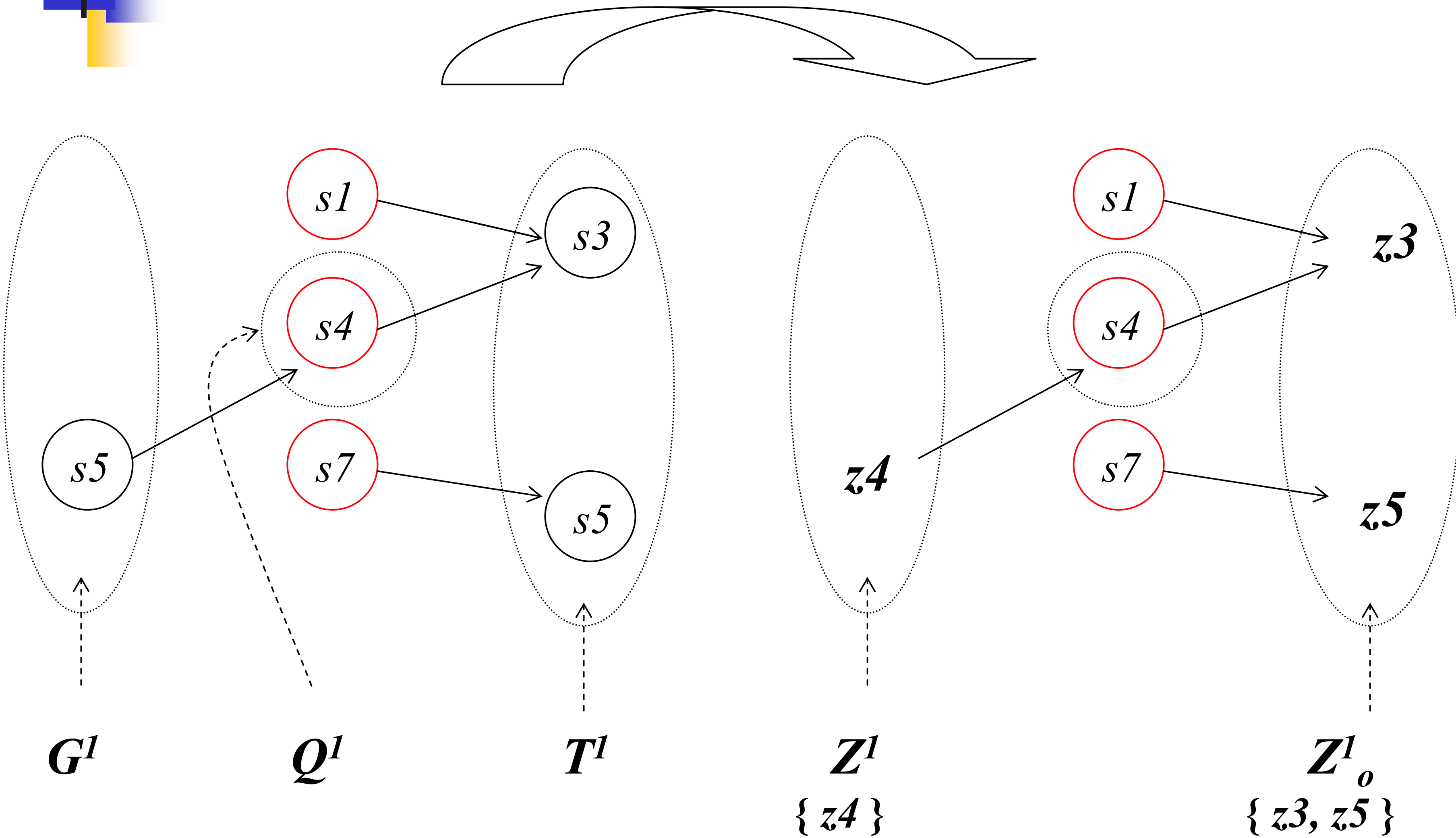
Additional variables for the first sub-FSM



$B^1 = \{ s_1, s_4, s_7 \}$

G^1 Q^1 T^1

Additional internal variables of the first sub-FSM



The set of input variables of component FSM

Now we can define the set of input variables X^m of m-th component FSM:

$$X^m = X(B^m) \cup Z^m$$

Here Z^m is the set of additional input variables which connect other component automata with this automaton A^m. To define let put the additional input variable z_k in accordance to each state in the set Q^m. So, the number of elements in Z^m is equal to the number of states in Q^m.

In our example:
$Z^1 = \{z_4\}$, $Z^2 = \{z_3\}$, $Z^3 = \{z_5, z_8\}$.

Additional output variables of sub-FSM

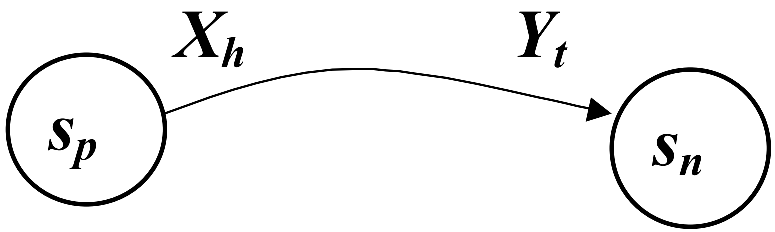
Similarly, we define the set of output variables Y^m:

$$Y^m = Y(B^m) \cup Z^m_o$$

Here Z^m_o is the set of additional output variables which connect the FSM B^m with other component FSMs. To define Z^m_o let put the additional output variable z_i in accordance to each state in the set T^m. So, the number of elements in Z^m *(outputs)* is equal to the number of states in T^m:

In our example:
$Z^1_o = \{z_3, z_5\}$, $Z^2_o = \{z_8\}$, $Z^3_o = \{z_4\}$.

Transition and output functions of sub-FSMs



Let

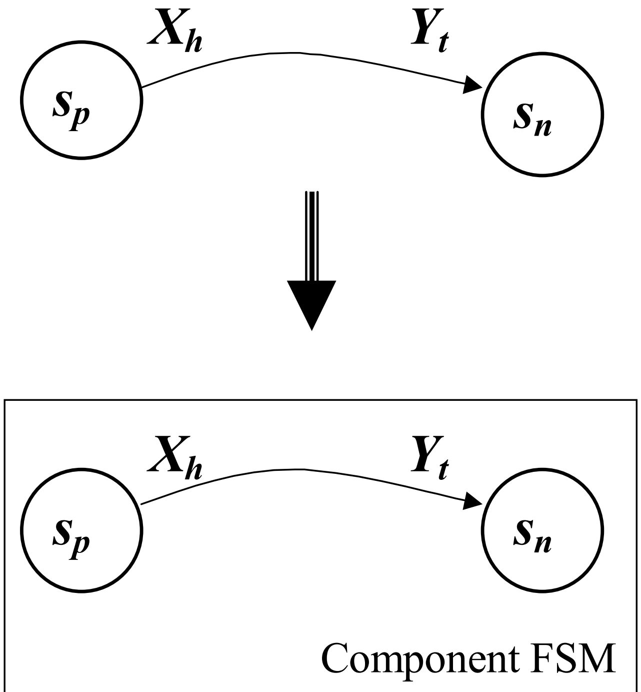
$$\delta(s_p, X_h) = s_n; \quad \lambda(s_p, X_h) = Y_t$$

be transition in the prototype FSM A.

There are two cases possible

- $s_p \in B^m$ and $\mathbf{s_n} \in \mathbf{B^m}$ (s_n and s_n are states of the same component FSM B^m)
- $s_p \in B^m$ and $s_n \notin B^m$ (s_p and s_n are states of two different component FSMs) .

Transition and output functions of sub-FSMs



a) $s_p \in B^m$ and $s_n \in B^m$, then

$$\delta^m(s_p, X_h^m) = s_n ;$$

$$\lambda^m(s_p, X_h^m) = Y_t$$

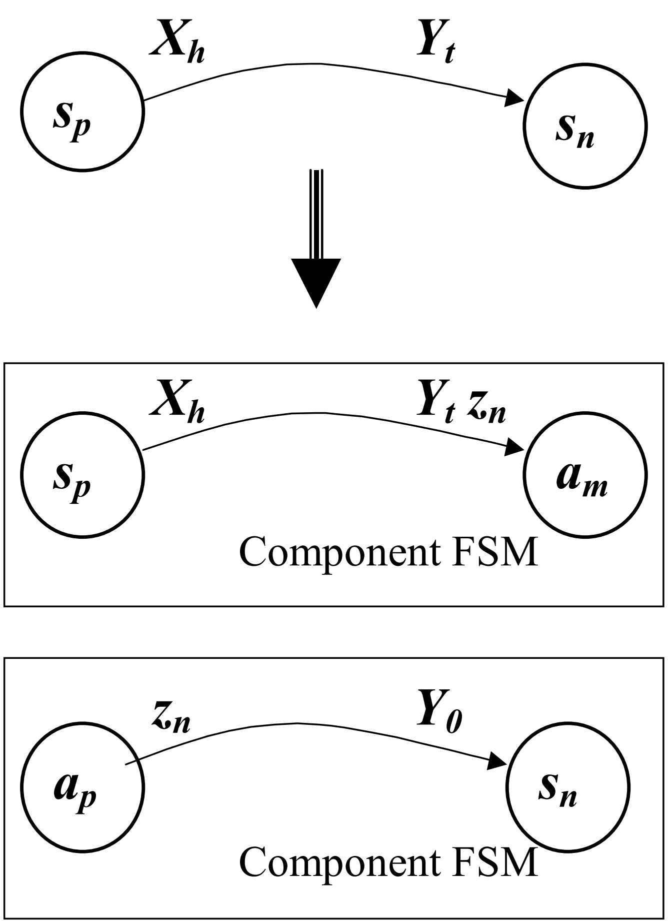
is the transition in component FSM B^m

Transition and output functions of sub-FSMs

Present state s_p	Next state s_n	Input condition $X_h(s_p, s_n)$	Output signals $Y_h(s_p, s_n)$	h
s_1	s_1	x_1	y_7	1
s_1	s_3	^x_1	-	2
s_2	s_3	*1*	y_{10} y_{11}	3
s_3	s_6	x_7	y_{10} y_{11}	4
s_3	s_8	^x_7 & x_8	-	5
s_3	s_2	^x_7 & ^x_8	y_2 y_5 y_{10}	6
s_4	s_1	x_1 & x_3	y_3 y_4	7
s_4	s_3	x_1 & ^x_3	y_1 y_3 y_4	8
s_4	s_7	^x_1 & x_2	y_3 y_4	9
s_4	s_4	^x_1 & ^x_2	y_1	10
s_5	s_4	x_4 & x_6	y_6 y_{13}	11
s_5	s_5	x_4 & ^x_6	y_6 y_{13}	12
s_5	s_8	^x_4	y_6 y_8	13
s_6	s_2	x_5	y_{10} y_{11}	14
s_6	s_3	^x_5 & x_7	y_{12}	15
s_6	s_8	^x_5 & ^x_7	y_{10} y_{11}	16
s_7	s_5	*1*	y_1	17
s_8	s_8	x_6	-	18
s_8	s_5	^x_6	y_9 y_{14}	19

Present state s_p	Next state s_n	Input condition $X_h(s_p, s_n)$	Output signals $Y_h(s_p, s_n)$	h
s_1	s_1	x_1	y_7	1
				2
				3
				4
				5
				6
s_4	s_1	x_1 & x_3	y_3 y_4	7
				8
s_4	s_7	^x_1 & x_2	y_3 y_4	9
s_4	s_4	^x_1 & ^x_2	y_1	10
				11
				12
				13
				14
				15
				16
s_7				17
				18
				19

Transition and output functions of sub-FSMs



b) $s_p \in B^m$ and $s_n \notin B^m$. Then, the component FSM B^m transists from state s_p to the additional state a_m with the output Y_t and additional output z_i:

$$\delta^m (s_p, X_h{}^m) = a_m ;$$

$$\lambda^m (s_p, X_h) = Y_t \cup z_i, z_i \in B^r$$

The additional output z_i of the component FSM B^m is the input of the component B^r . This input z_i produces the transition from the additional state a_r to the state s_n with the output $Y_0{}^r$ (all output variables are equal to 0) in the FSM B^r :

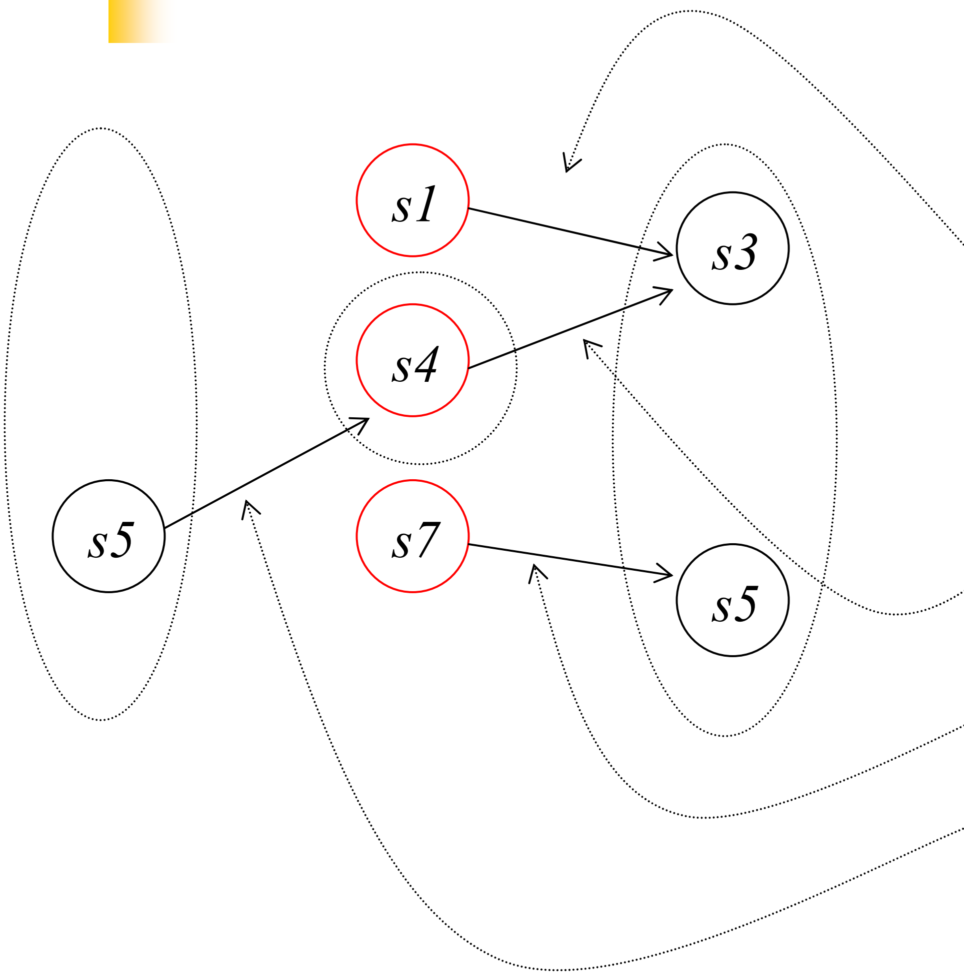
$$\delta^r (a_r, X_h) = s_n ; \lambda^r (a_r, X_h) = Y_0{}^r$$

Transition and output functions of sub-FSMs

Present state s_p	Next state s_n	Input condition $X_h(s_p, s_n)$	Output signals $Y_h(s_p, s_n)$	h
s_1	s_1	x_1	y_7	1
s_1	s_3	$\hat{}x_1$	-	2
s_2	s_3	*1*	y_{10} y_{11}	3
s_3	s_6	x_7	y_{10} y_{11}	4
s_3	s_8	$\hat{}x_7$ & x_8	-	5
s_3	s_2	$\hat{}x_7$ & $\hat{}x_8$	y_2 y_5 y_{10}	6
s_4	s_1	x_1 & x_3	y_3 y_4	7
s_4	s_3	x_1 & $\hat{}x_3$	y_1 y_3 y_4	8
s_4	s_7	$\hat{}x_1$ & x_2	y_3 y_4	9
s_4	s_4	$\hat{}x_1$ & $\hat{}x_2$	y_1	10
s_5	s_4	x_4 & x_6	y_6 y_{13}	11
s_5	s_5	x_4 & $\hat{}x_6$	y_6 y_{13}	12
s_5	s_8	$\hat{}x_4$	y_6 y_8	13
s_6	s_2	x_5	y_{10} y_{11}	14
s_6	s_3	$\hat{}x_5$ & x_7	y_{12}	15
s_6	s_8	$\hat{}x_5$ & $\hat{}x_7$	y_{10} y_{11}	16
s_7	s_5	*1*	y_1	17
s_8	s_8	x_6	-	18
s_8	s_5	$\hat{}x_6$	y_9 y_{14}	19

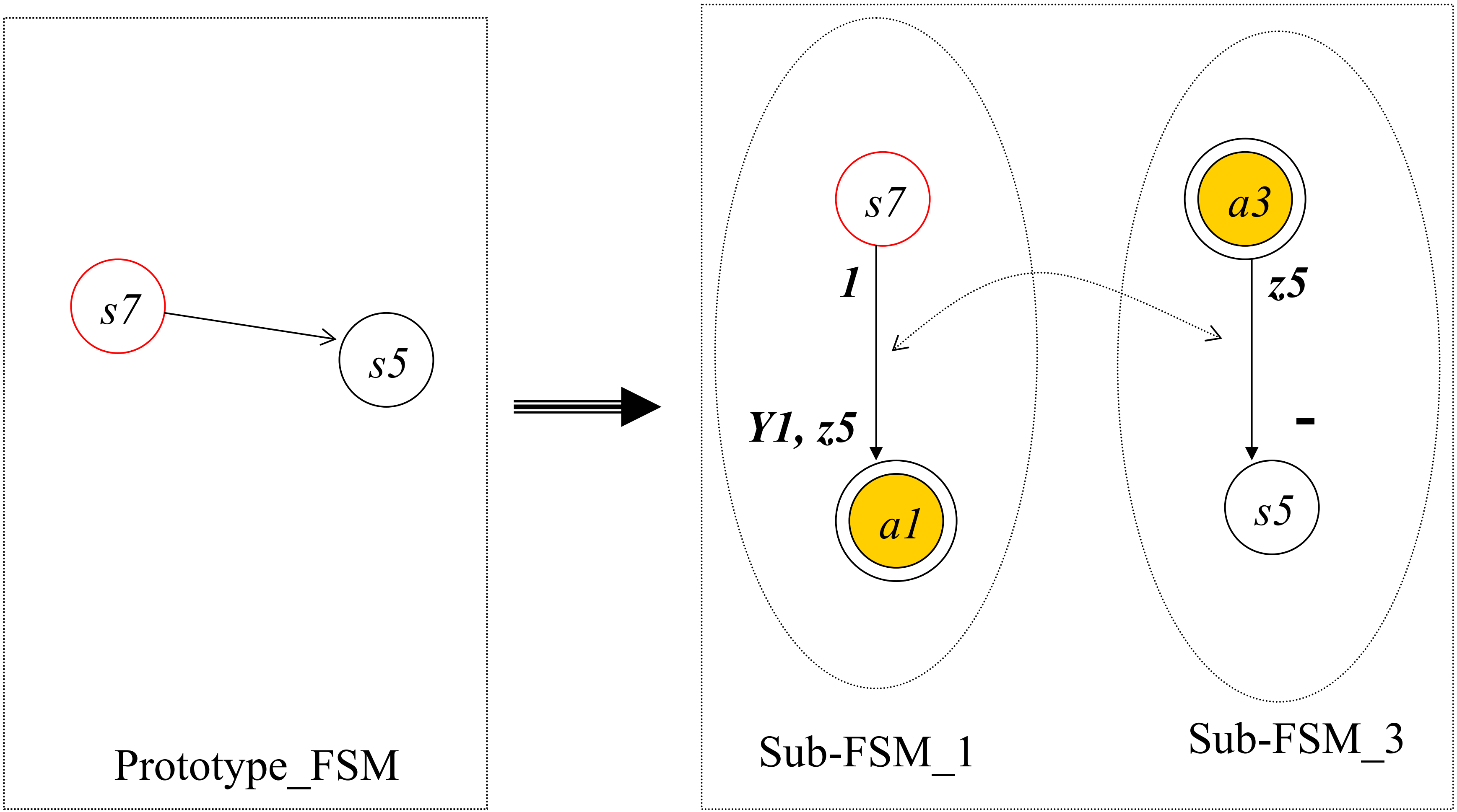
Present state s_p	Next state s_n	Input condition $X_h(s_p, s_n)$	Output signals $Y_h(s_p, s_n)$	h
s_1	s_1	x_1	y_7	1
s_1	*a1*	$\hat{}x_1$	*z3*	2
				3
				4
				5
				6
s_4	s_1	x_1 & x_3	y_3 y_4	7
s_4	*a1*	x_1 & $\hat{}x_3$	$y_1 y_3 y_4$ *z3*	8
s_4	s_7	$\hat{}x_1$ & x_2	y_3 y_4	9
s_4	s_4	$\hat{}x_1$ & $\hat{}x_2$	y_1	10
				11
				12
				13
				14
				15
				16
s_7	*a1*	*1*	y_1 *z5*	17
				18
				19

Transition table of the first sub-FSM



s_p	s_n	$X_h(s_p, s_n)$	$Y_h(s_p, s_n)$
s_1	s_1	x_1	y_7
	a_1	^x_1	$z3$
s_4	s_1	x_1 & x_3	y_3 y_4
	a_1	x_1 & ^x_3	y_1 y_3 y_4 z_3
	s_7	^x_1 & x_2	y_3 y_4
	s_4	^x_1 & ^x_2	y_1
s_7	a_1	1	y_1 z_5
a_1	a_1	^z_4	-
	s_4	z_4	-

Transformation of transition from s7 to s5

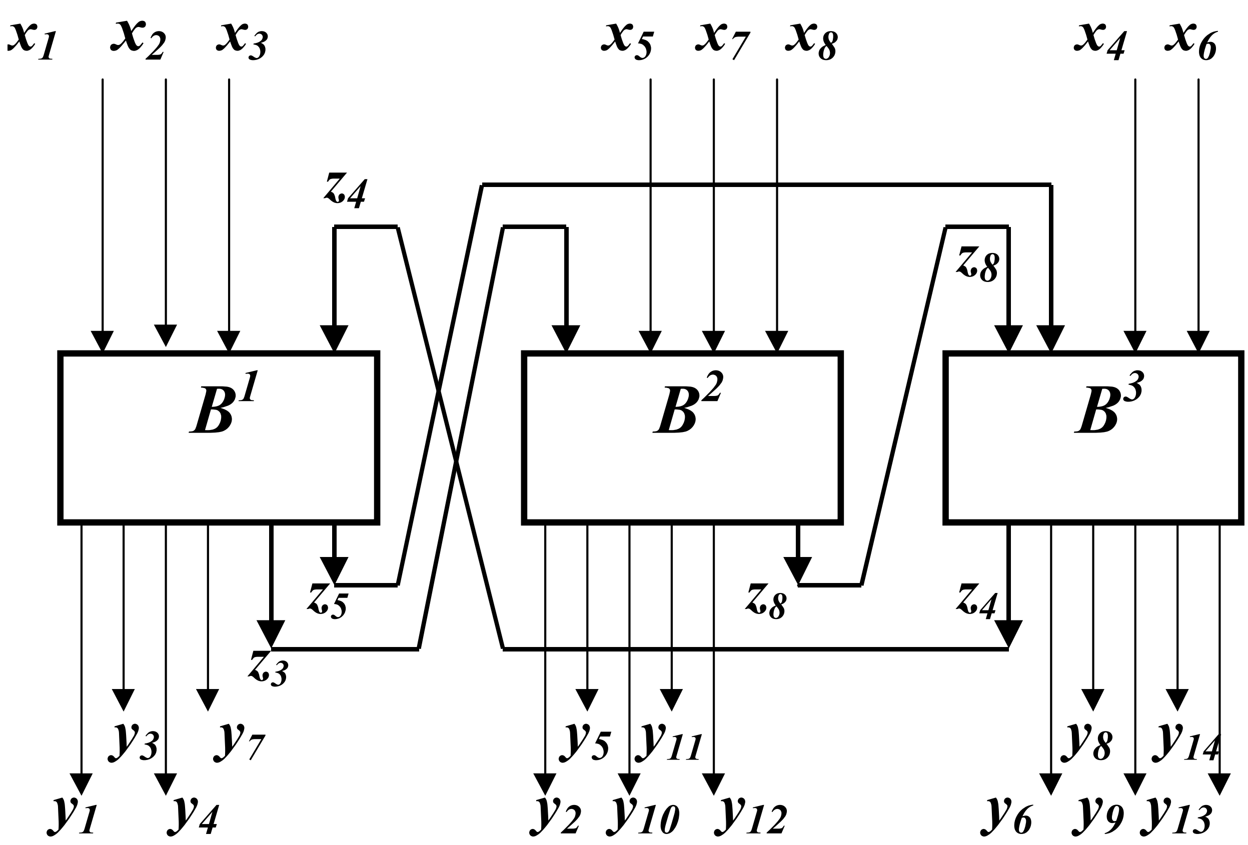


Component FSMs B2 and B3

Present state s_p	Next state s_n	Input condition $X_h(s_p, s_n)$	Output signals $Y_h(s_p, s_n)$	h
s_2	s_3	1	$y_{10}\ y_{11}$	1
s_3	s_6	x_7	$y_{10}\ y_{11}$	2
	a_2	$\wedge x_7 \& x_8$	z_8	3
	s_2	$\wedge x_7 \& \wedge x_8$	$y_2\ y_5\ y_{10}$	4
s_6	s_2	x_5	$y_{10}\ y_{11}$	5
	s_3	$\wedge x_5 \& x_7$	y_{12}	6
	a_2	$\wedge x_5 \& \wedge x_7$	$y_{10}\ y_{11}\ z_8$	7
a_2	a_2	z_3	-	8
	s_3	z_3	-	9

Present state s_p	Next state s_n	Input condition $X_h(s_p, s_n)$	Output signals $Y_h(s_p, s_n)$	h
s_5	a_3	$x_4 \& x_6$	$y_6\ y_{13}\ z_4$	1
	s_5	$x_4 \& \wedge x_6$	$y_6\ y_{13}$	2
	s_8	$\wedge x_4$	$y_6\ y_8$	3
s_8	s_8	x_6	-	4
	s_5	$\wedge x_6$	$y_9\ y_{14}$	5
a_3	s_8	z_8	-	6
	s_5	z_5	-	7
	a_3	$\wedge z_5\ \wedge z_8$	-	8

FSM Network



x_1
x_2
x_3
x_5
x_7
x_8
x_4
x_6
z_4
z_8
B^1
B^2
B^3
z_5
z_3
z_8
z_4
y_3
y_7
y_1
y_4
y_5
y_{11}
y_2
y_{10}
y_{12}
y_8
y_{14}
y_6
y_9
y_{13}

FSM B1 synthesis (realization)

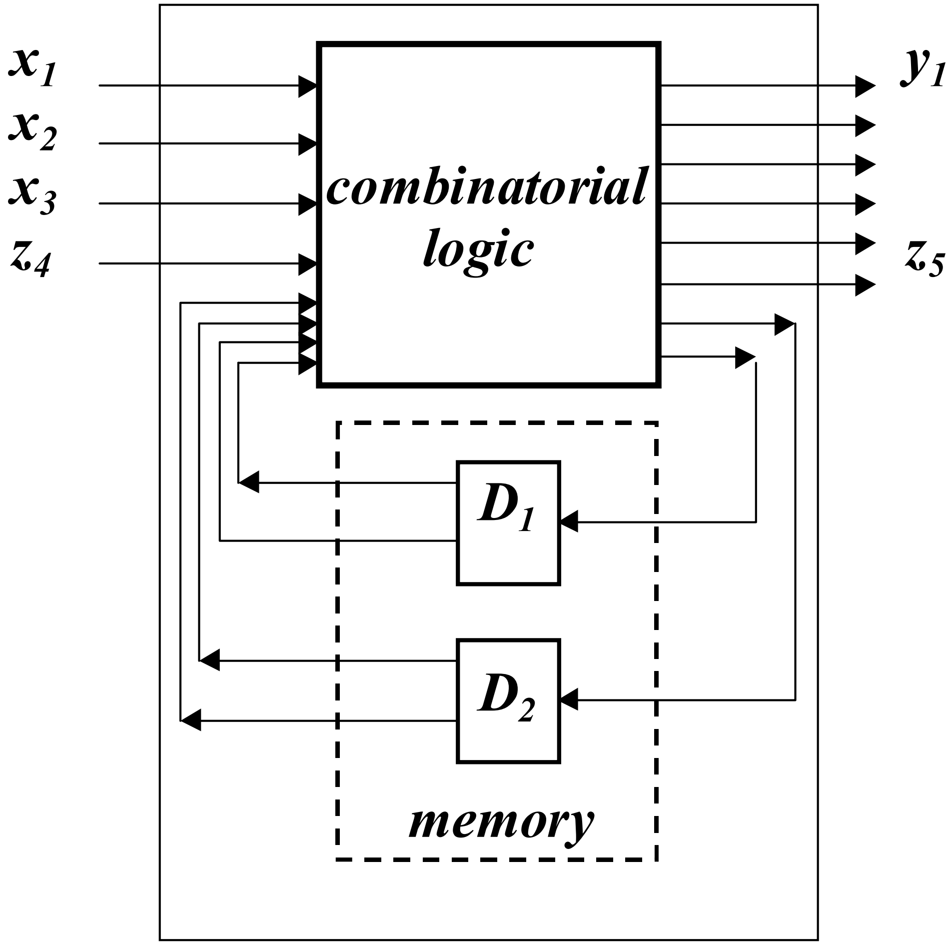
s_p	$K(s_p)$ d_1d_2	s_n	$K(s_p)$	$X_h(s_p, s_n)$	D_h	$Y_h(s_p, s_n)$
s_1	00	s_1	00	x_1	-	y_7
		a_1	11	${}^\wedge x_1$	D_1D_2	z_3
s_4	01	s_1	00	x_1 & x_3	-	y_3 y_4
		a_1	11	x_1 & ${}^\wedge x_3$	D_1D_2	y_1 y_3 y_4 z_3
		s_7	10	${}^\wedge x_1$ & x_2	D_1	y_3 y_4
		s_4	01	${}^\wedge x_1$ & ${}^\wedge x_2$	D_2	y_1
s_7	10	a_1	00	1	-	y_1 z_5
a_1	11	s_4	01	z_4	D_2	-
		a_1		${}^\wedge z_4$		

$$f_{D1}=({}^\wedge d_1\ {}^\wedge d_2\ {}^\wedge x_1) \vee ({}^\wedge d_1\ d_2\ x_1\ {}^\wedge x_3) \vee ({}^\wedge d_1\ d_2\ {}^\wedge x_1\ x_2)$$

$$f_{D2} = ({}^\wedge d_1\ {}^\wedge d_2\ {}^\wedge x_1) \vee ({}^\wedge d_1\ d_2\ {}^\wedge x_1\ {}^\wedge x_2) \vee (d_1\ d_2\ z_4) \vee ({}^\wedge d_1 d_2 x_1 x_3)$$

Excitation equations

FSM structure



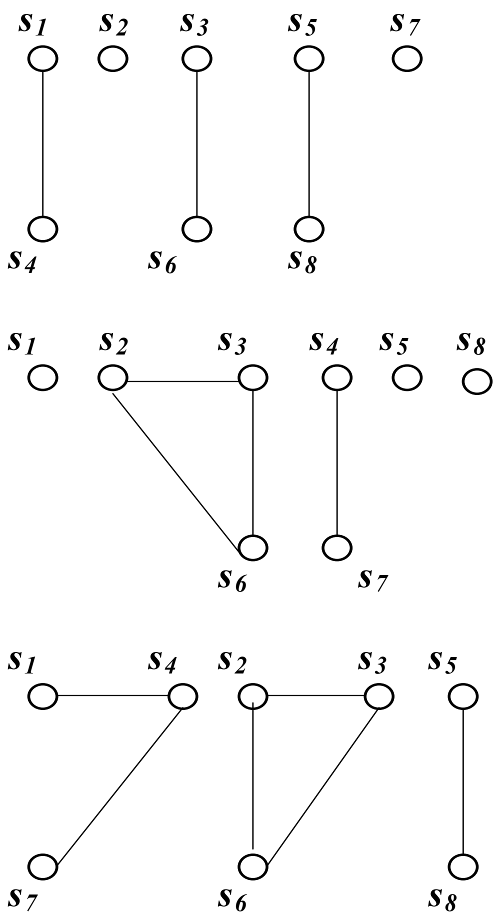
Component FSM B1 synthesis

s_p	$K(s_p)$ $d_1 d_2$	s_n	$K(s_n)$	$X_h(s_p, s_n)$	D_h	$Y_h(s_p, s_n)$
s_1	00	s_1	00	x_1	-	y_7
		a_1	11	$\hat{}x_1$	$D_1 D_2$	z_3
s_4	01	s_1	00	x_1 & x_3	-	y_3 y_4
		a_1	11	x_1 & $\hat{}x_3$	$D_1 D_2$	y_1 y_3 y_4 z_3
		s_7	10	$\hat{}x_1$ & x_2	D_1	y_3 y_4
		s_4	01	$\hat{}x_1$ & $\hat{}x_2$	D_2	y_1
s_7	10	a_1	00	1	-	y_1 z_5
a_1	11	s_4	01	z_4	D_2	-

- $y_1 = (\hat{}d_1\ d_2\ x_1\ \hat{}x_3) \vee (\hat{}d_1\ d_2\ \hat{}x_1\ \hat{}x_2) \vee (d_1\ \hat{}d_2)$
- $y_3 = (\hat{}d_1\ d_2\ x_1\ x_3) \vee (\hat{}d_1\ d_2\ x_1\ \hat{}x_3) \vee (\hat{}d_1\ d_2\ \hat{}x_1\ x_2)$
- $y_4 = (\hat{}d_1\ d_2\ x_1\ x_3) \vee (\hat{}d_1\ d_2\ x_1\ \hat{}x_3) \vee (\hat{}d_1\ d_2\ \hat{}x_1\ x_2)$
- $y_7 = (\hat{}d_1\ \hat{}d_2\ x_1)$
- $z_3 = (d_1\ d_2\ \hat{}x_1) \vee (\hat{}d_1\ d_2\ x_1\ \hat{}x_3)$
- $z_5 = d_1\ \hat{}d_2$

Output equations

Initial partition search



- Relation on the set of states

  si ω sj

iff there is at least one common variables in X(si) and X(sj)

((1,4), (2), (3,6), (5,8), (7))

- Relation on the set of states

  si φ sj

This relation we can use to divide output variables.

- Relation on the set of states

  si α sj